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Fertility and Family Leave Policies in Germany: Optimal Policy Design in a Dynamic Framework

Hanna Wang*

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Abstract

I develop and estimate a life-cycle discrete-choice model of fertility and female labor supply to study the optimal design of a range of child-related policies. First, I examine two German reforms that introduced wage-contingent parental leave payments and expanded access to low-cost public childcare. I find that both reforms raised completed fertility, with the parental leave reform having a particularly strong impact on highly educated women. Second, I solve for a budget-neutral optimal policy portfolio that maximizes either aggregate welfare or fertility, while ensuring that welfare and fertility do not decline for any education group. I consider four prominent child subsidies as well as the degree of tax jointness. My results show that optimal policy has the potential to increase welfare by 0.5% or fertility by 5.7%. While the solutions are qualitatively similar, they prioritize different policy instruments depending on the specific objective being targeted.

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1 Introduction

Low and declining fertility rates across developed countries have raised widespread concerns about the sustainability of social security and pension systems, as well as the risk of labor shortages and

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reduced economic growth. In response, policymakers are increasingly interested in developing and implementing programs that increase fertility while at the same time facilitate female employment. There are a number of commonly-used policy tools, often referred to as *family policies*. Childcare subsidies cover a part of childcare expenses for working parents, paid parental leave helps stay-at-home parents smooth consumption and fixed per-child subsidies, child tax deductions as well as joint taxation of couples provide additional income to families. Each has specific eligibility conditions regarding income, employment or marriage, resulting in varying effects for women depending on their characteristics. Although all developed countries make use of at least a subset of these policies, there is little knowledge of the trade-offs between them or how they interact.

Understanding how these policies work individually and in combination is crucial for policymakers, particularly when designing optimal policy. The aim of this paper is to investigate how different policies dynamically affect female labor supply and fertility, and how they can be optimally combined to achieve a specific policy objective. For this, I employ a rich life-cycle framework that accommodates all the above programs and heterogeneity across women's preferences, education level and marital status. The model captures key considerations in women's fertility and labor decisions. Children make it more costly for women to work, but reducing labor supply leads to losses in human capital and wage growth. This generates a dynamic trade-off for women between work and childbearing. Highly educated women face a particularly strong trade-off since they have the steepest wage profile.

The model is estimated using German household panel data. Germany is a particularly relevant case: it has low fertility, high spending on family policies, including generous joint taxation benefits, and underwent major reforms. These include a shift from flat to wage-contingent parental leave payments alongside a shorter payment duration, and an expansion of low-cost public childcare for children under three. I validate the estimated model by comparing its predicted short-run employment effects of the parental leave reform to estimates from administrative employment data. In the long run, both reforms raise fertility and welfare, but the parental leave reform generates larger fertility gains, while the childcare reform delivers greater welfare improvements. I also find that the short-run fertility effects of the parental leave reform are substantially larger than the long-run effects, underscoring the importance of evaluating long-term policy impacts using structural methods.

Next, I expand the analysis to a broader range of family policies and investigate optimal design of budget-neutral policy portfolios. In light of the growing policy focus on fertility in many countries, I consider specifications that account for birth rates alongside welfare of current women.[1] Since fertility can generate substantial positive externalities, such as increased future social security contributions, policy may be designed to prevent declines in birth rates or actively raise them. Accordingly, I first

[1] For instance, the fourth paragraph in the first section "Problem and Goal" of the parental leave bill points to Germany's low fertility. https://dserver.bundestag.de/btd/16/018/1601889.pdf.

analyze a specification that maximizes aggregate welfare and, as an alternative, one that maximizes the overall birth rate, both subject to the requirement that neither welfare nor fertility decreases for any education group. These education-group-specific constraints are imposed to ensure that any policy change constitutes a form of Pareto improvement relative to the baseline.

The set of instruments encompasses seven policy parameters: the parental leave replacement rate, minimum and maximum parental leave payments, childcare subsidies, fixed per-child subsidies, per-child tax deductions as well as tax jointness for married couples. Each instrument's effect on women's fertility, welfare and labor supply varies by heterogeneous characteristics such as marital status, education level and number of children. I show that the social planner faces a trade-off: policies that are effective at increasing fertility tend to have a smaller effect on welfare, and vice versa.

My findings show that optimal policy can achieve a remarkable welfare gain of 0.5% in consumption terms or an increase in birth rates by 5.7%. For either objective, it is optimal to eliminate joint taxation completely. This is because it is more efficient to let subsidies depend on children rather than marital status, which benefits single mothers, a low-income demographic group. Moreover, joint taxation tends to lower labor supply incentives of women as secondary earners. Both optimal solutions share other features: higher per-child subsidies, tax deductions, childcare subsidies and maximum parental leave pay, alongside a lower replacement rate. However, the specifications differ in their emphasis. The welfare-maximizing solution favors subsidies that boost female labor supply, such as childcare subsidies and tax deductions. In contrast, the fertility-maximizing solution prioritizes higher minimum parental leave payments and per-child subsidies to encourage higher-order births.

To evaluate the role of tax jointness, I also calculate solutions when restricting the taxation of couples to the current joint taxation system. Here, I find that while small improvements are still possible, removing joint taxation proves to be crucial. Moreover, I consider a welfare-maximizing specification that explicitly incorporates social security contributions of the next generation. In this setting, I remove fertility constraints and allow the model to determine the optimal fertility level based on both welfare and fiscal considerations. The results show that while welfare gains are larger than when maximizing welfare with fertility constraints, this comes at the cost of a decline in fertility.

This paper is one of the few to incorporate fertility decisions into optimal policy design and the first to consider a dynamic setting. Endogenous changes in the number of children not only have important implications for the current generation's actions and welfare but also for the government budget. Moreover, raising birth rates could be an important policy goal, if broader externalities or the welfare of future generations are considered. In previous studies, fertility is commonly assumed to be exogenous, even when designing family policies such as childcare subsidies (Haan and Wrohlich, 2010, Domeij and Klein, 2013, Ho and Pavoni, 2020, Bastani et al., 2020) and joint taxation (Kleven et al., 2009, Gayle and Shephard, 2019). Exceptions are Balestrino et al. (2002) and Kurnaz (2021) who employ static frameworks with endogenous fertility. In general, the existing optimal policy literature follows the Mirrleesian approach in designing non-linear taxation systems for welfare problems (e.g. Saez, 2002, Kleven et al., 2009, Blundell and Shephard, 2012).[2] Here, I instead focus on optimizing a restricted set of family policies to achieve a fertility objective using a life-cycle framework. This allows me to incorporate crucial dynamic considerations, such as the timing of fertility and its interaction with female employment and human capital accumulation.

My paper further adds to a broader literature that evaluates the effects of family policies on fertility and female labor supply. First, I expand the literature on tax jointness by studying the role of life-cycle fertility. Previous work examines effects on household labor supply (Kleven et al., 2009, Alesina et al., 2011, Guner et al., 2011 and Bronson et al., 2024) and marriage (Gayle and Shephard, 2019, Bronson et al., 2024). The only other study to jointly analyze the effect of tax jointness on fertility and female labor supply is Fehr and Ujhelyiova (2013). The authors calibrate an overlapping-generations model for Germany, but abstract from human capital accumulation and the possibility of divorce. While understanding the impact of marital tax-splitting on completed fertility is important in its own right, the implications for labor supply also matter. There is ample evidence that a high degree of tax jointness negatively affects the labor supply of married women,[3] which would tend to raise fertility, further reinforcing low female labor supply. Hence, it is crucial to allow for endogenous fertility decisions to capture the full scope of labor supply effects. Additionally, I study whether combining tax jointness with other child-related policies can enable policymakers to better achieve fertility and welfare objectives.[4]

Second, I contribute to the large literature on various child subsidies. Some examples for regression-based studies include Lefebvre and Merrigan (2008), Cohen et al. (2013), Bauernschuster and Schlotter (2015), Gathmann and Sass (2018) for childcare subsidies in Canada, Israel and Germany as well as Lalive and Zweimüller (2009) and Schönberg and Ludsteck (2014) for parental leave reforms in Germany and Austria. Studies using structural methods include Yamaguchi (2019) on parental leave in Japan, Mukhopadhyay (2012) on the Pregnancy Discrimination Act in the U.S and Kuehn and Garcia-Moran (2017) on childcare subsidies in Germany.[5] Several papers examine the two German policy changes studied here. Cygan-Rehm (2016) and Raute (2019) use difference-in-difference methods to measure fertility effects of the parental leave reform. Kluve and Tamm (2013), Bergemann and Riphahn

[2]For an extensive review see Piketty and Saez (2013).

[3]For example, Bick and Fuchs-Schündeln (2017) demonstrate this in a cross-country comparison, and Guner et al. (2011), Gayle and Shephard (2019) and Bronson et al. (2024) by simulating a switch from joint to individual taxation for the U.S.

[4]A similar idea was raised by Apps and Rees (2004), who show that countries with both individual taxation and high childcare benefits display both higher fertility and female labor supply.

[5]For a comprehensive cross-country comparison of family policies and summary of findings please see Olivetti and Petrongolo (2017). Brewer et al. (2009) further give an overview of studies on in-work benefit reforms for lower income families (such as the EITC in the U.S. and WFTC in the U.K.).

(2022) and Geyer et al. (2015) study employment responses to the same reform using various methods. For the childcare reform, Haan and Wrohlich (2011), Domeij and Klein (2013), Geyer et al. (2015) and Bick (2016) make use of structural frameworks to examine female labor supply and the latter two also fertility, while Bauernschuster et al. (2016) study fertility effects using a difference-in-difference design. I complement these studies by presenting a comprehensive evaluation of the long-run fertility and labor supply effects of the parental leave reform. I further expand the analysis of the childcare reform to single mothers, for whom I find particularly large fertility effects.

Regression-based and structural approaches each have distinct benefits and can complement one another; for instance, estimates from the former can serve to validate the latter such as in the current case. Life-cycle models offer crucial advantages when evaluating fertility effects. Due to the long time horizon, it is often difficult to distinguish changes in completed fertility from those in the timing of births with pre-post comparisons. Other factors in the economic environment affecting fertility could change over time, complicating measurement. In contrast, with a structural model, I can predict outcomes far into the future, compare responses to anticipated vs unanticipated policy changes, and disentangle effects of simultaneous reforms in parental leave and childcare. My model builds on a long line of life-cycle models of female labor supply (Mincer, 1962; Heckman and Macurdy, 1980; Eckstein and Wolpin, 1989). Since Francesconi (2002), a number of papers have included endogenous fertility in the model. Recently, Adda et al. (2017) and Gayle, Hincapié, and Miller (2025) evaluate counterfactual family policies and their effects on both fertility and female labor supply.[6] Given that my main goal is to study optimal policy design, my framework incorporates a variety of child-related policies, tax rules and other government budget items.

The following section describes the policy background, Section 3 discusses the data, Section 4 presents the structural model and Section 5 the estimated parameters and fit. Lastly, in Section 6, I show results for the evaluation of German policy reforms and optimal policy design.

2 Background

Low fertility and low female labor supply are persistent global challenges, particularly for the developed world. In Germany, women only have around 1.4 children in their lifetime compared to the OECD average of 1.5.[7] At the same time, women's careers are severely impacted by childbirth. For instance, in 2024, nearly all German fathers work full-time while, only half of German mothers with children

[6]For an exhaustive survey of the earlier dynamic female labor supply and fertility literature, see Keane et al. (2011).

[7]UN World Population Prospects for 2023.

below the age of six worked at all and of those more than two-thirds part-time. Children are particularly costly for highly educated women, reflected in the negative correlation between women's levels of education and fertility. In Germany, highly educated women have around 0.4 fewer children on average than women with low education.[8] The education fertility divide can have a number of negative consequences, such as unequal distribution of child-rearing resources across households.

To address these issues, Germany, like many countries, spends large amounts of public funds on fertility and family-related policies, amounting to roughly 2.5% of GDP.[9] These programs undergo constant reforms.[10] The publicly-funded paid parental leave system was substantially reformed in 2007, replacing a prior benefit of 300 EUR per month paid for up to two years. The new policy limits payment to one year and pays a two-thirds replacement of net pre-birth earnings. There are lower and upper limits on the payment of 300 EUR and 1800 EUR per month. For the majority of women, the reform substantially increased payments in the year of birth. For example, a woman with monthly net earnings of 2000 EUR is paid 1300 EUR under the new policy, 1000 EUR more than before.[11]

Moreover, public childcare provision for small children under the age of three was gradually expanded since 2005, relaxing prior shortages for this age group. The goal stipulated by the new law was to offer care to all children by 2013. Public childcare tends to be of high quality and is heavily subsidized while private care (nannies) are much more costly. The cost for full-time care to parents is roughly 300 EUR per month and fees are somewhat progressive in parents' income. Details can be found in Appendix A.2. As a result of the reform, public childcare enrollment for children under the age of three rose markedly, increasing from 16% in 2006 to 30% in 2013.

Other key family policies include fixed cash transfers and tax deductions per-child, as well joint taxation. In Germany, parents receive either fixed subsidies or tax deductions for each child, whichever benefit is higher. The former amounts to roughly 150 EUR per child per month and the latter to 4500 EUR per child per year. Fixed subsidies, as the name suggests, pay a fixed amount regardless of income or employment. However, benefits through tax deductions vary depending on household income. Par-

[8] German Statistical Office: www.destatis.de/DE/Themen/Gesellschaft-Umwelt/Bevoelkerung/Geburten/kinderlosigkeit-und-mutterschaft.html

[9] Percentage is for policies considered in this paper: childcare subsidies, parental leave payments, per-child subsidies, child tax deductions and joint taxation benefits. Numbers are calculated using official statistics from 2010: www.bpb.de/politik/innenpolitik/familienpolitik/193715/ familienpolitische-geldleistungen?p=all

[10] Other countries/states that also made changes to these policies include U.S. states New York and Washington which introduced paid parental leave (coming into effect in 2018 and 2020 respectively), and the Netherlands and the UK which increased the generosity of their publicly-provided childcare in 2005 and 2001 respectively.

[11] Part-time workers can receive two-thirds of the difference in pre and post-birth earnings. Parents are allowed to divide paid leave freely between themselves, but the vast majority of fathers took no leave prior to 2007. The reform created strong incentives for fathers to take two months of leave by earmarking it for them.Yet fewer than a third took exactly two months after the reform, often concurrently with the mother, and almost none took more than two months. Although I cannot rule out that policy changes could affect leave taken by fathers, given the low responsiveness, I abstract from it in my model.

ents are exempt from paying taxes on the deducted amount and also enjoy a lower tax rate due to progressive taxation. This implies that couples with incomes above 60000 EUR per year or single parents with incomes above 35000 EUR benefit more from tax deductions than fixed subsidies.[12]

Joint taxation is a prominent feature of the German tax system and can grant large tax benefits to married couples. Couples are taxed as if they each earned half of their joint income. Because of tax progressivity, this implies that by marrying, a couple can only pay less but not more taxes. Moreover, the second earner faces high marginal tax rates. For instance, a single woman who earns 2000 EUR per month pays a marginal tax rate of 28%. But, if she were married to a husband whose monthly income is 5000 EUR, her tax rate would increase to 37%. Reforming marital tax-splitting has been the subject of the political debate for decades, in particular due to its negative effects on female employment.[13]

3 Data

My data comes from the German Socio-Economic Panel (GSOEP), an ongoing annual household survey conducted by the German Institute for Economic Research (DIW). For each adult household member, current and retrospective information on education, marital status, employment, wages and children is collected. Summary statistics are shown in Table 1. My sample comprises West German women born between 1960 and 1984, aged 22-45. I restrict the sample period to 1990-2006, before the childcare and parental leave reforms took effect. This leaves 6386 women and on average 3124 observations per year of age.[14]

I divide the women into three education groups based on completion of school degrees and apprenticeships: women with at least a college degree, women with at least an intermediate school degree ("Realabschluss") and a completed apprenticeship, and the remainder. The first row in Table 1 shows that the average number of years of education increases with the education group as expected. There are also substantial differences in fertility and employment over the life-cycle across education groups.

[12] These values for subsidies and cutoffs are averages for my sample period 1990-2006.

[13] Examples of other countries with a similar joint taxation system are the U.S. and Poland. Countries that have switched from joint to individual taxation include the United Kingdom (1990), the Netherlands (1973) and Sweden (1971). Several reform attempts in Germany have failed in the past. In the 2021 federal election, the Social Democratic Party and the Green Party, now both forming part of the ruling coalition, have pledged to abolish joint taxation in favor of individual taxation coupled with additional child benefits ("Kindergrundsicherung"). See https://www.spd.de/fileadmin/Dokumente/Beschluesse/Programm/SPD-Zukunftsprogramm.pdf and https://www.gruene.de/themen/steuern.

[14] I focus on West-Germany because large cultural and earnings differences persist even after reunification. I exclude women who have children prior to the age of 22 and with high self-employment earnings (more than 400 EUR per month for two or more years). This is to better align samples, because self-employment is not observed in SIAB. While public servants are also absent from the SIAB, this information is unavailable for the retrospective employment and only 2.7% of women work as public servants for more than two years in the observed data.

Table 1: GSOEP Sample - Summary Statistics by Education

	Low	Med	High
Yrs of Education	10.5	12.3	16.3
	(1.0)	(1.3)	(2.0)
Age at First Birth	27.6	28.9	31.8
	(4.3)	(4.2)	(4.1)
No Child Aged 41+	0.26	0.18	0.33
	(0.44)	(0.38)	(0.47)
No. of Children:			
Age 24	0.22	0.13	0.02
	(0.48)	(0.37)	(0.13)
Age 34	1.30	1.23	0.70
	(1.02)	(0.99)	(0.91)
Age 44	1.45	1.55	1.24
	(1.09)	(1.00)	(1.16)
Hourly Wage (EUR):			
Age 24	9.54	11.09	12.50
	(3.77)	(4.66)	(8.96)
Age 34	14.85	14.01	17.43
	(10.56)	(5.91)	(6.74)
Age 44	12.80	14.29	18.20
	(4.94)	(6.09)	(8.14)
FT Share	0.39	0.45	0.65
	(0.49)	(0.49)	(0.48)
PT Share	0.23	0.26	0.18
	(0.42)	(0.44)	(0.39)
Employed One Child	0.54	0.61	0.67
	(0.50)	(0.49)	(0.47)
Married Share	0.54	0.56	0.42
	(0.50)	(0.50)	(0.49)
Share Mothers Single	0.28	0.25	0.27
	(0.45)	(0.43)	(0.44)
Husband's Monthly Earnings (EUR)	3461.7	4057.7	4796.1
	(1952.2)	(2059.9)	(2804.0)
No. of Obs.	16883	41009	17085

Standard errors in parentheses. Sample includes West German, non-self-employed women born 1960-1984, aged 22-45, years 1990-2006. All values weighted by population weights. Employed one child refers to mothers with exactly one child. Married share is the fraction of married women in the whole sample, share mothers single is the share of single mothers among mothers.

With increasing levels of education women tend to delay fertility. Roughly one third of the most highly educated women have no children when aged 41 or above and they have the lowest birth rate overall. Simultaneously, highly educated women are more likely to work full-time and less likely to work part-time. This is partially because they have fewer children, but, as can be seen from the fourth-to-last row, highly educated women are more likely to be employed conditional on the number of children, too.

Unsurprisingly, wages increase in education, evident from the third section of the table. Moreover, the education-wage gap widens with age. At the age of 24, women in the highest education group earn around three euros more per hour than their least educated counterpart, but by age 44 the difference increases to 5.4 EUR. The last row in section four further shows that, due to assortative marriage market matching, highly educated women also have husbands who earn substantially more, around 39%, compared to those in the lowest education group. Higher-educated women are less likely to be married, because they marry both later and less frequently.[15] However, the share of mothers who are single is relatively similar across all education groups.

I further use responses from three waves on the subjective importance of having children to capture heterogeneity in fertility preferences. To calculate expected childcare costs, I use information available in three waves. I further complement this information with data on childcare enrollment from the Federal Statistical Office. For the probability of conception, I make use of medical data on women's biological probability of successfully conceiving at a given age. This probability declines with age and falls drastically after the age of 35. More details are described in Appendices A.2 and A.4.

To validate my model, I use a large administrative dataset, the Sample of Integrated Labor Market Biographies (SIAB). This data, provided by the Institute for Employment Research (IAB) at the German Federal Employment Agency, is a 2% random sample of all individuals in Germany for whom mandatory social security contributions were made. It includes detailed information on employment, wages, education and age. However, information on births needs to be imputed using a method developed by Müller and Strauch (2017). I describe this in more detail in Appendix A.1.

4 Framework

4.1 Set-up

The model spans ages 22 to 60 and women can give birth to up to five children. Let t denote age and i the woman and her household. Variables referring to husbands carry a superscript h.[16]

[15] I assign cohabiting women to the single group but backdate the year of marriage by one year. This assumes women are likely in “marriage-like" stable relationships the year prior to getting married.

[16] To simplify the model, I eliminate fertility choices and marital transitions after age 45 and set terminal values at age 60 to zero. Only moments up to age 45 are used in the estimation, I present out-of-sample fit at age 50 in Appendix A.6.1.

4.1.1 Choices

Each period, a woman chooses whether to have a child, and whether to work full-time, part-time or not at all. The set of choice variables is $(f_t, \boldsymbol{l_t})$. The probability that a woman is fecund (biologically able to conceive) in period t is given by p_t^f.[17] This probability declines with age and is zero by age 45. If the woman is fecund and gives birth, f_t takes value one, otherwise it is equal to zero.

The employment choice is denoted by $\boldsymbol{l_t} = (l_t^F, l_t^P)$, where the first and second entries are indicators for full-time and part-time. Full-time work corresponds to 40 work hours per week and part-time to 20 hours, aligned with mean hours in the data. A woman can freely choose her employment level except when she was not employed in the previous period and is not covered by parental leave protection.[18] In that case, I assume she only receives a job offer with probability less than one. Denote $job_t = \{1,0\}$ as an indicator for whether the woman has a guaranteed job this period. If $job_t = 1$ then p_t^j takes value one otherwise it is determined as follows:

$$\ln\left(\frac{p_t^j}{1-p_t^j}\right) = \Sigma_{e=1}^3 \pi_{1e}\, ed_e + \pi_2 age_t + \pi_3 age_t^2 + \pi_4 exp_t + \pi_5 exp_t^2$$

where $e = 1,2,3$ denotes education levels and $\{ed_e\}_{e=1}^3$ are education indicators. exp_t is the level of human capital accumulated until period t. If no job offer is received, l_t^F and l_t^P both take values zero.

4.1.2 Preference

The period utility in period t, excluding preference shocks, for n_t children (including newborns) and labor supply decision $\boldsymbol{l_t}$ is given by:

$$u_t(\boldsymbol{l_t}, c_t; n_t, m_t, a_t, \boldsymbol{O}) = \underbrace{\alpha_1 \ln(c_t + \alpha_3)}_{\text{Utility from Consumption}}$$

$$+ \underbrace{(1 - O_1\alpha_6)\left(\Sigma_{k=1}^5 \alpha_{2k} kid_{kt} - (1 - m_t)\left(\alpha_4 \ln\left(\Sigma_{k=1}^5 kid_{kt} + 1\right) + \alpha_5 kid_{1t}\right)\right)}_{\text{Utility from Children}}$$

$$- \underbrace{\left(l_t^F + \gamma_4 l_t^P\right)\left(\gamma_1 + (1 - O_2\gamma_5)\left(\Sigma_{a=1}^5 \gamma_{3a} A_t^a\right)\left(\Sigma_{k=1}^5 \gamma_{2k} kid_{kt}\right)\right)}_{\text{Disutility from Work}}$$

[17]This is set using medical data detailed in Appendix A.4.
[18]In Germany, the right to part-time work is guaranteed by law.

where c_t denotes per-capita consumption, m_t is a marriage indicator, and a_t is the age of the youngest child. Utility depends on time-invariant heterogeneity in the preference for children and the disutility from working with children, (O_1, O_2). Indicators for the kth child are $\{kid_t^k\}_{k=1}^5$, and indicators for the youngest child's age group are $\{A_t^a\}_{a=1}^5$ corresponding to ages (0, 1-2, 3-6, 7-11, 12-18).

The utility function is concave in consumption, with the curvature determined by parameter α_3. The direct utility from children consists of the sum of valuations for each child and a psychic cost for single mothers depending on the number of children. $\alpha_5 kid_{1t}$ turns on for single women with at least one child. Women's binary fertility preference type O_1 scales the direct utility from children, with low-type women receiving only a fraction $\alpha_6 < 1$ of the utility enjoyed by high types.

The third line shows the disutility from working. γ_4 captures the differential cost of working part-time vs full-time. Individuals without children incur utility cost γ_1. Mothers incur additional costs which depend on the number of children that live in the household and the age of the youngest child through γ_{2k} and γ_{3a}.[19] The terms are scaled by γ_5 if the mother is of low disutility type $(O_2 = 1)$ while they are multiplied by one if she has high disutility $(O_2 = 0)$.

4.1.3 Human Capital, Wages and Husband's Income

The woman's wage shock ξ_t and her husband's income shock ξ_t^h are independently and normally distributed with mean zero and variances σ^2 and σ_h^2. Given a monthly wage w_t, the woman's gross labor income is $y_t = w_t\left(l_t^F + 0.5 l_t^P\right)$. Thus, part-time work earns half the salary of full-time work, conditional on education and human capital.[20] The wage function is education-specific and depends on work experience (or human capital):

$$\ln\left(w_{e,t}\right) = \phi_{0e} + \phi_{1e}\, exp_t + \phi_{2e}\, exp_t^2 + \xi_t$$

Work experience exp_t evolves according to:

$$exp_{t+1} = \left(exp_t + l_t^F + \Sigma_{e=1}^3 \lambda_{1e}\, l_t^P\right)\left(l_t^F + l_t^P + \delta\left(1 - l_t^F - l_t^P\right)\right)$$

[19]Similar additively separable utility functions have been employed in Eckstein et al. (2019) and Bick (2016). Moreover, the inclusion of the age of the youngest child to model work disutility of women is common, see e.g. Adda et al., 2017, Yamaguchi, 2019, and allows the model to fit lower employment rates of mothers with young children.

[20]This proportional-pay assumption, also used in Adda et al. (2017), is reasonable in the European, and in particular the German context, where empirical part-time wage penalties are relatively small.

It increases by one if the woman works full-time and decreases at rate δ, with $\delta < 1$, if the woman does not work. The effect of part-time work on experience accumulation varies by education level, reflecting that occupations typically held by lower-educated women may reward part-time work differently from those held by highly educated women.

A married woman's husband's income is modeled as a function of her age and education to account for assortative matching:

$$y_{te}^{h} = \psi_{0e} + \psi_{1e} age_t + \psi_{2e} age_t^2 + \xi_t^h$$

This avoids adding more variables to the state space as first proposed by Van der Klaauw (1996).

4.1.4 Budget

The budget constraint in period t is given by:

$$\max\Big(G\big(y_t, y_t^h, n_t\big) + \big(1-m_t\big)\, csupp_t\big(n_t\big)\,,\, S\big(m_t, n_t\big)\Big) + PL_t\big(\boldsymbol{l_t}, a_t, y_t^p\big)$$
$$= C_t + \zeta\big(n_t, a_t, y_t + y_t^h, \boldsymbol{l_t}, O_3\big) + A_t^1 NB$$

The equation states that total income on the left-hand side has to be equal to expenses on the right-hand side, ruling out savings.[21] G is a function mapping household labor income and number of children to after-tax income including per-child subsidies. $csupp_t$ denotes child support received by single mothers.[22] The maximum operator indicates that the government guarantees a minimum income to households. If the sum of after-tax income and child support payments falls below a threshold $S(m_t, n_t)$, which varies by marital status and the number of children, the household receives social assistance to cover the difference. Household consumption is denoted by C_t.[23]

[21]Modeling savings along with labor supply and fertility choices carries a high computational burden. Adda et al. (2017) and Blundell et al. (2016) provide insights into the role of savings in this class of models. The ability to save would allow individuals to partially self-insure, potentially mitigating the consumption-smoothing role of child subsidies.

[22]I set child support to be 200 EUR per child. This is roughly the amount that would be paid by the government if the father is not able to pay ("Unterhaltsvorschuss"). This serves as a lower bound. For government budget calculations in the optimal policy part, I assume that child support is paid by the father.

[23]Obtained by scaling per-capita consumption by the number of adults and children in the household such that $C_t = c_t(1 + 0.5m_t + 0.3n_t)$. This is in line with the modified OECD consumption scale. More information: http://www.oecd.org/eco/growth/OECD-Note-EquivalenceScales.pdf

Depending on eligibility, the woman might receive parental leave payments PL_t when not working full-time. Before the reform non-working mothers receive 300 EUR for the first two years after birth. After the reform, the amount depends on the mother's net pre-birth wage y_t^p and is paid only in the year of birth. Childcare costs are denoted by ζ and depend on number and age of children, parents' income, employment level of the mother and the availability of informal childcare O_3. Lastly, households with newborns incur a cost of NB set to 2400 EUR per year. Details on the German tax schedule, social assistance and childcare cost functions are provided in Appendix A.2 and A.3.

4.1.5 Marriage and Divorce

Marital transitions are taken as exogenous. The functions assume a logit form. Marriage probabilities depend on age, education and motherhood status. The probability that a single woman marries in a given period is:

$$\ln\left(\frac{p^{mar}_{e,t,n}}{1-p^{mar}_{e,t,n}}\right)=\theta_{01e}+\theta_{11e}\,age_t+\theta_{21}\,age_t^2+\theta_{31}\left(\Sigma_{k=1}^{5} kid_{kt}\right)$$

The probability of divorce additionally depends on the age of the youngest child and labor supply. Parameters θ_{30}-θ_{60} turn on if the youngest child is below six years old, above six years old, if the woman works full-time or part-time.

$$\ln\left(\frac{p^{div}_{e,t,n,a,\boldsymbol{l}}}{1-p^{div}_{e,t,n,a,\boldsymbol{l}}}\right)=\theta_{00e}+\theta_{10}age_t+\theta_{20}age_t^2+\theta_{30}\left(\Sigma_{a=1}^{3}A_t^a\right)+\theta_{40}\left(\Sigma_{a=4}^{5}A_t^a\right)+\theta_{50}\,l_t^F+\theta_{60}\,l_t^P$$

4.1.6 Household Problem

At the beginning of each period, the woman observes her fertility-preference shocks and her fecundity status. If fecund, she decides whether to conceive, with the child arriving immediately. She subsequently learns her labor-preference shock, her wage shock, her husband's income shock, and the availability of a job offer. In case of an offer, she chooses among full-time employment, part-time employment, and non-employment, otherwise she is non-employed by default. Following the realization of earnings, taxes, transfers, and childcare costs, the household allocates the remaining resources to consumption. At the end of the period, a marital-transition shock is realized, determining whether she marries, remains married, or divorces.

There are two fertility preference shocks $\boldsymbol{\epsilon_t^f} = \left(\epsilon_t^{f1}, \epsilon_t^{f0}\right)$ and three labor preference shocks $\boldsymbol{\epsilon_t^l} = \left(\epsilon^{lF}, \epsilon^{lP}, \epsilon^{lN}\right)$. I assume preference shocks are independent and have a type I extreme value distribution, allowing for expected utility to be partially expressed analytically. I denote the fecundity shock as ν_t, the job offer shock as η_t and the marital shock as μ_t.

Define $\boldsymbol{\epsilon_t} = \left(\boldsymbol{\epsilon_t^f}, \boldsymbol{\epsilon_t^l}\right)$ and $\boldsymbol{d_t} = \left(f_t, 1-f_t, \boldsymbol{l_t}, 1-l_t^F - l_t^P\right)$. Then the period utility, including preference shocks, for given fertility and employment choices, can be expressed as:

$$U_t\left(\boldsymbol{l_t}, c_t; n_t, m_t, a_t, \boldsymbol{O}, \boldsymbol{\epsilon_t}\right) = u_t\left(\boldsymbol{l_t}, c_t; n_t, m_t, a_t, \boldsymbol{O}\right) + \boldsymbol{d_t}\boldsymbol{\epsilon_t^{\intercal}}$$

Denote state variables at the beginning of period t after fertility preference shocks have been drawn as $\Omega_t^* = \left(e, \boldsymbol{O}, n_t^*, a_t^*, exp_t, m_t, y_t^p, job_t, \boldsymbol{\epsilon_t^f}\right)$, where e is education and $\boldsymbol{O} = (O_1, O_2, O_3)$ are individual heterogeneity in preferences and informal childcare availability. Define interim state variables at the employment choice stage after the realization of labor preference shocks, wage and income shocks, but before knowing η_t as $\Omega_t = \left(e, \boldsymbol{O}, n_t, a_t, exp_t, m_t, y_t^p, job_t, \boldsymbol{\epsilon_t}, \nu_t, \xi_t, \xi_t^h\right)$ where $n_t = n_t^* + f_t$ and $a_t = \left(1-f_t\right) a_t^*$ are updated depending on the fertility choice. Let $V_t^*\left(\Omega_t^*\right)$ and $V_t\left(\Omega_t\right)$ denote corresponding expected utilities for the initial and interim stage.

After making a fertility decision, the woman either receives a job offer and chooses her level of employment or is non-employed. Her expected value is:

$$V_t\left(\Omega_t\right) = p_t^j \left(\max_{\boldsymbol{l_t}} U_t\left(\boldsymbol{l_t}, c_t; n_t, m_t, a_t, \boldsymbol{O}, \boldsymbol{\epsilon_t}\right) + \beta E V_{t+1}^*\left(\Omega_{t+1}^* | \boldsymbol{l_t}, \Omega_t\right)\right)$$

$$+\left(1-p_t^j\right)\left(U_t\left(\boldsymbol{0}, c_t; n_t, m_t, a_t, \boldsymbol{O}, \boldsymbol{\epsilon_t}\right) + \beta E V_{t+1}^*\left(\Omega_{t+1}^* | \boldsymbol{0}, \Omega_t\right)\right)$$

where expectation $E V_{t+1}^*\left(\Omega_{t+1}^* | \boldsymbol{l_t}, \Omega_t\right)$ is taken over marital and fertility preference shocks in the next period. The woman takes into account the effect of her employment choice on next period's state variables Ω_{t+1}^* through human capital accumulation and job offers. She discounts the continuation value at rate $\beta < 1$.

At the start of a period, the woman considers that her fertility choice will affect this period's number of children and forms expectations over employment preference, wage, and spousal-income shocks. The expected value is:

$$V_t^*\left(\Omega_t^*\right) = p_t^f \left(\max_{f_t} E V_t\left(\Omega_t | f_t, \Omega_t^*\right)\right) + \left(1-p_t^f\right) E V_t\left(\Omega_t | 0, \Omega_t^*\right)$$

4.2 Identification and Estimation

I estimate parameters for husbands' incomes and childcare costs directly from the data. Income tax and social security contributions are approximated using piecewise linear functions. The details can be found in Appendices A.2, A.3 and A.4. I set the utility discount factor β to 0.97, similar to other studies in the literature.[24]

The remaining parameters of the model are estimated using the method of simulated moments (see McFadden, 1989, Gourieroux et al., 1993). The estimation targets a range of static and dynamic moments pertaining to fertility, employment and wages by age, education group and marital status, amounting to 240 moments in total. A full list is shown in Table 2 and data and model values can be found in Appendix A.6. While all moments contribute to identifying all model parameters, I briefly explain the intuition for how certain moments are more relevant for specific parameters.

Table 2: List of Moments

Moments	Number
Fertility	
Number of Children By Education and Age	9
Probability of Birth By Birth Order, Education, Age and Marital Status	45
Difference in Number of Children between Fertility Preference Types By Age	3
Population Share By Marital Status, Motherhood Status, Education and Age	27
Employment	
Full and Part-Time Rate by Education and Age	18
Work Transition Rates by Motherhood Status, Work Status, Education and Age	12
Full and Part-Time Rate by Age of Youngest Child, Education and Age	48
Employment Rate by Motherhood Status and Education	6
Full and Part-Time Rate of Mothers with Children Aged 3	2
Full and Part-Time Rate by Number of Children	8
Employment Rate by Marital Status, Age of Youngest Child, Education and Age	21
Wages	
Monthly Wage by Education and Age	9
Log Wage Regression Coefficients	9
Log Wage Increase by Education and Employment Status	6
Marital Transitions	
Linear Marriage Probability Regression Coefficients	8
Linear Divorce Probability Regression Coefficients	9
	240

[24] Blundell et al. (2016) assume a discount factor of 0.98 for women in the UK; Adda et al. (2017) estimate a discount factor of 0.96 using the GSOEP.

Identification of utility parameters depends crucially on moments that link the number and age of children to employment, age and marital status of the woman. For instance, the probabilities of giving birth to each child in different circumstances provides information on how much women value children by birth order. Since very few women have four or five children, I set the corresponding preference and disutility parameters equal to a fixed proportion of those for the third child, such that $\alpha_{24} = \alpha_{25} = \chi\alpha_{23}$ and $\gamma_{24} = \gamma_{25} = \chi\gamma_{23}$, where $0 \leq \chi \leq 1$ is a parameter to be estimated. Birth probabilities by marital status help pin down the disutility of single mothers. The difference in the number of children across fertility preference types help determine α_6. Work disutility parameters are identified by full and part-time employment rates by the number of children and age of the youngest child.

Log wage regression coefficients, average wage levels by age and education and changes in log wages for different employment choices allow identifying wage and human capital process parameters. For instance, the wage increase due to one additional year of full-time work by education group and the wage regression coefficients for years of full-time work help identify parameters ϕ_{1e} and ϕ_{2e}, which govern the wage return to experience. Targeting transitions from non-employment to employment pins down job finding rates, as well as employment rates for mothers with three-year-old children, the year in which job-protection expires. Lastly, coefficients from linear probability regressions for marriage and divorce identify the parameters in the logit probability functions.

Define Θ as the vector of the model parameters and $\hat{\Theta}$ as its estimate. For a given set of parameters, the model is solved backwards from the last period to obtain expected values at each state space point. For continuous variable exp_t I use interpolation based on eight grid points. Then the expected values are used to simulate life-cycle choices for a representative sample of 60000 women. The simulation begins at the age of 22 except for women in the highest education group, who complete higher education at an older age. I further account for initial conditions in marital and employment rates at the start of the simulation. The details can be found in Appendix A.4.

To estimate the model, I search for parameters that minimize the distance between moments calculated using simulated data from the model and those calculated from real data. The loss function uses a diagonal weighting matrix consisting of variances of the data moments.[25] Let M_k^d and M_k^m stand for the kth data and model moments respectively. Formally the estimation solves:

$$\min_{\hat{\Theta}} \sum_{k=1}^{K} \left(M_k^d - M_k^m(\hat{\Theta})\right)^2 / H\left(Var\left(M_k^d\right)\right)$$

[25]This type of weighting matrix is common, see e.g. Adda et al. (2017) and Blundell et al. (2016). To solve the optimization problem, I use the MIDACO solver, a high performance solver for numerical optimization problems. More information can be found in Schlueter et al. (2013).

5 Estimation Results

In this section, I present the parameter estimates and standard errors. All estimates are statistically significant at the 5% level except four. The first section of Table 3 shows that women's direct utility from children decreases in birth order. In a given period, when taking into account only the trade-off between a decrease in per-capita consumption and gain in contemporary consumption value of children, a non-working married woman with monthly per-capita consumption of 1500 EUR and high fertility preference derives a value equal to 716 EUR from a first child. For the second child, the value drops to 107 EUR while for the third child it becomes negative at −42 EUR. Single mothers incur sizable disutility costs, their value of a first child is only 98 EUR. These valuations are lower for working women and for women with low fertility preference, whose utility from children is reduced by 23%.

Table 3: Preference Parameter Estimates

Parameter	Estimate	Std. Error
Consumption and Children		
α_1 Cons. Slope	3.168	0.047
α_3 Cons. Shifter	0.262	0.034
α_{21} First Child	3.175	0.035
α_{22} Second Child	0.750	0.014
α_{23} Third Child	0.319	0.012
α_4 Single Mother Cost	2.755	0.043
α_5 Single Mother Cost Shifter	0.175	0.012
α_6 Low Fertility Preference	0.770	0.010
χ Fourth and Fifth Child Factor	0.344	0.054
Work Disutility		
γ_1 Constant	0.008	0.004
γ_{21} First Child	5.768	0.119
γ_{22} Second Child	2.027	0.082
γ_{23} Third Child	0.881	0.097
γ_{31} Age of Youngest Child: 0	1.727	0.078
γ_{32} Age of Youngest Child: 1–2	1.309	0.066
γ_{33} Age of Youngest Child: 3–6	1.368	0.058
γ_{34} Age of Youngest Child: 7–11	1.127	0.031
γ_4 Part-time	0.342	0.009
γ_5 Low Disutility for Working with Children	0.217	0.007
Low Disutility Type Distribution – By Fertility Pref.		
μ_1 Edu. Group 1, Low FP	0.146	0.033
μ_2 Edu. Group 2, Low FP	0.225	0.022
μ_3 Edu. Group 3, Low FP	0.369	0.037
μ_4 Edu. Group 1, High FP	0.568	0.019
μ_5 Edu. Group 2, High FP	0.732	0.034
μ_6 Edu. Group 3, High FP	0.439	0.134

The second section shows the work disutility estimates. The cost of working with children decreases in the birth order and also in the age of the youngest child for the most part. Part-time work or having low disutility for working with children reduces work disutility by 66% and 78% respectively. Section three shows the prevalence of work disutility types by education group and fertility preference type. Overall, since the majority of women in all education groups have low fertility preference (see Appendix A.4), around 35% of women are of low work disutility type. Among women with low fertility preference, few have low work disutility, but among those with high fertility preference, more than half do.

Table 4: Job Finding and Human Capital Parameter Estimates

Parameter	Estimate	Std. Error
Employment Probability		
π_{11} Intercept Low Ed.	−1.322	0.047
π_{12}-π_{11} Medium Ed.	−0.216	0.014
π_{13}-π_{11} High Ed.	−1.277	0.149
π_2 Age	−0.066	0.004
π_3 Age Sq.	−0.002	0.0002
π_4 Experience	0.556	0.007
π_5 Experience Sq.	−0.014	0.001
Human Capital		
λ_{11} Part-Time Low Ed.	−0.053	0.057
λ_{12} Part-Time Medium Ed.	−0.677	0.042
λ_{13} Part-Time High Ed.	0.148	0.070
δ Depreciation	0.985	0.002

Table 5: Wage Parameter Estimates

Parameter	Estimate	Std. Error
ϕ_{01} Intercept	2.703	0.009
ϕ_{02}-ϕ_{01} Medium Ed.	0.171	0.006
ϕ_{03}-ϕ_{01} High Ed	−0.165	0.013
ϕ_{11} Experience	0.037	0.001
ϕ_{12} -ϕ_{11} Exp. Medium Ed.	0.011	0.001
ϕ_{13} -ϕ_{11} Exp. High Ed.	0.064	0.002
ϕ_{21} Experience Sq.	−0.0005	0.00001
ϕ_{22} -ϕ_{21} Exp. Sq. Medium Ed.	−0.001	0.00005
ϕ_{23} -ϕ_{21} Exp. Sq. High Ed.	−0.002	0.00007
σ Variance	0.100	0.004

Table 6: Marriage and Divorce Rates

Parameter	Estimate	Std. Error
Marriage Probability		
θ_{011} Intercept.	−2.387	0.023
θ_{012}-θ_{011} Medium Ed.	0.135	0.014
θ_{013}-θ_{011} High Ed.	0.084	0.056
θ_{111} Age	0.066	0.003
θ_{112}-θ_{111} Age Medium Ed.	0.005	0.001
θ_{113}-θ_{111} Age High Ed.	−0.0005	0.004
θ_{21} Age Sq.	−0.005	0.0002
θ_{31} Mother	−0.181	0.010
Divorce Probability		
θ_{001} Intercept.	−3.947	0.030
θ_{002}- θ_{001} Medium Ed.	−0.164	0.021
θ_{003}- θ_{001} High Ed.	−0.649	0.034
θ_{10} Age	−0.010	0.001
θ_{20} Age Sq.	0.002	0.0001
θ_{30} Young Child	0.512	0.017
θ_{40} Older Child	−0.005	0.015
θ_{50} Full-time	0.548	0.024
θ_{60} Part-time	−0.231	0.012

Job finding and human capital accumulation estimates are presented in Table 4. In the simulated data, the job finding probability peaks at age 31 at around 54.8% on average.[26] The direct effect of age is negative, while for experience it is hump-shaped. Human capital depreciates by 1.5% if a woman does not work for one year and changes by -0.053, -0.677 and +0.148 if she works part-time for low to high levels of education. For women with medium levels of education, working part-time tends to be more detrimental for wages than not working, depending on the stock of human capital. This result is driven by the relatively flat wage profile observed for this education group (see Figure 3) and the negative wage change in the data associated with part-time work (Appendix Table A.40). Despite this penalty, women might choose part-time over not working to earn income and avoid search costs in the future. Table 5 shows how human capital affects wages for each education level. The slope is steepest for the highest educated. At the mean level of experience, highly educated women gain 181 EUR (6.1%) wages and women in the lowest education group 58 EUR (2.8%) for one additional year of experience.[27]

The last set of estimates in Table 6 are for the marriage and divorce probabilities. These apply to ages 45 and below, as I fix marital status thereafter. Here, three estimates are not significantly different

[26]These numbers roughly match with data from the employment agency which shows that, in 2000, about 50% of unemployed 25-49 year-olds searched for more than a year.

[27]Francesconi (2002) finds similar magnitudes. He estimates increases of 7.5% and 3.8% for women with zero and 20 years of experience respectively.

from zero: the difference between the intercept of highly educated vs less educated women $\theta_{013} - \theta_{011}$, the difference between the age coefficient of highly educated vs less educated women $\theta_{113} - \theta_{111}$ and the impacts of having an older child θ_{40}. Having a child has a moderate negative impact on the probability of marrying. For example, for a woman in the lowest education group at age 30, the probability is lowered from 9.4% to 7.9%. The probability of divorce is higher for women with a young child or working full-time. For a 30-year-old woman in the lowest education group who does not work and has no children, the probability increases from 2.0% to 3.3% with a young child and to 3.4% if working full-time.

I present Marshallian and Frisch labor supply elasticities as well as fertility elasticities in Table 7 for a 1% increase in wages. Labor supply is measured as hours of work including zeros. Marshallian and fertility elasticities are calculated for a permanent wage increase, while the Frisch elasticities for an anticipated increase at a given age are measured in that particular year. I estimate an overall Marshallian elasticity of around 0.8, which is within the range in the literature for Germany.[28] Frisch elasticities tend to be lower but move in the same direction as Marshallian elasticities. Generally speaking, women who supply less labor respond more to wage incentives. In particular, elasticities are highest for women with lower levels of education, married women and women aged 32 and 40 who are more likely to be mothers.[29] When decomposing elasticities into intensive and extensive margins, I find that the latter to be more important (see Appendix A.5).

Table 7: Labor Supply and Fertility Elasticities

	Frisch	Marshallian	Fertility
Overall.	0.24	0.77	−1.25
Low Ed.	0.25	0.92	−1.34
Medium Ed.	0.26	0.79	−1.19
High Ed.	0.17	0.51	−1.27
Age 25	0.13	0.37	−2.10
Age 32	0.33	1.11	−1.00
Age 40	0.29	0.9	−0.99
Married	0.35	1.46	−1.06
Single	0.17	0.36	−2.71

Marshallian and fertility elasticities calculated for permanent 1% wage increase. Frisch elasticities for anticipated one-period 1% wage increase, at ages 25, 32 or 40 only for the period of wage increase, by education and family status averaged over the three ages.

[28] For instance, Haan and Wrohlich (2011) estimate an elasticity of 0.5, Kaiser et al. (1993) of 1.0.

[29] Blundell et al. (2016) find the same education gradient for the U.K. and Eckstein et al. (2019) also find larger responses of married women for the U.S.

Fertility elasticities are shown in Table 7 for percentage changes in the number of children born in response to a wage increase. The results confirm the common finding that fertility is decreasing in women's wages: higher wages increase myopic work incentives, but also the return to human capital. On average, I find an elasticity of −1.4, similar to Francesconi (2002) who finds elasticities from −1.0 to −1.5 for the U.S.[30] Young and single women react most strongly. The difference across education groups is minor. In addition, I report fertility elasticities for various child policies included in optimal policy design in Table 12 in Section 6.2.2. The comparability with other studies is limited given that settings vary considerably. Studies that examine reforms normally calculate immediate responses in birth rates and not completed fertility. For fixed child subsidies, the most commonly-studied subsidy, I find that completed fertility increases by about 0.03 or 2.3% for an additional 10 EUR in monthly subsidies under joint taxation. Estimates in the literature vary vastly. Converting the results to be proportional for an increase in subsidy of the same size, Laroque and Salanié (2014) find an increase of about 1.4% for France, Cohen et al. (2013) find 3.2% for Israel and Adda et al. (2017) find close to no long-term effects for Germany.[31]

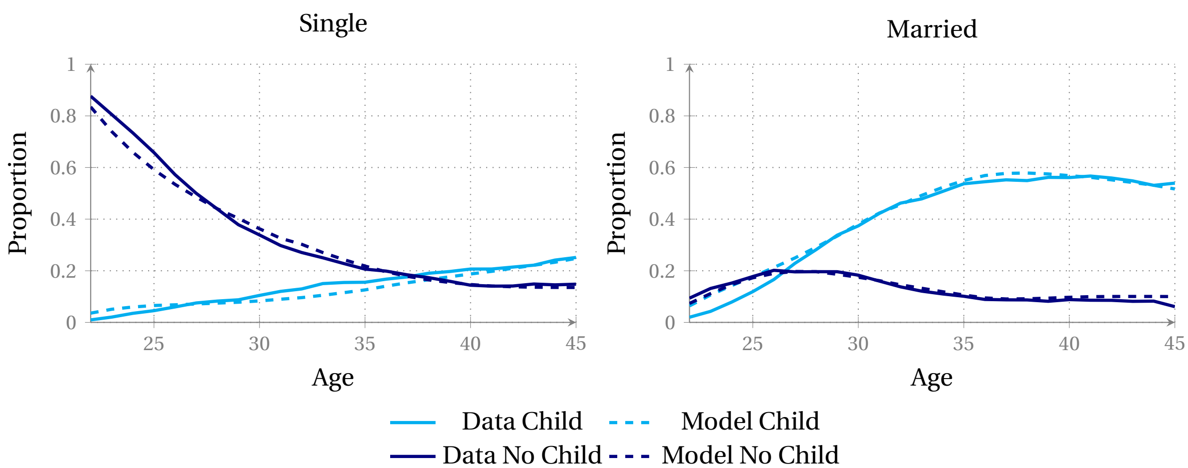


Figure 1: Proportion of Women by Marital and Motherhood Status Over the Life-Cycle

I present the detailed fit for all targeted moments as well as a number of non-targeted moments in Appendix A.6. Overall, the model fits the data well. The following figures illustrate how family status,

[30] Geyer et al. (2015) estimate elasticities about half the size for Germany for a one period wage change, while Butz and Ward (1979) report a larger estimate of about −1.73 for the U.S.

[31] Laroque and Salanié (2014) find a 21% increase in birth rates for a simulated 150 EUR increase in monthly subsidies, Cohen et al. (2013) finds that an increase in subsidies for the third child by NIS 150 (about 30 EUR) increases the yearly birth probability of mothers with two children by about 10%, Adda et al. (2017) estimate a structural model with savings and simulate effects for a 6000 EUR payment at birth. This roughly corresponds to an increase of 35 EUR in monthly subsidies until the child turns 18. The increase in fertility is less than 0.2%. Two other studies, Geyer et al. (2015) and Milligan (2005), estimate very large elasticities for Germany and Canada. Geyer et al. (2015) find a payment increase of 30 EUR per month for children under the age of three increases fertility by 4.6%, while Milligan (2005) finds a payment of 1000 CAD (669 EUR) at birth does so by 16.9%.

full and part-time employment and wages evolve over the life-cycle in the estimated model and data. The data is represented with solid lines and the model with dashed lines. Figure 1 shows the distribution of women who are single or married and have or do not have children. The vast majority of women are unmarried at age 22, while at later ages most women are married and have children.

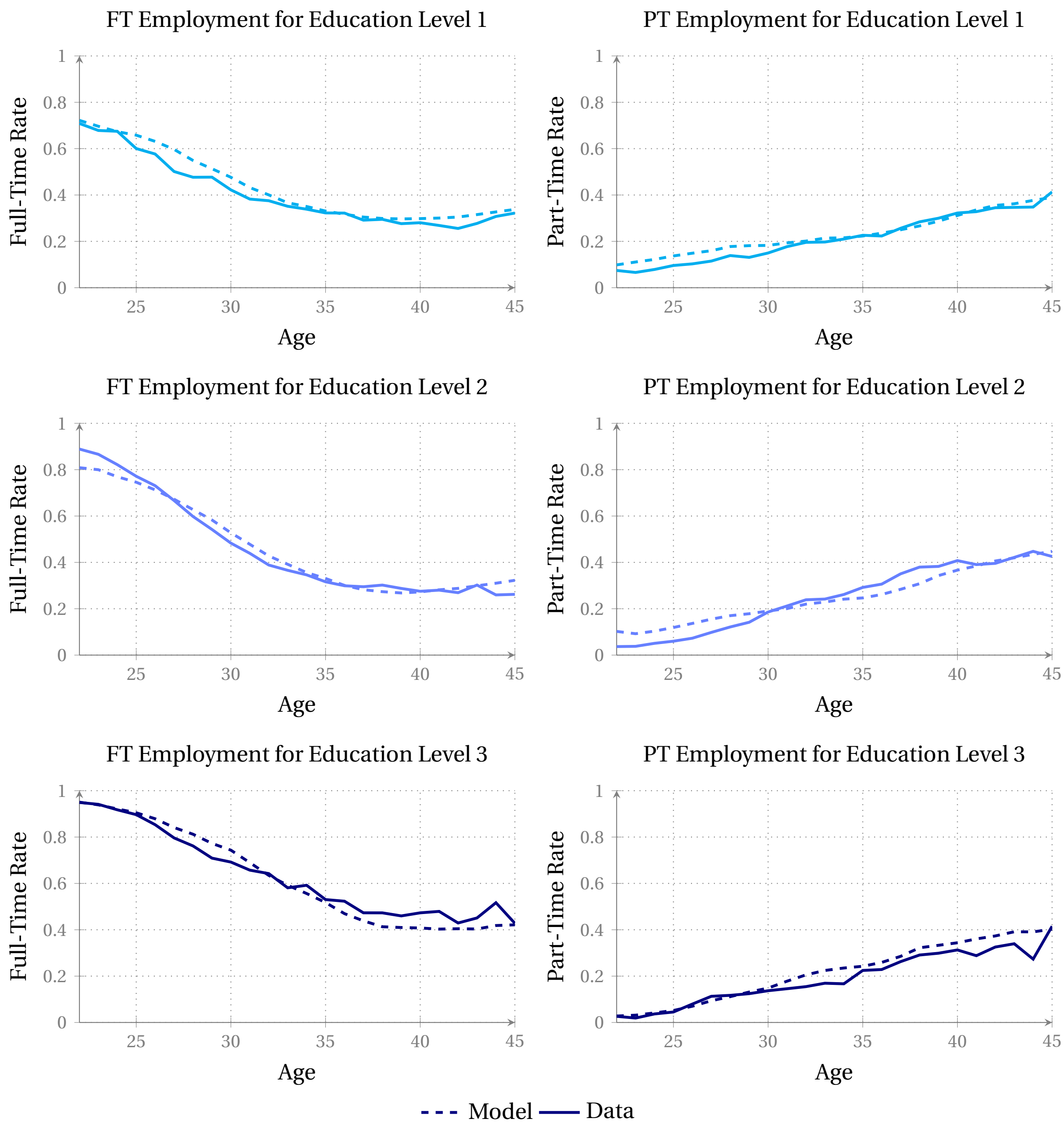


Figure 2: Full-Time and Part-Time Rates Over the Life-Cycle By Education

Figure 2 shows the model matches full and part-time work rates for women by each education level. Full-time employment decreases steadily until around age 37, while part-time employment in-

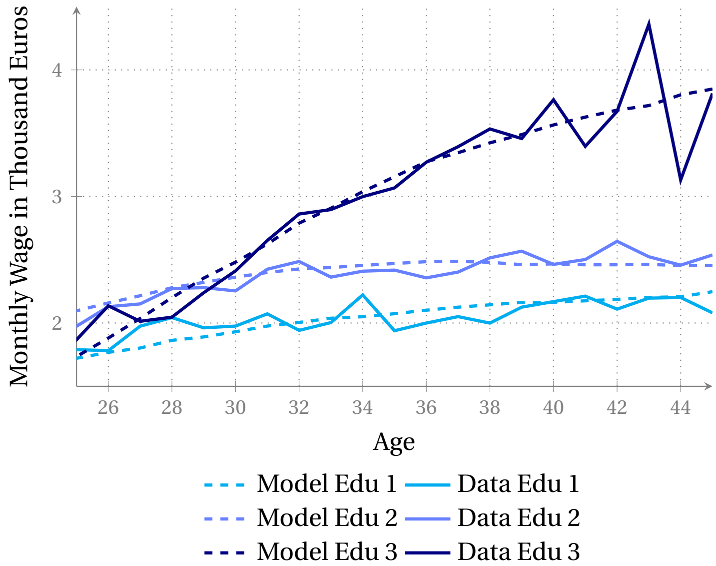


Figure 3: Average Monthly Wage By Education Over the Life-Cycle

creases as women give birth. Furthermore, more educated women have uniformly higher full-time rates. Lastly, Figure 3 shows the fit for life-cycle wage profiles for women with high, medium and low education. The GSOEP wage data is somewhat noisy, but the model captures the levels and overall shapes of the profiles. In particular, wage growth is steepest for highly educated women and relatively flat for the lower two education groups.

6 Policy Simulation

Equipped with the estimated model, I can analyze counterfactual policies and solve for optimal policy.[32] First, I make use of the sharp change in the parental leave policy to validate the model and give more credibility to my results. For this, I simulate short-term employment and fertility effects and compare them to regression results using the SIAB dataset and estimates from the literature. Using natural experiments to validate structural models has gained traction since first proposed by Todd and Wolpin (2006). For the parental leave reform setting, Geyer et al. (2015) have performed a similar exercise comparing employment effects from their model with findings from the Mikrozensus, a large cross-sectional dataset.

[32]To do so, I simulate the choices of 60000 representative women. I further calculate discounted values of lifetime earnings and government spending using an interest rate of 4%, the average rate in Germany over the sample period (approximated using data from the Federal Reserve Bank of St. Louis available at https://fred.stlouisfed.org/series/intgstdem193n).

Next, I evaluate the long-term effects of the parental and childcare reforms on fertility, welfare and female employment. A structural model is essential for this task, also because of the overlap in timing of the two reforms. The results illustrate distinct mechanisms of each policy and reveal how effects vary across women with different characteristics. These insights provide the foundation for the next step of the analysis: solving for the optimal design of an extensive set of family and child policies.

6.1 Parental Leave and Childcare Reforms

6.1.1 Model Validation and Short-Run Effects of Parental Leave Reform

To simulate the parental leave reform, I reduce the payment period to one year, set a replacement rate of 66.7% and fix lower and upper bounds at 300 EUR and 1800 EUR. For the validation exercise, I assume the reform was unanticipated.[33] From the SIAB data, I select women whose children were born in the first four months of both 2006 and 2007. For children born early in 2007, it is unlikely that mothers anticipated the reform at the time of conception, hence this avoids problems of selection into becoming a first-time mother. I regress the outcomes of interest on a reform dummy, effectively comparing means. To compute comparable effects with the model, I simulate two scenarios: (i) no reform implemented; (ii) the reform is implemented unexpectedly at a random age between 22 and 45.[34] Then I compare outcomes for the same women in (i) and (ii) who became mothers in the year in which the reform was implemented under (ii).

Table 8 shows the results from both datasets and methods for the changes in the full-time rate, part-time rate and daily earnings, by the mother's education level and for the first and second year the child is born. The regression effects on part-time work and earnings for all education groups in the first year are significant at the 5% level.[35] The model produces similar estimates to the regressions for the most part. In line with policy incentives, the estimated effect on all employment outcomes in the year of birth is negative for both. The magnitude increases with education, consistent with payments growing more for women with higher income. Furthermore, both methods find this to be primarily driven by part-time employment. Most structural estimates lie within the 95% confidence interval of the regression

[33] Kluve and Tamm (2013) reach this conclusion after examining Google searches around the timing of public announcements and finding the earliest date mothers could have been aware of the impending reform to be May 2006.

[34] For (ii), I obtain two sets of expected values for the pre and post-reform regimes. In the years prior to the reform, women make decisions based on expected values under the old policy. After the reform they then switch to expected values under the new policy, assuming it remains in effect permanently. In the year of the reform, I assume women first decide on fertility, then learn about the reform, and subsequently choose employment. Due to constraints given by the SIAB data detailed in Appendix A.1 I focus on employment effects following the birth of the first child.

[35] The first-year effect on full-time work of women in the second education group is also significant.

estimates.

In terms of magnitude, the first-year estimates from the model are smaller than those from the regressions. For instance, the model predicts a decrease of 10.0 percentage points in the part-time rate of highly educated women, while the regression finds a decrease of 19.7 percentage points. This discrepancy can partially be attributed to cohort effects. The estimation sample is composed of women from earlier birth cohorts who tend to exhibit lower maternal labor supply, thus providing a smaller margin for reducing work hours in response to the parental leave policy. In the second year after birth, full-time employment and earnings increase for both model and regressions, and more so for the highly educated. The effect on part-time work is mostly negative. However, none of the second-year regression results are significant.

This set of results is very similar to those presented in Kluve and Tamm (2013), who examine data from a commissioned survey, and in Geyer et al. (2015), who use census data. Kluve and Tamm (2013) report an overall decrease in employment of 15% for first-time mothers. Moreover, the magnitude of the effect increases with education. Geyer et al. (2015) also find significant negative results for part-time but not full-time work. They estimate a 5.0% decrease in overall employment, lower than the 8.9% from my model simulation. However, this is likely because their sample includes women who have more than one child and had lower income before birth. Lastly, Bergemann and Riphahn (2022) perform duration analysis and do not find a significant decrease in employment in the first year, but find that women return to work more often in the second year.

In addition to employment and earnings, I compute short-run and long-run effects on fertility. For that, I compare the yearly number of births in three cases: (i) no reform, (ii) the year in question is the first year of the unexpected reform (iii) the new policy has been in place since age 22 for all women. Table 9 shows that the reform causes the number of births to increase both in the short and long-run. This is true for all education groups, but the effects increase with education. A striking result is that the short-run effects are 40.5% larger than those in the long-run. This difference is driven by older women responding in the short run, evident from the increase in average age at birth in the last row. These women do not have many remaining fertile years to take advantage of the higher parental leave benefits. In the long-term, women anticipate the policy from a young age and respond earlier. This stark discrepancy in fertility responses depending on the time horizon illustrates the merit of structural models to make long-term predictions. It also cautions against approximating long-term effect estimates with those for the short-term. In this case, fertility effects would have been severely overestimated.

The closest regression-based estimates in the literature are provided in Raute (2019) who employs a difference-in-difference approach around a five-year window around the reform with less educated women as the control group. She finds larger fertility effects of 23% for highly educated women, more

Table 8: Parental Leave Reform Effect on Maternal Employment and Earnings by Child's Age

	Child's Age	**Simulation**/Regression		
		low	med	high
Full–Time	<1	**−0.4**	**−1.4**	**−2.5**
		−4.7	−5.5	−5.3
		(−9.5,.2)	(−10.6,−0.4)	(−13.6,3.0)
	1–2	**2.3**	**2.8**	**3.4**
		0.7	2.6	3.3
		(−4.9,6.3)	(−3.1,8.3)	(−6.5,13.0)
Part–Time	<1	**−6.7**	**−7.2**	**−10.0**
		−7.3	−14.3	−19.7
		(−12.6,−1.8)	(−19.5,−9.0)	(−29.0,−10.4)
	1–2	**−1.8**	**−1.8**	**−2.2**
		4.6	−2.6	−0.6
		(−2.8,11.9)	(−9.7,4.4)	(−12.1,11.0)
Daily Earnings	<1	**−2.6**	**−4.2**	**−7.8**
		−6.9	−7.9	−15.8
		(−10.5,−3.2)	(−11.7,−4.0)	(−25.9,−5.7)
	1–2	**0.9**	**1.6**	**2.1**
		−0.5	3.0	7.6
		(−4.6,3.5)	(−1.3,7.3)	(−3.6,18.7)

Upper row: simulated effects from structural model in bold; second row: regression coefficients using SIAB; 95% CI of regression estimates in parentheses; employment in %, earnings in EUR/day.

than double my estimate of 10.4%. Apart from differences in sample selection, part of the discrepancy may reflect the influence of other contemporaneous changes during the five-year period. One potential confounder is the gradual increase in childcare availability since 2005. In line with this, I show in the next subsection that the childcare reform has particularly large fertility effects for highly educated women.[36]

6.1.2 Long-Run Effects of Parental Leave and Childcare Reforms

To comprehend the impact of the parental and childcare reforms on life-cycle fertility, employment and welfare, I calculate changes in key outcomes for each reform separately and also for implementing both jointly. For the childcare reform, I increase the fraction of government-funded childcare to one.

[36]Descriptive evidence in Raute (2019) shows a marked increase in births roughly two years after the reform, possibly coinciding with improvements in childcare availability. For women with medium levels of education, finds an increase of about 9%, similar to my estimate of 8.8%. Raute (2019) uses the Mikrozensus and, like my analysis, excludes civil servants and self-employed in her sample. However, I further restrict the sample to women born in West Germany, whereas hers includes foreign-born women and those born in East Germany.

Table 9: Parental Leave Reform – Short-run and Long-run Effect on Births

	Education Group	Pre-Reform	PL SR	Diff	PL LR	Diff
	1	817.9	857.7	+4.9%	844.1	+3.2%
		(26.9)	(27.5)	(1.6%)	(27.8)	(2.2%)
No. of Births	2	1697.7	1846.8	+8.8%	1800.1	+6.0%
		(39.1)	(40.4)	(1.2%)	(39.8)	(1.5%)
	3	422.2	465.9	+10.4%	459.3	+8.8%
		(20.1)	(20.8)	(2.3%)	(20.5)	(2.9%)
Age at Birth		30.9	31.0	+0.1	30.8	-0.1
		(0.1)	(0.1)	(0.04)	(0.1)	(0.06)

Yearly births per 60 000 women. PL SR: short-run, second year after unanticipated reform; PL LR: long-run, new policy anticipated from age 22. Standard errors in parentheses calculated using 1000 simulations.

The results are depicted in Table 10.

The new parental leave policy leads to an overall fertility increase of 3.5%. For the highest education group the increase is 7.1%, while for women with low levels of education the effect is modest at 0.6%. Less-educated women receive a smaller increase in payments due to lower income and are affected more by the reduction in payment duration since they tend to take longer leaves. Table 11 shows the reform is able to substantially reduce the fraction of highly educated women who never have children, by four percentage points, accomplishing one of the intended policy goals. This is because first-time mothers experienced the greatest raise in payments, as they have the highest pre-birth earnings. Women with more children rarely work full-time and benefit less.

Figure 4 shows the changes in benefit amount and labor supply around the birth of the first child. Before women have children, they are slightly more likely to work full-time in anticipation of income-dependent leave benefits. Hours in the second year after birth also increase due to paid leave no longer being available. Nevertheless, because women have more children and work less during the first year, overall employment decreases by about two months in total. There is considerable heterogeneity in education in terms of changes in lifetime earnings. Because highly educated women decrease labor supply the most and have the largest fertility response, their earnings decrease by 8 746 EUR or 2.0%. In contrast, for less educated women, earnings increase by 0.2%. In terms of welfare, all education groups experience a gain equivalent to between 0.1% and 0.3% permanent increase in consumption.

In contrast, I find that childcare subsidies affect women of all education groups relatively evenly as shown in column "CC" of Table 10. The increase in average fertility is around 1.7%, while for highly educated women the effect is somewhat higher at 3.1%. A priori the sign of the effect is not obvious, since there is an income and a substitution effect. Women receive a higher benefit but only conditional on

Table 10: Parental Leave and Childcare Reform Effects

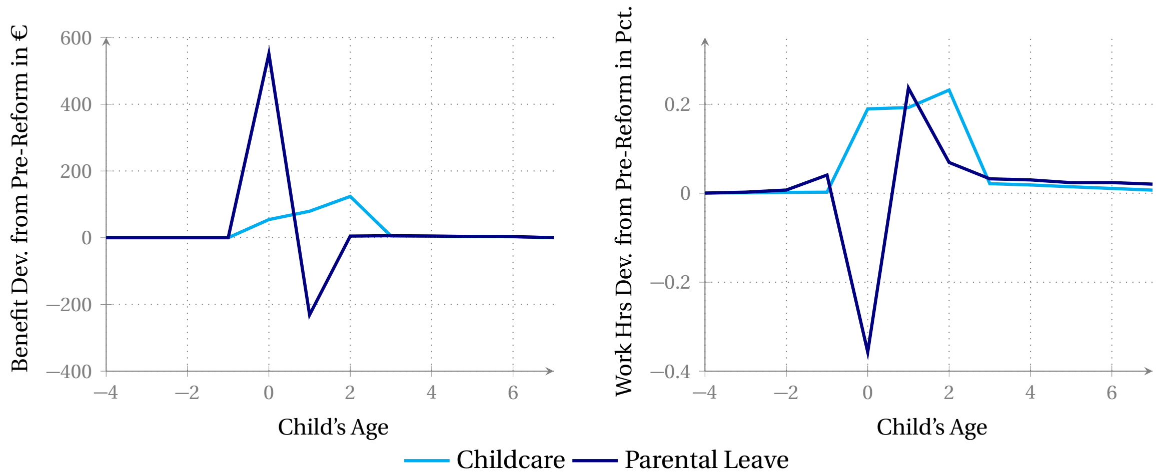
	Educ.	Pre-Reform	PL	CC	PL+CC
Periods Full-Time	-	11.6	−0.1	0.0	−0.1
Periods Part-Time	-	5.6	−0.2	0.0	−0.1
Lifetime Earnings	1	295.5	0.5	0.1	1.0
(in 000s)	2	361.5	−4.1	0.7	−3.2
	3	438.2	−8.7	−0.6	−8.5
All Fertility	-	1.40	+0.05	+0.02	+0.07
By Education	1	1.51	+0.01	+0.02	+0.03
	2	1.48	+0.06	+0.02	+0.08
	3	1.04	+0.07	+0.03	+0.10
Born To Single Mother	-	0.13	−0.01	+0.01	+0.00
Welfare	-	-	+0.2	+0.3	+0.5
By Education	1	-	+0.1	+0.3	+0.4
	2	-	+0.3	+0.3	+0.6
	3	-	+0.1	+0.4	+0.4
Δ Budget	-	-	−1.00	−0.68	−1.64

Individual and joint effects of parental leave and childcare reforms. First two rows: average years worked full-time and part-time for ages 22-45. Third section: discounted lifetime earnings in thousands of EUR. Fourth section: average number of children per woman and proportion born to single women. Fifth section: Discounted lifetime utility changes in % of consumption. Last row: Change in budget is proportion relative to cost of parental leave reform.

Table 11: Percentage of Women Without Children at Age 45 by Education

Education	Pre-Reform	PL	Diff	CC	Diff
low	23.7	22.3	−1.5	23.2	−0.5
med	18.9	16.3	−2.7	18.6	−0.3
high	36.3	32.3	−4.0	34.8	−1.5

Effects of parental leave and childcare reforms on % of women who never have children.



Model simulation for women who have exactly one child.

Figure 4: Parental Leave and Childcare Reform - Change in Benefits and Employment Effects By Child's Age

working, which increases the opportunity cost of additional children. Furthermore, it is not clear how effects vary by education. Although more educated women are more likely to work and use childcare, the amount of costs saved is more significant for lower-educated women.

Haan and Wrohlich (2011) find positive fertility effects for highly educated women while effects are negative for less educated women. Neither Haan and Wrohlich (2011) nor Bick (2016) find an increase in average fertility. On the contrary, Bauernschuster et al. (2016), using a difference-in-differences design, do find a positive fertility effect, a 2.8% increase in births for a 10 pp. increase in childcare coverage. My model generates fertility effects that closely align with this estimate.[37] One possible explanation is that the two former papers only examine married mothers. In fact, I find that the number of children born to single mothers increases by 11.6% while the number born to married mothers only by 0.2%. Since single women work substantially more and they benefit more from childcare subsidies.[38]

The childcare reform leads to a modest increase in lifetime earnings of approximately 0.1%. By lowering the cost of childcare, the policy incentivizes greater labor force participation among mothers of young children (see Figure 4), which in turn fosters human capital accumulation and wage growth over time. However, these positive effects are partially offset by an increase in fertility among women. The reform raises welfare by around 0.3% of lifetime consumption, roughly one-third more than the

[37] In my simulation, the childcare reform led to a 6.1 pp. increase in childcare coverage for under three-year-olds (number of children attending childcare divided by the total number of children). Thus, a 10 pp. increase would raise births by 2.7%. Like Bauernschuster et al. (2016), I also find that the reform had a larger effect on higher-order births.

[38] Furthermore Bick (2016) simulates a simultaneous increase in labor taxes to finance the childcare subsidies and he attributes the lack of a positive response in fertility to the decrease in income.

welfare gain from the parental leave reform. This greater impact is attributable to the childcare policy promoting female labor supply and raising household consumption. As a result, the childcare reform also requires only 68% of the public expenditure of the parental leave reform. When adjusting for cost, the latter exerts a stronger influence on fertility while the former yields larger welfare gains.

The last column of Table 10 displays the joint effects of implementing both reforms at once. Most effects are of similar size as the sum of the individual reform effects, however, there are some differences. For instance, the joint public cost is 2.6% smaller than the sum of individual costs and the joint effect on average fertility is also 2.5% smaller than the sum. In principle, there is no reason for the effects to be additive, given the non-linear nature of the model, hence it is important to analyze different policies in conjunction to capture interaction effects.

6.2 Optimal Family Policy Design

The analysis of individual policy reforms reveals that different policies have distinct effects on key outcomes, such as welfare and fertility. Furthermore, the effects of these policies are not uniform across the population and interact in complex ways. An optimal policy design framework is therefore necessary to determine the most efficient combination of instruments to achieve a given goal while accounting for important trade-offs.

6.2.1 Framework

Solving for optimal policy in a setting with endogenous fertility requires formulating the planner's problem to account for the broader social value of children. The vast majority of the existing literature on optimal policy design abstracts from fertility choice and maximizes aggregate welfare of the current generation. However, a simple welfare-maximizing objective may be insufficient in my context if fertility carries societal implications beyond its direct impact on individual utility. In particular, the social value of newborn children may diverge from parents' private utility for two key reasons. First, children may generate positive externalities for the current generation. For example, in a pay-as-you-go pension system, contributions by the next generation finance transfers to the elderly. Moreover, severe fertility decline can lead to labor shortages and dampen economic growth and innovation.[39] Second, the planner might directly value the welfare of the next generation and, consequently, the number of individuals born into it. While parents may be partially altruistic toward potential children, they might

[39] Jones (2022) argues that population decline poses substantial risks to innovation and long-term growth.

not fully internalize their children's welfare.[40]

Fully quantifying the social value of future children—including externalities and the welfare of yet-unborn individuals—poses complex normative and empirical challenges, such as how to weigh the value of future children's lives relative to the well-being of individuals in the current generation (see Golosov et al., 2007). Rather than take a stand on these contentious issues, I search for policy solutions that constitute a form of improvement over the status quo, both in terms of welfare and fertility levels.

Specifically, I require that average welfare within each education group under the optimal policy must not be lowered. This follows prior work, e.g. Blundell and Shephard (2012) who impose that the solution must offer Pareto improvements across observable types. Education is the only permanent observable characteristic in my model and a strong predictor of income, marriage, and fertility. Moreover, I impose the restriction that fertility rates must not decline for any education group. This preserves any positive externalities at the group level.[41] In particular, if children's expected welfare can be approximated by their parents' education-group average, then, combined with the welfare constraints, the resulting allocation guarantees weakly improved welfare for both current and future generations at the education-group level.[42]

I consider two specifications with distinct objectives, both subject to the aforementioned education-specific welfare and fertility constraints as well as a budget constraint. The first maximizes aggregate welfare of the current generation. The second maximizes aggregate fertility, motivated by the positive externalities of a larger future population and by the explicit goals of many recent pro-natalist policies.[43] The contrast between these two objectives illustrates the trade-off between prioritizing current women's direct welfare and prioritizing fertility externalities, including the welfare of future generations. Formally, the two specifications are stated as follows:

$$\begin{aligned} \max_x \quad W(x) &= \sum_i w_i(x) \\ \text{s.t.} \quad \mathscr{B}(x) &\geq \mathscr{B}(x^b) \\ W_e(x) &\geq W_e(x^b) \ \ \forall e = 1,2,3 \\ N_e(x) &\geq N_e(x^b) \ \ \forall e = 1,2,3 \end{aligned} \tag{1}$$

$$\begin{aligned} \max_x \quad N(x) &= \sum_i n_i(x) \\ \text{s.t.} \quad \mathscr{B}(x) &\geq \mathscr{B}(x^b) \\ W_e(x) &\geq W_e(x^b) \ \ \forall e = 1,2,3 \\ N_e(x) &\geq N_e(x^b) \ \ \forall e = 1,2,3 \end{aligned} \tag{2}$$

[40] Parental utility from children, captured by parameters $\alpha_{21} - \alpha_{23}$, can reflect both private and altruistic motives. However, the estimated sharp decline in marginal utility across birth order suggests that parents might mostly enjoy "selfish" consumption value from children.

[41] Assuming the size of externalities only depends on fertility levels and does not vary otherwise with policy.

[42] This aligns with the concept of P-efficiency from Golosov et al. (2007), which requires that no current or *potential* agent can be made better off without harming another. Assuming agents prefer being born over not being born, the optimal allocation is a P-improvement at the education group level. Approximating children's welfare with parents' by education group is credible under conditions of limited social mobility in Germany.

[43] Given very low current fertility rates, it can be argued that at least locally, increasing fertility is optimal.

where W_e and N_e denote welfare and the number of births by education group, x_b are the baseline policy parameters. Formulation (1) abstracts from direct redistribution motives beyond those implicit in the concavity of the utility function.

The government budget under policy vector x consists of the following items:

$$\mathscr{B}(x) = T(x) - F(x) - CC(x) - PL(x) - S(x)$$

$T(x)$ denotes revenue from income taxes and social security contributions,[44] $F(x)$ are total per-child subsidies, $CC(x)$ and $PL(x)$ are expenses on childcare subsidies and parental leave pay respectively. $S(x)$ are social assistance payments. Based on government reports, I set the gross cost of a full-time childcare spot for a child under the age of three to 1000 EUR per month and for a child aged 3-6 to 700 EUR per month.[45] $CC(x)$ is equal to the total gross cost of public childcare minus total private costs borne by parents. All items are discounted to present value.

The set of family and child-related policy instruments includes parental leave, childcare subsidies, a fixed per-child subsidy, child-related tax deductions, and the degree of tax jointness. Parental leave is characterized by three parameters: the replacement rate and the minimum and maximum benefit amounts. Each of the remaining instruments is governed by a single scaling parameter. To set tax jointness continuously, I let each spouse's marginal tax rate depend on their own income plus a varying fraction of the income difference with their partner.

In Appendix A.8, I solve specifications that maintain the current system of joint taxation, allowing for a clearer assessment of the impact of tax jointness on optimal policy design. I also consider a welfare-maximizing specification that directly internalizes a key externality of children by accounting for fiscal contributions of the next generation in the government budget. In this case, I remove the fertility constraints from the main analysis, allowing the model to determine the optimal (possibly lower) fertility level endogenously, when considering both the current generation's welfare and the fiscal externalities of fertility. In addition, I examine a concave transformation of individual utilities to illustrate how the optimal policy changes under inequality aversion. Together, these alternative specifications help assess the robustness of the main results and clarify the policy trade-offs involved.[46]

[44] In line with German regulations, I set firm contributions equal to worker contributions at 20% of labor income. $T(x)$ also reflects tax benefits given for children or based on joint taxation.

[45] www.bmfsfj.de/blob/94182/763244389dd4e093fa22d4788bbaddeb/kosten-betrieblich-unterstuetzter-kinderbetreuung-data.pdf

[46] Since my framework is dynamic with endogenous fertility, it is difficult to benchmark my approach against existing studies, which typically rely on static settings with fixed fertility (e.g. Blundell and Shephard (2012)). Moreover, the optimal-taxation literature primarily centers on the design of redistributive income tax schedules, whereas my analysis focuses on child-related subsidies and the trade-off between fertility and welfare.

6.2.2 Effects of Policy Instruments

To illustrate the effects and mechanisms of different policy tools, I first analyze how key outcomes respond to increasing the generosity of subsidies through each policy parameter. I vary each parameter such that the government budget is lowered by 1%. Table 12 reports these effects.[47] As expected, all instruments yield positive welfare effects (see the third section). Notably, almost all instruments also raise fertility (section two) with the exception of per-child tax deductions for the lowest education group.

Overall, Table 12 shows that the effects of each policy instrument vary widely, and their magnitudes depend on characteristics such as education, marital status, and number of children. For instance, fixed subsidies have a strong positive impact on both fertility and welfare of less-educated women, but much less so for highly educated women. In contrast, maximum leave pay has the opposite effect. This is because fixed subsidies primarily benefit women with lower incomes and less labor supply, whereas maximum leave pay is more advantageous for women with higher incomes. Additionally, only child tax deductions reduce the proportion of women with three or more children, since benefits increase in income, yet working becomes more costly as family size grows. Higher tax jointness benefits married couples and lowers the proportion of children born to single mothers.

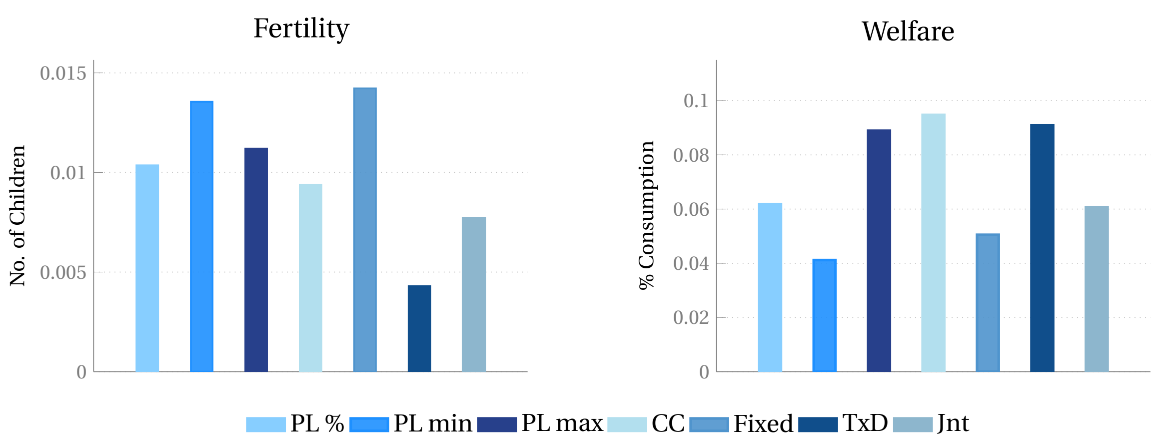
Furthermore, the results reveal a clear trade-off: policy instruments tend to have a strong effect on either fertility or welfare. Figure 5 shows the effects on the average number of children per woman and welfare for each instrument. For instance, tax deductions and childcare subsidies have the largest effects on welfare, but are the worst-performing policy tools to induce higher fertility. Similarly, minimum leave payments and fixed per-child subsidies have the largest impact on fertility, but for welfare, they are among the worst-performing policy tools. The key reason for this trade-off is that increasing welfare is most effectively achieved by boosting female labor supply, not fertility. When increasing labor supply one also raises the opportunity cost of childbearing and tends to suppress birth rates. Encouraging women to work more is a cost-effective strategy to improve welfare because it builds human capital, raises lifetime earnings and consumption, and generates higher tax revenues. In contrast, while policies that raise fertility also positively affect welfare, this is at the cost of reducing labor supply and earnings while increasing public spending on child-related transfers.

[47] For tax jointness, effects are simulated for a decrease in jointness leading to a 1% budget increase and multiplied by -1. Additionally, Table A.48 in Appendix A.7.1 shows the effects of the first six instruments for the fully individual taxation system.

Table 12: Effect of Policy Parameters

Outcome	Ed.	PL %	PL min	PL max	CC	Fixed	TxD	Joint
Years Full-Time	-	−0.04	−0.04	+0.00	−0.01	−0.03	−0.03	−0.04
Years Part-Time	-	+0.00	−0.01	−0.03	+0.03	−0.01	+0.00	−0.01
Lifetime Earn.	1	−0.68	−1.70	+0.37	+0.01	−1.31	−0.15	−1.03
(in 000s)	2	−0.96	−1.13	−0.06	+0.15	−1.06	−0.75	−1.25
	3	−1.63	−0.80	−2.01	−0.10	−0.40	−2.60	−1.32
Earn. No Child	-	−0.89	+0.92	−0.33	−1.03	+0.20	−0.78	−0.31
Earn. Mothers	-	−1.87	−4.22	−1.65	+7.41	−2.26	−1.26	−3.76
All Fertility	-	+0.10	+0.14	+0.11	+0.09	+0.14	+0.04	+0.08
By Education Level	1	+0.09	+0.21	+0.02	+0.08	+0.20	−0.02	+0.07
	2	+0.10	+0.12	+0.12	+0.09	+0.15	+0.03	+0.08
	3	+0.13	+0.06	+0.23	+0.11	+0.04	+0.19	+0.07
Pct. Born Single	-	+0.00	+0.20	−0.10	+0.26	+0.01	−0.03	−0.05
Pct. No children	-	−0.30	−0.14	−0.37	−0.15	−0.18	−0.42	−0.29
Pct. 3 Children	-	+0.18	+0.29	+0.17	+0.32	+0.34	−0.17	+0.13
Pct. 4+ Children	-	+0.02	+0.19	+0.03	+0.04	+0.19	−0.09	+0.05
Agg. Welfare	-	+0.06	+0.04	+0.09	+0.09	+0.05	+0.09	+0.06
Welfare by Educ.	1	+0.05	+0.04	+0.05	+0.07	+0.07	+0.08	+0.07
	2	+0.06	+0.03	+0.10	+0.10	+0.05	+0.12	+0.07
	3	+0.08	+0.07	+0.11	+0.12	+0.01	+0.03	+0.03

Effects for increasing each policy parameter such that government budget decreases by 1%. Effects simulated using deviations from baseline parameters ("PL+CC" in Table 10). Left to right: Parental leave replacement rate (%), minimum and maximum payment in EUR/mo., childcare subsidies (% decrease of cost), fixed per-child subsidies in EUR/mo., per-child tax deductions in EUR/yr, reducing tax jointness in %. All outcomes for ages 22-45 except discounted earnings and utility which are summed across all ages and discounted to age 22. Lifetime earnings are discounted and in thousands of EUR. Earnings by motherhood status in EUR/mo. First four rows second section are total number of children per woman multiplied by 10. Pct Born Single is the percentage of children born to single mothers. Pct. No Children and Three Children are measured at age 45. Lifetime utility changes in % of consumption.



Effects for increasing each policy parameter such that government budget decreases by 1%. Left to right: Parental leave replacement rate (%), minimum and maximum payment in EUR/mo., childcare subsidies (% decrease in cost), fixed per-child subsidies in EUR/mo., per-child tax deductions in EUR/yr. Fertility measured as the total number of children per woman. Welfare changes in % of consumption.

Figure 5: Effect of Policy Parameters - Fertility vs. Welfare

Table 12 shows that welfare-focused policy instruments like childcare subsidies and tax deductions only have small negative or even positive effects on mothers' earnings (+7.41 and −1.26 EUR) and hours-weighted work years (+0.01 and −0.03). However, minimum leave pay and per-child subsidies, fertility-focused tools, have large negative effects (−4.22 and −2.26 EUR, and −0.05 and −0.04 respectively).[48] Some policies are also ineffective on both fronts. For example, increasing tax jointness is dominated by a convex combination of per-child and childcare subsidies in terms of fertility and welfare for all education groups. Moreover, the parental leave replacement rate is also dominated by a convex combination of other child subsidies.[49]

6.2.3 Problem Solution

Turning to the solutions for optimal policy, these are displayed in Table 13. The subscript denotes the first (welfare-focused) or second (fertility-focused) specification. I show key outcomes for the current scenario and changes in Table 14. Remarkably, even while maintaining both welfare and fertility at or above baseline levels for each education group, both solutions deliver substantial gains over the status quo. The first solution x_1^* achieves an increase in overall welfare equivalent to 0.5% of consumption with the highest educated benefitting the most. They see a welfare gain of 10.0% while the lowest education group only experiences negligible improvements. Birth rates are also slightly higher by 0.7%.[50] Under the second solution x_2^*, the fertility gains are 0.08 children per woman or 5.7%. This is primarily driven by the lowest education group, whose birth rates increase by 9.6% because this education group has lower opportunity costs of childbearing. There is a small welfare increase of 0.1%.

Table 13: Optimal Policies

	PL %	PL min	PL max	CC	Fixed	TxD	Joint
x_1^* Welfare	0.53	87	6790	−100	221	10.9K	0.0
x_2^* Fertility	0.55	302	4962	−9	226	8.9K	0.0
Baseline	0.67	300	1800	0.0	150	4.5K	1.0

x_1^*: solution for maximizing welfare; x_2^*: solution for maximizing fertility; baseline policy parameters for reference. First three columns are parental leave parameters: replacement rate (%), monthly min. and max. amount in EUR; next four columns: childcare subsidies (% decrease in cost), monthly fixed subsidy in EUR, yearly tax deductions in EUR, tax jointness (one for fully joint).

[48] Hours-weighted work years are calculated as the sum of years worked full-time plus half the years worked part-time.

[49] Decreasing tax jointness is dominated by 0.75×*Fixed*+0.25×*CC* in the joint taxation case. The replacement rate is dominated by 0.31×*Fixed*+0.35×*CC*+0.34×*PL max* in the joint taxation case and by 0.007×*PL max*+0.767×*Fixed*+0.08×*CC* +0.146×*PL min* in the individual taxation case for example.

[50] Percentage effects calculated relative to baseline fertility, e.g. $0.01/1.47 = 0.7\%$.

Table 14: Optimal Policies - Main Outcomes

Outcome	Ed.	Baseline	Indv.	x_1^*- Welfare	x_2^* - Fertility
Years Full-Time	–	11.61	+2.02	+0.58	+0.50
Years Part-Time	–	5.61	+0.35	+0.09	+0.07
Lifetime Earnings	-	354.52	+55.33	+16.63	+14.49
By Education Level	1	296.53	+49.06	+15.15	+7.23
	2	358.22	+57.53	+18.55	+16.03
	3	429.70	+58.37	+13.39	+20.86
Earn. No Child	–	2184	+8	+6	+14
Earn. Mothers	–	786	+199	+97	+67
All Fertility	–	1.47	−0.35	+0.01	+0.08
By Education Level	1	1.54	−0.31	+0.00	+0.15
	2	1.56	−0.38	+0.00	+0.08
	3	1.15	−0.32	+0.05	+0.00
Born to Single Mother	–	0.13	+0.03	+0.01	+0.01
No Children	–	0.21	+0.18	+0.01	+0.04
Three Children	–	0.12	−0.04	−0.01	+0.03
Four+ Children	–	0.02	+0.00	+0.01	+0.03
Consumption	–	1610	+93	+39	+25
Cons. Married No Child	–	2337	+54	+1	+28
Cons. Married w/ Child	–	1579	−23	+42	−9
Cons. Single No Child	–	1542	+8	+1	+3
Cons. Single w/Child	–	1268	+42	+80	+66
Aggregate Welfare	–		−1.83	+0.51	+0.09
By Education Level	1	–	−2.03	+0.06	+0.00
	2	–	−1.94	+0.59	+0.03
	3	–	−1.23	+0.95	+0.42
Δ Budget	–	–	+9.36	0.00	0.00

Changes relative to baseline reported for individual taxation (Indv.), and solutions for first (x_1^*) and second (x_2^*) specification. All outcomes for ages 22-45 except discounted earnings and utility which are summed across all ages and discounted to age 22. First section: average years worked full-time and part-time, lifetime earnings in thousands of EUR. Second section: average number of children per woman, proportion of children born to single women, proportion of women at age 45 with zero, three and four or more children. Third section: monthly per-capita consumption of households by marital status and presence of children, welfare changes in % of consumption. Last row: Change in budget is proportion relative to cost of parental leave reform.

A key feature shared by both optimal policy solutions is the shift to fully individual taxation. To understand its role, it is instructive to first isolate the effect of eliminating tax-splitting before examining other policy parameters. The results are presented in column "Indv." in Table 14 and show a pronounced fertility decline, reducing the average number of children per woman by 0.35. This reflects both an income and a substitution effect. Married couples suffer a substantial loss in after-tax income on average, while the reduction in marginal tax rates on second earners raises the opportunity cost of childbearing.[51] There is a positive employment effect for all education groups, most notably among less-educated women who exhibit low baseline labor supply. The lower two education groups see an increase in lifetime earnings of over 16%. In contrast, the response is more muted for college-educated women, whose baseline employment is already high. Not surprisingly, welfare declines across all groups, with an average loss equivalent to 1.8% of consumption and the least educated most adversely affected.[52] There is a substantial budget surplus which exceeds the cost of the parental leave reform by a factor higher than 9, owing to higher tax revenues and reduced public spending on family benefits.

By adjusting the child policy parameters the planner can reinvest the budget surplus from switching to individual taxation to counterbalance household income losses and simultaneously raise fertility and welfare. This approach allocates subsidies based on parenthood rather than marital status, generating greater incentives to have children. Moreover, it effectively shifts public resources from married couples with no children to single mothers. As shown in Table 14, the former have nearly double the per-capita consumption of the latter, implying that this redistribution is welfare-enhancing. Importantly, work incentives also remain strong and for both policy solutions, employment and lifetime earnings are higher compared to baseline. For instance, years in full-time employment increase by at least half a year.

The solutions share several other key characteristics, partially because they must satisfy the same set of welfare and fertility constraints. Compared to baseline, they feature higher tax deductions, higher per-child subsidies, higher parental leave maximum payments and lower replacement rates. As noted previously, the replacement rate emerges as a relatively less effective instrument across the board. This is likely because it particularly benefits first-time mothers as their pre-birth earnings are the highest.[53] Since these women already enjoy higher levels of consumption, compared to women with many children, this policy tool has a relatively low impact on welfare. Additionally, the first child is more detrimental to women's income than the second or third. So, from a public spending perspective, it is more effective to incentivize either zero or multiple births per woman. On the other hand, per-child subsidies for example, pay the same amount for all children and have a relatively large positive impact on higher order births.

Notably, there are also discrepancies between solutions. Not surprisingly, each emphasizes tools that are relatively better at achieving the respective goals. When maximizing welfare, subsidies that

[51] In Appendix A.7.2, I present a breakdown of the substitution and income effects.

[52] Monthly per-capita consumption among married couples without children increases by 54 EUR, driven by compositional changes: more couples have no children under individual taxation. Controlling for age, this effect becomes negative. Conversely, consumption rises for single mothers due to higher labor supply and accumulated human capital.

[53] This is evident from the large negative effect on the proportion of women who do not have children and modest increase in women with three or more children.

encourage labor supply are prioritized. Childcare costs are fully eliminated, while they are only reduced by 9% in the fertility-maximizing solution. Tax deductions are raised to 10892 EUR per child, a 2019 EUR larger increase than for x_2^*. Parental leave maximum payments also increase by a larger amount. In contrast, instruments more effective at raising fertility are higher in the fertility solution. The parental leave minimum is 302 EUR per month, while the welfare solution decreases the amount substantially to 87 EUR. Fixed per-child transfers increase to 226 EUR, 5 EUR more than in the welfare solution.

In Appendix A.8, I present policy solutions for additional specifications. When tax jointness is excluded from the set of instruments, the solutions for other policy parameters remain qualitatively similar. The parental leave replacement rate is reduced, freeing resources to expand childcare subsidies, raise the parental leave maximum benefit, and increase per-child transfers. The main insight is that gains are modest in this setting. Maximizing welfare, a 0.09% increase can be achieved with negligible effects on fertility, while focusing on fertility results in a 0.8% rise in births alongside a 0.03% welfare gain.

The solution for the specification that incorporates future fiscal contributions of children into the government budget constraint is shown in Appendix A.8.2. Despite accounting for these future benefits of fertility, the optimal policy prioritizes current welfare, as the fiscal and utility gains associated with children do not outweigh the consumption loss. Welfare increases by 0.96%, 0.45% more than in the main welfare-maximizing specification, while births fall by 19.3%. The welfare gains are driven primarily by policies that stimulate labor supply, achieved through similar adjustments as in the main welfare-maximizing specification: a shift to fully individual taxation, elimination of childcare costs, and a sharp increase in tax deductions. When inequality aversion is introduced, welfare becomes slightly more equal. The policy solution changes only modestly as child-related instruments have only limited scope for redistribution. A key caveat is that this version disregards other positive externalities of fertility, such as innovation and the welfare of children.

6.3 Discussion

While the analysis offers important insights into optimal policy design for fertility and welfare, several limitations should be acknowledged. First, the model abstracts from household savings decisions, which may influence fertility choices and labor supply by providing an alternative channel for consumption smoothing. For instance, Adda et al. (2017) show that households accumulate savings prior to having children. Such behavior could dampen the effectiveness of policy instruments and reduce the magnitude of welfare and fertility responses. These considerations are likely to be particularly relevant for highly educated women, who generally have greater capacity to save.

Second, marital transitions and divorce probabilities are modeled in a simplified manner. Modeling endogenous marriage decisions could alter the evaluation of varying tax jointness. Marriage or divorce rates and also the degree of assortative matching could change in that case. A shift to individual taxation, for instance, would likely increase assortative sorting, although Gayle and Shephard (2019) find that this effect is relatively small.

Finally, an important direction for further research is the impact of family policies on child development. The analysis could be extended to examine how children's skills are influenced by maternal or parental time, financial investments, and time spent in childcare. A substantial literature highlights that the returns to these various inputs can interact in complex ways and often depend on parental skill levels. In addition, parents may take child development into account when making decisions about fertility and work, which in turn is relevant for the optimal design of family policies.

7 Concluding Remarks

In this paper, I present and estimate a dynamic life-cycle model of fertility and female labor supply to study two German policy reforms and the optimal design of family policy. The framework incorporates a rich portfolio of policy instruments and, after validation against the reforms, is used to solve for budget-neutral policies that maximize either fertility or welfare while ensuring that current outcome levels are at least maintained. The analysis reveals that policy interventions affect demographic groups unevenly and create varied impacts across outcomes.

Carefully calibrated combinations of policies, notably a shift from joint taxation to individual taxation, can deliver substantial improvements over the status quo. The optimal policy mix depends on whether the priority is to enhance welfare or to increase birth rates. The results indicate that it is possible to achieve either a 0.5% increase in welfare or a 5.7% increase in the birth rate. This demonstrates the scope for well-designed policy interventions, offering critical guidance for policymakers aiming to address demographic challenges.

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A.1 Identifying Births in the IAB Sample

Here, I describe the procedure developed by Müller and Strauch (2017) to identify births in the IAB data. In Germany, virtually all women enter into mandatory maternity leave for 8 weeks after giving birth ("Mutterschutz"). Unemployed women going into maternity leave are directly recorded as such but employed women are recorded as being on sickness or maternity leave. In order to filter out births from sickness leaves, a series of restrictions are imposed. First, the leave gap must exceed three months; this includes the mandatory two months that usually begin 6-8 weeks prior to the predicted day of birth and one month of leave following birth. Furthermore, the woman has to be below the age of 40 and if two gaps are spaced more closely than 224 days, the second gap is deleted.

There are some limitations. This method can only identify first births reliably since many women might not work in between births and large gaps in which two children were born would be classified as one birth. Also, some extended illnesses could be falsely labeled as births. However, Schönberg (2009) proposes a similar method and finds that births are identified with nearly 90% accuracy by performing a check on a sub-sample that can be linked to administrative birth records.

Table A.15: SIAB Sample

	Low Ed.		Med. Ed.		High Ed.	
	Pre	Post	Pre	Post	Pre	Post
Age at Birth	29.5	29.1	30.0	29.6	32.3	32.0
	(4.6)	(4.4)	(4.2)	(4.1)	(4.0)	(3.5)
Daily Earnings Before Birth	39.4	42.7	52.5	52.0	68.7	65.3
	(36.4)	(34.6)	(40.4)	(36.7)	(57.5)	(51.6)
Full-Time Before Birth	0.49	0.56	0.60	0.62	0.60	0.63
	(0.50)	(0.50)	(0.49)	(0.49)	(0.49)	(0.48)
Part-Time Before Birth	0.12	0.15	0.11	0.12	0.11	0.15
	(0.33)	(0.36)	(0.32)	(0.32)	(0.37)	(0.36)
Observations	298	283	360	339	139	142

Standard errors in parentheses. Sample includes West-German, non-self-employed women born 1960-1984, aged 22-45. Pre and Post indicate women who gave birth to a child in the first four months of either 2006 or 2007. Education groups have the same criteria as in the GSOEP sample. Daily earnings in EUR.

For the regression analysis, I transform the IAB dataset from spell to panel data using code provided by Eberle et al. (2013) and select women who had births in the first four months of 2006 and 2007. To increase the likelihood that the birth year was imputed correctly, I exclude women who gave birth in the first two weeks of January. A summary of the sample can be found in Table A.15. The difference in age at birth, daily earnings, full-time and part-time employment in the year prior to giving birth between the pre and post-reform samples are not significant at the 5% level.

A.2 Childcare Costs

Parents with children aged zero to six (prior to entering primary school) may incur childcare costs. For children under the age of three, the expected childcare costs are modeled as a weighted average of the public (ζ^{pu}), private (ζ^{pr}) and informal (free) care costs. Some but not all households have access to informal care. Access to informal childcare is treated as a permanent binary unobserved characteristic that is orthogonal to other characteristics. For simplicity, the amount of informal care is set to a fixed number of hours. I assume that households will use the cheapest care available. That means, they prefer informal care if available, followed by public care and then private care. The childcare cost can be expressed as follows:

$$\begin{aligned}\zeta(n_t, a_t < 3, y_t^f + y_t^h, \boldsymbol{l_t}, O_3) = &\Big(\big(\iota_{1f}\, \zeta^{pu}(a_t < 3, y_t^f + y_t^h) + (1 - \iota_I O_3 - \iota_{1f})\, \zeta^{pr}\big) \times l_t^f \\ &+ \big(\iota_{1p}\, \zeta^{pu}(a_t < 3, y_t^f + y_t^h) + (0.5 - \iota_I O_3 - \iota_{1p})\, \zeta^{pr}\big) \times l_t^p\Big) \\ &\times \left(kid_t^1 + 0.66\, kid_t^2 + 0.5\, kid_t^3\right)\end{aligned}$$

where ι_I is the portion of childcare covered by informal care if available ($O_3 = 1$), ι_{1f} and ι_{1p} are actual hours of public childcare used if the mother works full or part-time respectively. Denote the maximum portion covered by public childcare as ι_{pu}, then $\iota_{1f} = \min\,(1 - \iota_I O_3, \iota_{pu})$ and $\iota_{1p} = \min\,(0.5 - \iota_I O_3, \iota_{pu})$. The cost is discounted for siblings which is also reflected in the public childcare cost schedule. Moreover, the public cost increases in parental income.

I estimate the availability and usage intensity of childcare for under three-year olds using detailed childcare information in the GSOEP which is available for children born after 2003. The results are presented in Table A.16. The data show that around 54% of mothers use informal care, I approximate this by setting the proportion of the informal care type to 0.6.[54] Mothers work 19.2 hours on average and use 8.4 hours of informal care per week if any. I therefore assume that informal care, when available, covers approximately half of part-time hours and a quarter of full-time hours, implying $\iota_I = 0.25$.

Children of working mothers spend on average 7.0 hours in public childcare. Since the vast majority of mothers are employed part-time, I approximate the share of hours covered by public care with $\iota_{pu} =$

[54] In the data, for a given individual, informal care hours do not systematically vary with maternal employment: regressing informal childcare on hours worked with an individual fixed-effect yields insignificant results. This is robust to expanding the dataset to 3-6 year olds or until year 2009. This suggests that the observed proportion of women using informal care is also approximately the fraction with access.

0.25. This number is roughly consistent with official enrolment rates. Figure A.6 below, created using data from the Federal Agency for Civic Education for years 2006-2012 and Hank and Kreyenfeld (2003) for years 1990-1999, shows that prior to the childcare reform, around 6-8% of children below the age of three were in public childcare. Given that roughly 25% of mothers were employed (primarily part-time), this roughly translates to a range of $0.24-0.32$ for ι_{pu}. As a result, this implies that children whose mothers do not have access to informal care and work part-time spend the remaining portion of time (0.25) in private care.[55]

According to official government reports, there is no shortage of public childcare slots for children aged three to six. The GSOEP data (expanding years to 2009 for sufficient sample size) shows that working women work 21.9 hours and use 19.3 hours of public childcare. There is close to no private care used. So public childcare can cover mother's work hours almost entirely. Notably, as Bick (2016) finds, many non-working mothers use public childcare, roughly 13.4 hours on average. Although, some women also use informal care, this does not seem to crowd out public childcare. Based on this evidence, I approximate the usage of public childcare as follows: full-time working mothers rely on public care for the entirety of working hours, part-time workers for half of their hours and non-working mothers for one quarter. The equation for the cost is therefore:

$$\zeta(n_t, a_t \geq 3, y_t^f + y_t^h, \boldsymbol{l_t}) = \zeta^{pu}\big(a_t \geq 3, y_t^f + y_t^h\big) \times \big(kid_t^1 + 0.66kid_t^2 + 0.5kid_t^3\big) \times \big(l_t^f + 0.5 l_t^p + 0.25\,\big(1 - l_t^f - l_t^p\big)\big)$$

I estimate public childcare costs using data from the 1996, 2002, and 2005 waves of the GSOEP, which include detailed childcare-related questions. Due to limited data availability for children under the age of three, I estimate costs based on children aged three to six and assume that costs for younger children are twice as high. This approximation is supported by findings in (Wrohlich, 2011 and Hank and Kreyenfeld, 2003). The estimated monthly fee is given by the following equation:

$$\zeta^{pu}(a_t \geq 3, y_t^f + y_t^h) = 100 + 0.008\,(y_t^f + y_t^h)$$

Where $y_t^f + y_t^h$ is the monthly household labor income. Private monthly cost ζ^{pr} is set to 840 EUR (Haan and Wrohlich, 2011 and Wrohlich, 2011).

[55] Wrohlich (2011) estimates a larger value of about 0.63 for ι_{pu}. There are only 35 observations for the number of private care hours reported for working mothers without informal care before 2007, but when Including years up to 2009 the mean is 5.8 hours.

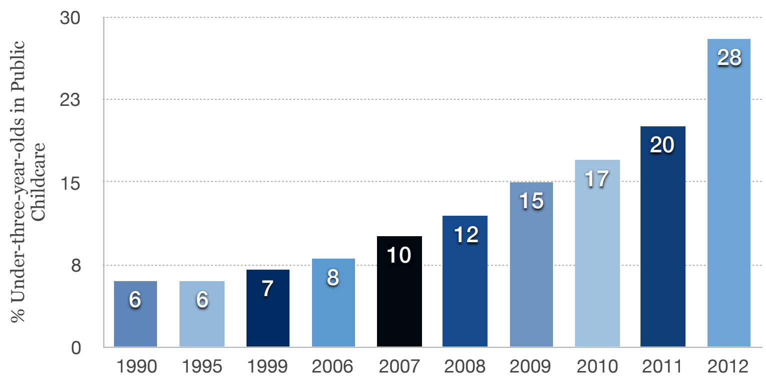

Figure A.6: Public Childcare enrollment for Children Below the Age of Three

Table A.16: Childcare Usage by Child's Age, Mother's Employment Status and Informal Care Availability

	Child's Age	Subgroup	Hours	Obs.
Informal Care	1-2	Any Inf. Care	8.41	257
			(8.37)	
Public Care	1-2	Working	7.00	162
			(13.19)	
Work Hours	1-2	Working	19.23	161
			(12.63)	
Private Care*	1-2	Working & No Inf. Care	5.76	104
			(10.45)	
Private Care*	1-2	Working & Any Inf. Care	0.98	212
			(4.45)	
Public Care*	3-6	Working	19.82	414
			(14.51)	
Public Care*	3-6	Working & Any Inf. Care	19.34	246
			(14.57)	
Private Care*	3-6	Working	0.87	414
			(3.89)	
Work Hours*	3-6	Working	21.93	414
			(13.12)	
Public Care*	3-6	Not Working	13.47	320
			(13.00)	

Estimated using the GSOEP Mother and Child sample 2003-2006, outcomes with * estimated using years 2003-2009. Standard deviations in parentheses.

A.3 Income Tax, Social Security and Social Assistance

Tax Rate and Social Security Contributions Employees pay income tax and social security contributions on gross labor income. The share of social security contributions is 20% of wages for most, however, there is a cap for very high earners and workers with monthly income of less than 400 Euros are exempt.

I use the following piece-wise linear function of monthly individual labor income y to approximate the share:

$$ss(y) = \begin{cases} 0 & y < 400 \\ 0.00025y & 400 \leq y < 800 \\ 0.2 & 800 \leq y < 4300 \\ \frac{860}{y} & 4300 < y \end{cases}$$

The income tax rate is determined by the amount of taxable income. The tax schedule is progressive and further depends on marital status. Married couples are taxed jointly as if each earned half of the joint income ("Splitting"). This gives them a tax advantage, especially when the difference between the spouses' income is large, for instance if one spouse does not work. I represent the tax rate on monthly taxable labor income of single individuals y_s as follows:

$$\tau(y_s) = \begin{cases} 0 & y_s < 700 \\ -0.09 + 0.00015y_s & 700 \leq y_s < 1700 \\ 0.075 + 0.000045y_s & 1700 \leq y_s < 5000 \\ 0.2 + 0.00002y_s & 5000 \leq y_s < 15000 \\ 0.5 & 15000 < y_s \end{cases}$$

The tax rate for married couples is $\tau(\frac{y_s + y_s^h}{2})$.

Social Assistance. German households are guaranteed a subsistence level depending on how many adults and children there are. I model this as a consumption floor using the following function:

$$S(m, n) = (1 + 0.75m + \big(0.65 + 0.2(1 - m)\big)n) * 600$$

The amount is in monthly EUR. Note that a single parent receives higher benefits per child than a married household. Social assistance acts as a floor for disposable income if the sum of after-tax labor income, per-child benefits and child support payments falls below. One exception is parental leave pay, which is not included in the calculation.

A.4 Simulation Starting Values and Husband's Income

Observed Heterogeneity in Fertility Preference. Waves 1990, 1992, 1995 in the GSOEP include the survey question how important respondents think having children is, with answers on a scale from 1 (very important) to 4 (not at all important). The answers are relatively constant for each individual across waves, although the importance of children tends to increase slightly over time. Hence, I focus on answers provided during ages 20-25. I categorize women into two groups, high fertility preference for answers 1 or 2 and low fertility preference for answers 3 and 4.

Table A.17 shows that roughly 76% of all women have low fertility preference. The proportion increases with the level of education. Table A.18 further demonstrates that the preference stated during ages 20-25 is highly predictive of future fertility levels above age 38.

Table A.17: Fraction of Employed and Married Women at Age 21 by Education Group

	Low Ed.	Med. Ed.	High Ed.
Low Fertility Pref.	0.69	0.71	0.79
Married | Low FP	0.06	0.08	0.07
Married | High FP	0.36	0.24	0.00
Employed	0.84	0.94	0.96

Table A.18: Summary Statistics for High and Low Fertility Preference Women

	High	Low
No. of Children Ages 22-28	0.48	0.11
	(0.70)	(0.35)
No. of Children Ages 38-45	1.58	1.04
	(0.94)	(0.91)

Standard errors in parentheses.

Employment, Marital Status and Education. There are differences in employment and marital status at age 21 across education groups which I mimic in the simulation. Marriage rates further vary with fertility preference. Table A.17 shows the distributions. Most women with the highest education level have not completed schooling at age 25. Because almost no women have children when still in school, I begin the simulation when schooling is complete (this does not affect any women of lower education levels). Table A.19 shows the initial age distribution of women in the highest education group when they first enter the simulation. I assume that during schooling women accumulate human capital as if working full-time. For example, a woman who finishes her education when turning 28 has 6 units of human capital.

Table A.19: Age Distribution at Start of Simulation of Women in Highest Education Group

Age	22	23	24	25	26	27	28	29	30	31	32
Fraction	0.34	0.00	0.03	0.09	0.10	0.11	0.08	0.10	0.02	0.04	0.09

Fecundity by Age. To capture that women's biological fecundity declines with age, I set p_t^f according to the following data taken from Khatamee and Rosenthal (2002)[56], which has been used in many other studies such as Adda et al. (2017). This is the probability of conceiving in one year given the attempt to become pregnant.

Table A.20: Yearly Probability of Conception by Woman's Age

Age	22	23	24	25	26	27	28	29	30	31	32	33
p_t^f	0.85	0.83	0.82	0.80	0.78	0.76	0.74	0.72	0.69	0.65	0.60	0.58
Age	34	35	36	37	38	39	40	41	42	43	44	45
p_t^f	0.56	0.54	0.52	0.50	0.47	0.44	0.39	0.35	0.28	0.21	0.13	0.05

First-Step Estimates for Husbands' Incomes. The following table, Table A.21, shows estimates for the parameters governing husbands' incomes for each education group. After the age of 45, profiles flatten, so I apply the value at age 45.

[56] The Fertility Sourcebook by Sara Rosenthal MS and Masood Khatamee MD.

Table A.21: Husband's Income by Education Group

	low	med	high
ψ_{0e} Intercept	21.691	20.593	10.908
	(1.886)	(1.387)	(3.858)
ψ_{1e} Age	0.410	1.422	2.686
	(0.344)	(0.233)	(0.633)
ψ_{2e} Age Sq.	0.007	-0.028	-0.046
	(0.014)	(0.009)	(0.024)

Standard errors in parentheses.

A.5 Labor Supply Elasticities

The following table shows the intensive and extensive labor supply elasticities both for a transitory (Frisch) and permanent (Marshallian) increase in wages. The extensive margin seems to matter more for Marshallian elasticities while it is the opposite for Frisch elasticities.

Table A.22: Intensive and Extensive Labor Supply Elasticities

	Frisch		Marshallian	
	Intensive	Extensive	Intensive	Extensive
All	0.22	0.15	0.43	0.63
Low Ed.	0.26	0.14	0.59	0.73
Medium Ed.	0.23	0.17	0.44	0.64
High Ed.	0.15	0.11	0.2	0.44
Age 25	0.12	0.08	0.09	0.34
Age 32	0.29	0.21	0.45	0.94
Age 40	0.37	0.17	1.08	0.63
Married	0.35	.24	1.86	1.05
Single	0.21	0.07	0.14	0.29

Marshallian elasticities calculated for permanent 1% wage increase, responses averaged across all observations. elasticities for anticipated one-period 1% wage increase at ages 25, 32 or 40, responses by age only for the period of wage increase, by education and family status averaged over the effects of implementing the increase at each of the three ages. Intensive elasticities calculated as percentage change in hours per worker, extensive as percentage change in employment.

A.6 Model Fit

Here, I list the complete set of data moments, their simulated counterparts, the data standard deviation and the difference normalized by the standard deviations. Tables A.23-A.26 list moments relating to fertility, Tables A.27-A.37 contain moments on employment and A.38-A.40 show the moments relating to wages.

Table A.23: Number of Children By Education and Age

Education Level	Ages	Model	Data	Std. Dev	Norm. Diff.
	22−29	0.4505	0.4080	0.0246	1.728
1	30−36	1.1186	1.2000	0.0435	1.874
	37−45	1.4555	1.4546	0.0336	0.028
	22−29	0.3504	0.3012	0.0139	3.547
2	30−36	1.0357	1.1113	0.0262	2.880
	37−45	1.4293	1.4868	0.0213	2.699
	22−29	0.1177	0.0778	0.0111	3.599
3	30−36	0.5556	0.6095	0.0384	1.403
	37−45	0.9979	1.1822	0.0430	4.285

Table A.24: Difference in Number of Children between High and Low Fertility Preference by Age

Age	Model	Data	Std. Dev	Norm. Diff.
22-29	0.504	0.448	0.071	0.787
30-36	0.714	0.602	0.137	0.818
37-45	0.872	0.446	0.142	3.002

Table A.25: Probability of Birth by Birth Order, Education, Marital Status and Age

Ed. Level	Number of Children, Marital Status	Age	Model	Data	Std. Dev	Norm. Diff.
	None	22−29	0.255	0.204	0.015	3.280
	Married	30−36	0.238	0.126	0.019	5.992
		37−45	0.035	0.024	0.008	1.375
	One	22−29	0.249	0.188	0.018	3.489
	Married	30−36	0.137	0.137	0.016	0.004
		37−45	0.033	0.014	0.004	4.465
	Two	22−29	0.100	0.065	0.020	1.804
1	Married	30−36	0.036	0.038	0.007	0.250
		37−45	0.019	0.011	0.003	2.399
	None	22−29	0.026	0.034	0.004	1.912
	Single	30−36	0.012	0.037	0.008	3.393
		37−45	0.002	0.002	0.002	0.090
	One	22−29	0.075	0.062	0.019	0.669
	Single	30−36	0.028	0.030	0.010	0.169
		37−45	0.006	0.006	0.004	0.009
	None	22−29	0.213	0.181	0.008	3.731
	Married	30−36	0.241	0.194	0.013	3.751
		37−45	0.049	0.032	0.007	2.497
	One	22−29	0.206	0.201	0.014	0.332
	Married	30−36	0.147	0.160	0.010	1.361
		37−45	0.034	0.021	0.004	3.250
	Two	22−29	0.066	0.069	0.015	0.189
2	Married	30−36	0.040	0.039	0.005	0.127
		37−45	0.014	0.007	0.001	4.325
	None	22−29	0.015	0.019	0.002	1.991
	Single	30−36	0.020	0.027	0.004	1.794
		37−45	0.014	0.010	0.003	1.185
	One	22−29	0.046	0.057	0.014	0.767
	Single	30−36	0.015	0.047	0.009	3.484
		37−45	0.003	0.007	0.003	1.247
	None	22−29	0.144	0.115	0.015	2.005
	Married	30−36	0.198	0.183	0.015	0.996
		37−45	0.047	0.044	0.009	0.281
	One	22−29	0.157	0.183	0.041	0.620
	Married	30−36	0.168	0.200	0.020	1.571
		37−45	0.034	0.090	0.016	3.575
	Two	22−29	0.067	0.052	0.060	0.251
3	Married	30−36	0.055	0.071	0.016	0.971
		37−45	0.014	0.030	0.007	2.304
	None	22−29	0.007	0.007	0.002	0.213
	Single	30−36	0.014	0.018	0.004	1.079
		37−45	0.014	0.010	0.004	0.983
	One	22−29	0.074	0.091	0.050	0.339
	Single	30−36	0.011	0.061	0.019	2.603
		37−45	0.001	0.006	0.005	1.030

Table A.26: Population Share By Marital Status, Motherhood Status, Age and Education

Marital/Family Status	Ed. Level	Age	Model	Data	Std. Dev	Norm. Diff.
Married with No Child		22−29	0.137	0.143	0.007	0.815
	1	30−36	0.115	0.106	0.008	1.139
		37−45	0.091	0.082	0.010	0.866
		22−29	0.132	0.151	0.005	4.267
	2	30−36	0.131	0.112	0.005	3.419
		37−45	0.081	0.066	0.007	2.152
		22−29	0.056	0.093	0.006	6.735
	3	30−36	0.159	0.168	0.011	0.844
		37−45	0.179	0.149	0.017	1.717
Married with Children		22−29	0.244	0.259	0.009	1.730
	1	30−36	0.537	0.533	0.013	0.327
		37−45	0.614	0.592	0.018	1.222
		22−29	0.181	0.189	0.005	1.534
	2	30−36	0.536	0.522	0.009	1.494
		37−45	0.620	0.609	0.013	0.828
		22−29	0.031	0.059	0.005	5.491
	3	30−36	0.292	0.330	0.014	2.617
		37−45	0.480	0.500	0.026	0.779
Single with Children		22−29	0.106	0.090	0.006	2.786
	1	30−36	0.133	0.165	0.009	3.388
		37−45	0.173	0.204	0.013	2.378
		22−29	0.047	0.052	0.003	1.314
	2	30−36	0.087	0.130	0.006	7.120
		37−45	0.171	0.181	0.010	0.946
		22−29	0.040	0.021	0.003	6.235
	3	30−36	0.079	0.086	0.008	0.885
		37−45	0.131	0.154	0.017	1.337

Table A.27: Full-time Rate By Education and Age

Education Level	Age	Model	Data	Std. Dev	Norm. Diff.
1	22−29	0.63	0.586	0.018	2.397
	30−36	0.382	0.358	0.020	1.173
	37−45	0.309	0.286	0.013	1.743
2	22−29	0.715	0.731	0.010	1.619
	30−36	0.402	0.376	0.013	2.043
	37−45	0.289	0.281	0.010	0.743
3	22−29	0.878	0.854	0.011	2.047
	30−36	0.6	0.605	0.020	0.222
	37−45	0.413	0.466	0.019	2.741

Table A.28: Part-time Rate By Education and Age

Education Level	Age	Model	Data	Std. Dev	Norm. Diff.
1	22−29	0.142	0.101	0.010	4.042
	30−36	0.209	0.198	0.015	0.770
	37−45	0.326	0.331	0.012	0.383
2	22−29	0.132	0.079	0.005	9.734
	30−36	0.227	0.249	0.011	2.023
	37−45	0.377	0.400	0.010	2.357
3	22−29	0.070	0.070	0.007	0.049
	30−36	0.213	0.174	0.015	2.709
	37−45	0.356	0.310	0.017	2.781

Table A.29: Work Transition Rates By Education, Motherhood Status and Current Work Status

Ed. Level	Transition Type	Model	Data	Std. Dev	Norm. Diff.
1	No Child No Work-to-Work	0.331	0.194	0.022	6.220
	No Child Work-to-Work	0.943	0.926	0.005	3.252
	Child No Work-to-Work	0.140	0.117	0.007	3.500
	Child Work-to-Work	0.880	0.883	0.008	0.376
2	No Child No Work-to-Work	0.403	0.277	0.025	5.038
	No Child Work-to-Work	0.939	0.946	0.003	2.646
	Child No Work-to-Work	0.160	0.143	0.005	3.292
	Child Work-to-Work	0.902	0.898	0.005	1.069
3	No Child No Work-to-Work	0.352	0.426	0.043	1.710
	No Child Work-to-Work	0.972	0.961	0.003	3.991
	Child No Work-to-Work	0.198	0.172	0.013	2.026
	Child Work-to-Work	0.900	0.889	0.010	1.113

Table A.30: Full and Part-Time Rate By Age of Youngest Child - Education Level 1

Child's Age	Woman's Age	Emp. Level	Model	Data	Std. Dev	Norm. Diff.
	22-24	FT	0.846	0.763	0.020	4.053
		PT	0.082	0.068	0.011	1.254
No Child	25-30	FT	0.872	0.789	0.021	3.932
		PT	0.107	0.086	0.015	1.458
	30-45	FT	0.813	0.684	0.022	5.934
		PT	0.159	0.150	0.016	0.599
0	all	FT	0.018	0.016	0.017	0.080
		PT	0.147	0.123	0.029	0.838
1−2	all	FT	0.037	0.038	0.012	0.049
		PT	0.182	0.211	0.027	1.057
3−6	all	FT	0.106	0.125	0.016	1.200
		PT	0.283	0.308	0.019	1.283
7−11	all	FT	0.144	0.143	0.015	0.016
		PT	0.359	0.413	0.020	2.740
11+	all	FT	0.336	0.274	0.021	2.918
		PT	0.489	0.355	0.019	6.952

Table A.31: Full and Part-Time Rate By Age of Youngest Child - Education Level 2

Child's Age	Woman's Age	Emp. Level	Model	Data	Std. Dev	Norm. Diff.
	22-24	FT	0.909	0.904	0.008	0.635
		PT	0.067	0.037	0.005	5.505
No Child	25-30	FT	0.913	0.884	0.009	3.104
		PT	0.078	0.058	0.007	2.833
	30-45	FT	0.790	0.782	0.014	0.529
		PT	0.186	0.141	0.012	3.614
0	all	FT	0.032	0.031	0.010	0.116
		PT	0.181	0.122	0.016	3.752
$1-2$	all	FT	0.060	0.053	0.009	0.744
		PT	0.210	0.290	0.017	4.729
$3-6$	all	FT	0.139	0.113	0.009	2.714
		PT	0.352	0.402	0.013	3.880
$7-11$	all	FT	0.178	0.149	0.011	2.578
		PT	0.422	0.514	0.015	6.182
11+	all	FT	0.365	0.314	0.016	3.119
		PT	0.500	0.437	0.016	4.000

Table A.32: Full and Part-Time Rate By Age of Youngest Child - Education Level 3

Child's Age	Woman's Age	Emp. Level	Model	Data	Std. Dev	Norm. Diff.
	22-24	FT	0.971	0.940	0.010	3.176
		PT	0.026	0.027	0.006	0.279
No Child	25-30	FT	0.931	0.851	0.014	5.787
		PT	0.064	0.096	0.011	2.880
	30-45	FT	0.801	0.810	0.018	0.478
		PT	0.194	0.138	0.017	3.278
0	all	FT	0.056	0.070	0.019	0.744
		PT	0.244	0.140	0.025	4.100
$1-2$	all	FT	0.095	0.095	0.019	0.022
		PT	0.289	0.374	0.030	2.822
$3-6$	all	FT	0.181	0.248	0.028	2.404
		PT	0.410	0.408	0.028	0.083
$7-11$	all	FT	0.231	0.349	0.032	3.682
		PT	0.477	0.453	0.032	0.764
11+	all	FT	0.456	0.415	0.044	0.925
		PT	0.460	0.356	0.041	2.531

Table A.33: Employment By Motherhood Status and Education

Education Level	Motherhood Stat.	Model	Data	Std. Dev	Norm. Diff.
1	Child	0.438	0.462	0.015	1.594
	No Child	0.964	0.847	0.012	9.464
2	Child	0.517	0.539	0.010	2.156
	No Child	0.981	0.936	0.005	9.531
3	Child	0.583	0.596	0.020	0.651
	No Child	0.995	0.951	0.006	7.866

Table A.34: Full and Part-Time Rate of Mothers of Children Aged Three

Employment Level	Model	Data	Std. Dev	Norm. Diff.
FT	0.143	0.092	0.011	4.708
PT	0.352	0.427	0.016	4.629

Table A.35: Full and Part-Time Rate by Number of Children

No. of Children	Emp. Level	Model	Data	Std. Dev	Norm. Diff.
0	FT	0.863	0.826	0.006	6.360
	PT	0.118	0.094	0.005	5.231
1	FT	0.234	0.287	0.011	5.076
	PT	0.340	0.316	0.010	2.506
2	FT	0.109	0.122	0.008	1.692
	PT	0.369	0.396	0.009	2.842
3	FT	0.059	0.069	0.011	0.956
	PT	0.328	0.293	0.019	1.890

Table A.36: Employment Rate of Single Women By Education and Age of Youngest Child

Education Level	Woman's Age	Child's Age	Model	Data	Std. Dev	Norm. Diff.
	$21-23$	No Child	0.923	0.838	0.020	4.257
1	$24-45$	No Child	0.982	0.839	0.016	9.192
	all	≤ 3	0.263	0.192	0.042	1.697
	all	> 3	0.595	0.627	0.026	1.211
	$21-23$	No Child	0.974	0.945	0.006	4.616
2	$24-45$	No Child	0.992	0.929	0.007	9.171
	all	≤ 3	0.395	0.426	0.035	0.915
	all	> 3	0.693	0.724	0.017	1.869
	$21-23$	No Child	0.997	0.970	0.007	3.746
3	$24-45$	No Child	0.996	0.950	0.007	6.298
	all	≤ 3	0.593	0.534	0.073	0.793
	all	> 3	0.798	0.770	0.034	0.823

Table A.37: Employment Rate of Married Women By Education and Age of Youngest Child

Education Level	Child's Age	Model	Data	Std. Dev	Norm. Diff.
1	No Child	0.960	0.862	0.023	4.306
	≤ 3	0.278	0.279	0.028	0.048
	> 3	0.549	0.541	0.019	0.441
2	No Child	0.970	0.937	0.008	3.994
	≤ 3	0.329	0.333	0.018	0.208
	> 3	0.616	0.640	0.012	1.953
3	No Child	0.991	0.941	0.010	4.775
	≤ 3	0.423	0.430	0.029	0.246
	> 3	0.666	0.752	0.025	3.398

Table A.38: Monthly Wage By Education and Age

Education Level	Age	Model	Data	Std. Dev	Norm. Diff.
1	22−29	1,733.47	1,814.88	42.86	1.900
	30−36	2,021.48	2,084.54	54.63	1.154
	37−45	2,182.42	2,141.93	32.26	1.255
2	22−29	2,104.80	2,079.97	31.27	0.794
	30−36	2,430.58	2,389.27	32.29	1.279
	37−45	2,464.29	2,515.31	28.13	1.814
3	22−29	1,934.58	2,015.26	64.07	1.259
	30−36	2,782.31	2,880.51	71.17	1.380
	37−45	3,618.07	3,606.36	77.27	0.151

Table A.39: Log Wage Regression Coefficients

Regressor	Model	Data	Std. Dev	Norm. Diff.
Constant	2.693	2.668	0.019	1.346
Ed. Level 2 Constant	0.160	0.152	0.014	0.578
Ed. Level 3 Constant	0.260	0.294	0.020	1.712
Full-time Exp.	0.042	0.040	0.003	0.626
Full-Time Exp. Sq.	0.000	−0.001	0.000	1.238
Part-time Exp.	−0.009	0.020	0.004	7.183
Part-Time Exp. Sq.	0.000	0.000	0.000	2.031
Non-Empl	−0.004	−0.022	0.004	4.547
Non-Empl Sq.	0.000	0.001	0.000	2.099

Table A.40: Log Wage Increase By Education and Past Employment

Education Level	Emp. Level	Model	Data	Std. Dev	Norm. Diff.
1	FT	0.021	0.028	0.006	1.062
	PT	0.020	0.002	0.010	1.790
2	FT	0.027	0.024	0.004	0.667
	PT	−0.003	−0.001	0.006	0.241
3	FT	0.050	0.054	0.008	0.478
	PT	0.026	0.037	0.017	0.628

Table A.41: Marriage Regression Coefficients

Regressor	Model	Data	Std. Dev	Norm. Diff.
Constant	0.083	0.073	0.008	1.181
Ed. Level 2	0.016	0.019	0.008	0.311
Ed. Level 3	−0.055	−0.032	0.009	2.573
Age	0.005	0.006	0.001	0.963
Ed. Level 2 Age	0.000	0.000	0.001	0.287
Ed. Level 3 Age	0.003	0.004	0.001	1.590
Age Sq.	0.000	0.000	0.000	2.962
Motherhood Status	−0.012	−0.023	0.004	2.434

Table A.42: Divorce Regression Coefficients

Regressor	Model	Data	Std. Dev	Norm. Diff.
Constant	0.018	0.027	0.008	1.122
Ed. Level 2	−0.005	−0.008	0.004	0.810
Ed. Level 3	−0.018	−0.019	0.005	0.199
Age	0.000	0.001	0.001	0.945
Ed. Level 2 Age	0.000	0.000	0.000	0.339
Ed. Level 3 Age	0.015	−0.006	0.005	4.303
Age Sq.	−0.001	0.002	0.006	0.460
Young Child	0.016	0.002	0.004	3.335
Old Child	−0.006	0.007	0.004	3.116

A.6.1 Non-Targeted Moments

In the following tables, I show how the model performs for a selection of moments that are not targeted in the estimation. I first consider age, marital status and work experience at first birth by women's education groups. The model fits most moments well. It slightly overestimates age at first birth but captures that it is increasing in education. The model slightly underpredicts the share of married mothers among the least educated group, but it replicates the share for the two higher education groups well. The number of years worked full and part-time also behave similarly in the model and the data.

Next, Table A.44 shows that the model can also replicate monthly earnings (conditional on working) by family status. The model matches the data for women without children and married mothers. The earnings of married mothers are underestimated by around 200 EUR, while for single mothers, they are overestimated by roughly 300 EUR, which is driven by a higher full-time rate.

Table A.45 shows that cumulative years worked full and part-time until age 45 are very similar in model and data. In particular, years full-time increase with education, and women in the second-highest education group work the most part-time. Here, I use data for all years (not only within my sample period) of all women in the estimation sample to have a sufficiently long panel for ages 22-45.

To check model fit beyond women's fertile ages, I examine employment outcomes at age 50 by education group in Table A.46. Here, I again use all years of observations for the women in my sample. Given that women in my sample are born from 1961 onward, there are insufficient observations to examine ages greater than 50. The model performs very well for all outcomes except for the employment rate of women in the lowest education group. The model underestimates it by 15 percentage points but correctly predicts that it is the lowest among all education groups.

Lastly, I investigate employment and earnings by fertility preference types. Given the smaller size of the sample of women that have provided a response regarding fertility preference, I focus on women below the age of 32. The model fits the data remarkably well, although only three moments were targeted for fertility preference types, the differences in cumulative fertility by type across three age groups. The largest discrepancy between model and data is for the part-time work rate of women with high fertility preferences. The model overestimates this by 15 percentage points, which also causes it to underestimate earnings conditional on working by roughly 50 EUR.

Table A.43: Outcomes at First Birth

	Ed. Level	Model	Data
Age at First Birth	1	27.9	27.1
	2	29.1	28.3
	3	32.0	30.9
Married at First Birth	1	0.81	0.68
	2	0.85	0.80
	3	0.81	0.80
Years FT at First Birth	1	5.0	4.7
	2	6.4	6.1
	3	9.2	8.4
Years PT at First Birth	1	0.8	0.4
	2	0.8	0.4
	3	1.0	0.6

Table A.44: Monthly Earnings When Working by Family Status

	Model	Data
Married No Children	2157	2245
Single No Children	2066	2129
Married with Children	1331	1516
Single with Children	1954	1637

Table A.45: Periods Worked until Age 45

	Ed. Level	Model	Data
Years FT until Age 45	1	10.5	10.3
	2	11.1	11.5
	3	14.9	15.4
Years PT until Age 45	1	5.5	4.8
	2	6.0	6.0
	3	5.3	4.7

Table A.46: Employment and Earnings at Age 50

	Ed. Level	Model	Data
Monthly Earnings When Working	1	1974	1939
	2	2087	2081
	3	3332	3253
Employment Rate	1	0.81	0.64
	2	0.85	0.80
	3	0.87	0.80

Table A.47: Employment and Earnings By Fertility Type, Ages 22-31

	Fertility Preference	Model	Data
Monthly Earnings When Working	High	1697	1749
	Low	1955	1909
Full-Time Employment	High	0.55	0.48
	Low	0.71	0.71
Part-Time Employment	High	0.27	0.12
	Low	0.08	0.10

A.7 Additional Evidence on Policy Effects

A.7.1 Policy Effects Under Joint and Individual Taxation

Here I present the marginal effects for the policy instruments for parental leave, childcare subsidies, fixed per-child subsidies and child tax deductions under both fully joint and individual taxation. The effects for joint taxation are identical to those presented in the main text in Table 12, while the effects for individual taxation are reported in the shaded columns. Marginal effects differ considerably depending on the taxation system. Nevertheless, the same instruments tend to be effective in raising welfare or fertility under both regimes.

Table A.48: Effect of Policy Parameters

Outcome	Ed.	PL %		PL min		PL max		CC		Fixed		TxD	
		Jnt	Indv.	Jnt	Indv.	Jnt	Indv.	Jnt	Indv.	Jnt	Indv.	Jnt	Indv.
Years Full-Time	-	−0.04	−0.05	−0.04	−0.05	+0.00	−0.03	−0.01	−0.01	−0.03	−0.06	−0.03	−0.03
Years Part-Time	-	+0.00	−0.01	−0.01	−0.01	−0.03	−0.03	+0.03	+0.02	−0.01	+0.00	+0.00	−0.02
Lifetime Earn.	1	−0.68	−1.49	−1.70	−1.91	+0.37	−0.12	+0.01	−0.18	−1.31	−1.37	−0.15	−0.31
(in 000s)	2	−0.96	−1.43	−1.13	−1.11	−0.06	−1.36	+0.15	−0.08	−1.06	−1.63	−0.75	−0.52
	3	−1.63	−0.79	−0.80	−0.50	−2.01	−1.38	−0.10	−0.15	−0.40	−0.67	−2.60	−2.27
Earn. No Child	-	−0.89	−0.20	+0.92	+1.61	−0.33	−0.07	−1.03	−0.48	+0.20	+0.47	−0.78	−0.90
Earn. Mothers	-	−1.87	−5.52	−4.22	−6.68	−1.65	−5.08	+7.41	+5.43	−2.26	−6.02	−1.26	+0.46
Fertility													
Total Number	1	+0.09	+0.15	+0.21	+0.27	+0.02	+0.04	+0.08	+0.10	+0.20	+0.13	−0.02	+0.04
Children ×10	2	+0.10	+0.14	+0.12	+0.11	+0.12	+0.17	+0.09	+0.09	+0.15	+0.16	+0.03	+0.07
	3	+0.13	+0.07	+0.06	+0.06	+0.23	+0.14	+0.11	+0.09	+0.04	+0.06	+0.19	+0.17
Pct. Single	-	+0.02	+0.05	+0.19	+0.26	+0.03	+0.04	+0.04	+0.06	+0.19	+0.06	−0.09	−0.03
Pct. No children	-	−0.30	−0.52	−0.14	−0.22	−0.37	−0.61	−0.15	−0.16	−0.18	−0.47	−0.42	−0.66
Pct. 3 Children	-	+0.18	+0.20	+0.29	+0.15	+0.17	+0.14	+0.32	+0.26	+0.34	+0.19	−0.17	−0.08
Pct. 4+ Children	-	+0.02	+0.05	+0.19	+0.26	+0.03	+0.04	+0.04	+0.06	+0.19	+0.06	−0.09	−0.03
Agg. Welfare	-	+0.06	+0.03	+0.04	+0.06	+0.09	+0.03	+0.09	+0.08	+0.05	+0.05	+0.09	+0.09
Welfare by Educ.	1	+0.05	+0.05	+0.04	+0.09	+0.05	+0.04	+0.07	+0.06	+0.07	+0.05	+0.08	+0.05
	2	+0.06	+0.02	+0.03	+0.04	+0.10	+0.03	+0.10	+0.08	+0.05	+0.05	+0.12	+0.10
	3	+0.08	+0.03	+0.07	+0.05	+0.11	+0.01	+0.12	+0.13	+0.01	+0.02	+0.03	+0.12

Effects for increasing each policy parameter such that government budget decreases by 1%. Jnt and Indv. stand for joint and individual taxation. Effects are for post-reform baseline. Left to right: Parental leave replacement rate (%), minimum and maximum payment in EUR/mo., childcare subsidies (% decrease of cost), fixed per-child subsidies in EUR/mo., per-child tax deductions in EUR/yr, reducing tax jointness in %. All outcomes for ages 22-45 except discounted earnings and utility which are summed across all ages and discounted to age 22. Lifetime earnings are discounted and in thousands of EUR. Earnings by motherhood status in EUR/mo. First three rows in the fertility section are the total number of children per woman multiplied by 10. Pct. No Children and Three Children are measured at age 45. Life-time utility changes in % of consumption.

A.7.2 Individual Taxation and Income Effect

Here, I decompose the effects of switching from joint to individual taxation into an income and a substitution effect. Specifically, the former operates through lower marginal tax rates for secondary earners, which increase female labor supply incentives and the opportunity cost of child-rearing. The latter is the loss of joint-filing benefits, which reduces after-tax income of married couples. Assuming both children and leisure are normal goods, this further lowers fertility and increases labor supply.

To examine the substitution effect while holding income constant, I simulate a switch to individual taxation where the resulting change in tax revenue is returned to married households via a budget-neutral lump-sum transfer. The results are shown in the last column of Table A.49, together with the uncompensated switch in column "Indv.". As expected, compared to baseline, fertility declines while lifetime earnings and years worked (particularly full-time) increase across all education groups. Consistent with economic theory, the fertility decline is substantially smaller than in the uncompensated case (0.26 compared to 0.35). Similarly, the increase in full-time work is nearly halved (1.10 years vs. 2.02 years) when the negative income effect is removed. Welfare increases substantially with the lump-sum subsidy. A key caveat to this exercise is that a uniform subsidy does not perfectly eliminate the income effect for all households. Because joint taxation mainly benefits couples with unequal incomes, often those with young children and low female labor supply, a fixed, budget-neutral transfer tends to undercompensate them.

Table A.49: Switch to Individual Taxation

Outcome	Ed.	Baseline	Indv.	Indv.+LS
Years Full-Time	–	11.61	+2.02	+1.10
Years Part-Time	–	5.61	+0.35	+0.47
Lifetime Earnings	1	296.53	+49.06	+26.88
By Education Level	2	358.22	+57.53	+33.95
	3	429.70	+58.37	+42.07
All Fertility	-	1.47	−0.35	−0.26
By Education Level	1	1.54	−0.31	−0.22
	2	1.56	−0.38	−0.28
	3	1.15	−0.32	−0.26
Born to Single Mother	–	0.13	+0.03	+0.02
No Children	–	0.21	+0.18	+0.11
Three Children	–	0.12	−0.04	−0.03
Four+ Children	–	0.02	+0.00	−0.00
Aggregate Welfare	–	–	−1.81	+3.93
By Education Level	1	–	−2.00	+4.21
	2	–	−1.92	+4.07
	3	–	−1.22	+3.07
Δ Budget	–	–	+9.36	0.00

All outcomes for ages 22-45 except discounted earnings and utility which are summed across all ages and discounted to age 22. Outcomes in last two columns are changes relative to baseline for implementing individual taxation and individual taxation plus a lump-sum transfer. First section: average years worked full-time and part-time, lifetime earnings in thousands of EUR. Second section: average number of children per woman, proportion of children born to single women, proportion of women at age 45 with zero, three and four or more children. Third section: welfare changes in % of consumption. Last row: Change in budget is relative to cost of parental leave reform.

A.8 Alternative Problem Specifications

Here, I present results for four alternative specifications. For the first two, I eliminate the possibility of varying tax jointness and apply the current joint system for both the welfare and fertility-maximizing objectives. These solutions illustrate the effects of the remaining policy instruments allowing for comparison with the benchmark in the main analysis. For the third and fourth specification, I maximize welfare but embed positive fiscal externalities of fertility in the budget constraint and eliminate education-specific fertility constraints. This approach highlights the trade-off between lowering fertility to increase current generation's welfare and increasing fertility to raise tax contributions. I explore both the linear welfare function as well

A.8.1 Solutions Under Joint Taxation

Without reducing tax jointness, the possible gains from optimizing child subsidies alone are substantially smaller. The solutions are presented as x_1^*' for the welfare-focused and x_2^*' for the fertility-focused specification in Table A.50 and key outcomes are shown in Table A.36. When maximizing welfare, x_1^*' achieves a gain of 0.09%, equivalent to 18.2% of the effect of x_1^*. Fertility effects are positive but close to zero. x_2^*' boosts births by 0.8%, only 15% of the effect of x_2^*. Welfare increases slightly by 0.03%.

Despite smaller effects, the solutions are qualitatively similar to those in the main text. For example, they both lower the replacement rate, which is dominated by other policies. Moreover, they cut around 90% of the remaining childcare cost and increase the parental leave maximum pay. The key difference between x_1^*' and x_2^*' is that one focuses on increasing welfare by promoting employment via higher tax deductions and lowers the parental leave minimum pay, while the other does the opposite. Minimum parental leave payments have a large impact on fertility because they benefit women with many children who tend to work little prior to giving birth and are likely to receive the minimum amount.

Both solutions increase lifetime earnings for all education groups and generate the highest welfare benefits for college-educated women just like in the main specifications. Furthermore, when the goal is to increase fertility, women in the lowest education group are targeted.

Table A.50: Optimal Policies

	PL %	PL min	PL max	CC	Fixed	TxD	Joint
x_1^*' Fertility Jnt.	0.59	318	4269	−90	156	4.1K	1.0
x_2^*' Welfare Jnt.	0.53	256	4367	−89	160	4.6K	1.0
$x^*_{3,\theta=0.0}$ Welfare Fisc. Ext.	0.23	20	4363	−100	162	19.0K	0.0
$x^*_{3,\theta=-0.4}$ Welfare Fisc. Ext.	0.23	17	4523	−97	172	18.8K	0.0
Baseline	0.67	300	1800	0.0	150	4.5K	1.0

x_1^*' and x_2^*' solutions for maximizing welfare and fertility under joint taxation; $x^*_{3,\theta=0.0}$ and $x^*_{3,\theta=-0.4}$ are solutions taking into account fiscal externalities of children and inequality aversion. First three columns are parental leave parameters: replacement rate (%), monthly min. and max. amount in EUR; next three columns: childcare subsidies (% decrease in cost), monthly fixed subsidy in EUR, yearly tax deductions in EUR.

A.8.2 Internalizing the Fiscal Externality of Fertility

In this specification, instead of imposing fertility constraints, I incorporate the next generation's net fiscal contributions into the government budget. These are approximated using current average contri-

Table A.51: Optimal Policies – Main Outcomes for Additional Specifications

Outcome	Ed.	Baseline	x_1^*' W. Jnt.	x_2^*' F. Jnt.	$x^*_{3,\theta=0.0}$ W. Ext.	$x^*_{3,\theta=-0.4}$ W. Ext.
Years Full-Time	–	11.61	+0.10	+0.14	+0.75	+0.74
Years Part-Time	–	5.61	−0.01	−0.05	+0.37	+0.36
Lifetime Earnings	1	296.53	+1.10	+0.80	+31.35	+30.92
By Education Level	2	358.22	+3.00	+3.49	+29.11	+28.85
	3	429.70	+3.01	+4.51	−1.23	−1.59
Earn. No Child	–	2184	+3	+5	−19	−19
Earn. Mothers	–	786	+10	+8	+187	+185
All Fertility	–	1.47	+0.00	+0.01	−0.28	−0.28
By Education Level	1	1.54	+0.00	+0.02	−0.44	−0.43
	2	1.56	+0.00	+0.01	−0.32	−0.32
	3	1.15	+0.00	+0.00	+0.05	+0.06
Born to Single Mother	–	0.13	+0.00	+0.00	+0.01	+0.02
No Children	–	0.21	+0.00	+0.01	−0.03	−0.02
Three Children	–	0.12	+0.00	+0.01	−0.10	−0.10
Four+ Children	–	0.02	+0.00	+0.01	−0.02	−0.02
Cons. Married No Child	–	2337	+7	+12	−71	−70
Cons. Married w/ Child	–	1579	+4	+1	+168	+163
Cons. Single No Child	–	1542	+1	+2	−6	−6
Cons. Single w/ Child	–	1268	+13	+20	+86	+87
Aggregate Welfare	–	–	+0.09	+0.03	+0.96	+0.96
By Education Level	1	–	+0.09	+0.01	+0.19	+0.20
	2	–	+0.07	+0.01	+1.02	+1.02
	3	–	+0.15	+0.14	+1.95	+1.94

Fourth and fifth columns report welfare- and fertility-maximizing specifications restricting policy to joint taxation. Sixth column maximizes welfare accounting for fiscal externalities of children, seventh column maximizes a concave transformation of individual utilities. All outcomes are for ages 22–45, except discounted earnings and utility, which are summed over the life cycle and discounted to age 22. Lifetime earnings are discounted and reported in thousands of EUR. Earnings and consumption by motherhood and marital status are in EUR per month. No Children, Three Children, and Four+ Children refer to completed fertility. Welfare changes are expressed as percentages of consumption.

butions of the respective education group and discounted to present value.[57] The government budget constraint under policy vector x is:

$$B(x) = \mathscr{B}(x) + (w_1 * N_1(x) + w_2 * N_2(x) + w_3 * N_3(x))/(1+r)^{T^*}$$

Here, $\mathscr{B}(x)$ denotes the net contributions of the current generation just like in the main specifica-

[57]This education-group-level approximation is justifiable given relatively low intergenerational mobility in Germany. For instance, in 2021, the fraction of children of college-educated parents attending college was 52 pp. higher than the fraction among children of non-college educated parents (79% vs. 27%) according to a study of the "Stifterverband für die Deutsche Wissenschaft e.V." available at https://www.bpb.de/themen/bildung/dossier-bildung/515729/von-der-grundschule-zur-hochschule-wer-kommt-an/

tions. The second term captures the discounted value of future fiscal contributions by children, where w_e is the average lifetime contribution of individuals in education group e and $N_e(x)$ is the number of children born to mothers in that group. I set w_e to the current level of lifetime contributions of 105.5k EUR, 161.8k EUR, and 243.0k EUR for low, middle, and high education levels, respectively. I use a discount rate $r = 4\%$ and average maternal age at birth $T^* = 29$.[58]

Note that child-related policies have both fiscal effects on the current generation, through affecting labor supply and net revenue, and future generation, through increasing the number of contributors. Because w_e is increasing in education, policies that raise fertility among more educated women yield greater future fiscal returns from the next generation. However, the opportunity cost of childbearing is also higher for these women, affecting $\mathscr{B}(x)$.

As in the main specification, I require that the average welfare within each education group under the policy x is at least as high as under the baseline policy x_b.[59] I consider maximizing aggregate welfare, with a possible concave transformation of individual utilities to capture inequality aversion. Specifically, I apply the same functional form as Blundell and Shephard (2012):

$$W(x;\theta) = \sum_i \frac{\exp(\theta\, w_i(x)) - 1}{\theta}$$

where w_i is normalized discounted lifetime utility.[60] When $\theta = 0$, this represents the linear case as in equation (1) in Section 6.2.1. I also consider the case of moderate inequality aversion when $\theta = -0.4$.

The problem can thus be written as:

$$\begin{aligned} \max_x \quad & W(x;\theta) \\ \text{s.t.} \quad B(x) & \geq B(x_b) \\ W_e(x) & \geq W_e(x_b) \;\; \forall e = 1,2,3 \end{aligned}$$

The penultimate column of Table A.51 shows that fertility rates sharply decrease under the linear solution by 19.3%. This implies that the fiscal benefits of having more future taxpayers and individual utility from children are outweighed by the loss in consumption due to more children of the cur-

[58]Fixing w_e and T^* limits the number of moving parts in the optimization.

[59]This requires the solution to be a Pareto improvement for current women and relates to the concept of A-efficiency in Golosov et al. (2007) for settings with endogenous fertility when only welfare of existing (alive) individuals is considered but not of potentially unborn ones.

[60]Normalization is implemented by dividing lifetime utility by the number of periods and subtracting 6.0, approximately the mean utility. In this dynamic setting, the welfare criterion captures the planner's aversion to lifetime inequality, but does not map directly into per-period consumption.

rent generation. To boost welfare, the policy bundle is adjusted to increase employment with similar changes as in the main welfare-maximizing specification. Taxation is fully individual, childcare costs are eliminated and tax deductions are lifted drastically from 4 500 to 19 004 EUR (see penultimate row in Table A.50). At the same time, parental leave becomes less generous on average with lower minimum pay and replacement rate, however the upper limit is raised to encourage more labor supply.

The welfare gain is 0.96%, just below twice as large as the increase in the welfare-maximizing main specification with fertility constraints. All education groups experience welfare improvements, implying that the welfare constraints are non-binding. Interestingly, fewer women have no children, likely because households need at least one child to benefit from per-child tax deductions. Also, only the lowest two education groups see a decline in births, highly educated women experience a 4.6% increase in births. This is partly because many highly educated women have no children in the baseline and few have more than one child.

When imposing explicit inequality aversion and setting $\theta = -0.4$, the results change only slightly (see the last column of Table A.51). As expected, welfare becomes slightly more equalized: women with the lowest education experience a welfare gain of 0.01% (from 0.19% to 0.20%), while college-educated women incur a small welfare loss of 0.02% (from 1.95% to 1.94%). Aggregate welfare falls marginally. Fertility increases modestly across all education groups, while lifetime earnings fall somewhat. By placing greater weight on lower-income households and shifting the policy mix toward higher fixed subsidies and lower deductions, the planner trades off welfare for higher fertility. Overall, the small magnitude of the adjustments reflects the modest redistributive capacity of the child-related policy instruments considered here, particularly when compared with the possibility of redesigning the entire tax schedule, as in the optimal taxation literature.

A.8.3 Sensitivity Analysis

This section presents results from modest variations in the role of fertility in the problem. For the welfare- and fertility-maximizing objectives with education-specific fertility constraints, I consider tighter and looser versions of the fertility constraints by scaling the fertility lower bounds by 99% and 101%, respectively. For the linear welfare-maximizing objective with child-related fiscal externalities, I explore alternative parameterizations that similarly scale children's fiscal contributions by 99% and 101%.

The resulting policy solutions are reported in Table A.52, while the corresponding effects on fertility and welfare outcomes are shown in Table A.53. Across all cases, both policy solutions and outcomes vary smoothly with modest changes in the fertility constraints and the weighting of child-related externalities, supporting the robustness of the main results.

For the welfare-maximizing solution x_1^*, only the fertility constraint of the lowest education group is binding. Accordingly, when lower-bound fertility is increased from 99% to 101% of the baseline level, the solution reallocates resources toward fixed childcare subsidies (by about 7 EUR per month) to stimulate fertility among low-education women. In return, tax deductions are lowered. As a result, overall welfare declines (from a gain of +0.55% to +0.44%), while fertility increases (from a gain of 0.00 to +0.02). In contrast, for the fertility-maximizing solution x_2^*, the fertility constraint binds for the highest education group, so these patterns are reversed when moving from looser to tighter fertility bounds.

When maximizing welfare in the presence of fiscal externalities due to fertility, increasing the weight on the externalities generates both income and substitution effects. Under x_3^*, fiscal externalities from children are lower than in the baseline, while revenues collected from current women are higher in order to balance the budget. Raising the weight on the negative term creates a negative income effect. This limits the scope for child-related subsidies of any kind and lowers both welfare and fertility. At the same time, greater importance is attached to fertility, particularly of higher income women whose children drive externalities in the budget constraint. Overall, the negative income effect dominates for fertility, leading to a slight decline in the average number of children, especially in the lower two education groups. Consistent with this, the minimum parental-leave benefit is reduced. Welfare falls by roughly 0.02%, from a gain of 0.97% to 0.96%.

Table A.52: Optimal Policies

	PL %	PL min	PL max	CC	Fixed	TxD	Joint
Pareto-Improving:							
Welfare							
$x_{1,.99}^*$	0.53	95	6679	−100	218	11.1K	0.0
x_1^*	0.53	87	6790	−100	221	10.9K	0.0
$x_{1,1.01}^*$	0.53	78	7041	−100	225	10.6K	0.0
Fertility							
$x_{2,.99}^*$	0.56	295	4702	−6	227	8.5K	0.0
x_2^*	0.55	302	4962	−9	226	8.9K	0.0
$x_{2,1.01}^*$	0.55	296	5255	−37	225	9.2K	0.0
Welfare Fisc. Ext.							
$x_{3,.99}^*$	0.23	21	4466	−100	166	19.0K	0.0
x_3^*	0.23	20	4363	−100	162	19.0K	0.0
$x_{3,1.01}^*$	0.23	10	4171	−100	163	19.0K	0.0

Policy solutions for specifications with small variations in the fertility constraints (x_1^* and x_2^*) and weights on fiscal fertility externalities (x_3^*). First three columns are parental leave parameters: replacement rate (%), monthly min. and max. amount in EUR; next three columns: childcare subsidies (% decrease in cost), monthly fixed subsidy in EUR, yearly tax deductions in EUR.

Table A.53: Optimal Policies - Sensitivity

Outcome	Ed.	Baseline	$x^*_{1,.99}$	x^*_1	$x^*_{1,1.01}$	$x^*_{2,.99}$	x^*_2	$x^*_{2,1.01}$	$x^*_{3,.99}$	x^*_3	$x^*_{3,1.01}$
All Fertility	–	+1.47	0.00	+0.01	+0.02	+0.09	+0.08	+0.07	−0.28	−0.28	−0.29
By Educ.	1	+1.54	−0.01	0.00	+0.03	+0.16	+0.15	+0.13	−0.43	−0.44	−0.44
	2	+1.56	−0.01	0.00	+0.02	+0.09	+0.08	+0.07	−0.32	−0.32	−0.33
	3	+1.15	+0.06	+0.05	+0.04	−0.01	0.00	+0.01	+0.05	+0.05	+0.05
Welfare	–		+0.56	+0.51	+0.44	+0.07	+0.09	+0.14	+0.97	+0.96	+0.96
By Educ.	1	–	+0.06	+0.06	+0.02	0.00	0.00	0.00	+0.21	+0.19	+0.18
	2	–	+0.67	+0.59	+0.51	0.00	+0.03	+0.08	+1.02	+1.02	+1.02
	3	–	+1.01	+0.95	+0.86	+0.36	+0.42	+0.52	+1.98	+1.95	+1.93

Changes relative to baseline reported for specifications with small variations in the fertility constraints (x^*_1 and x^*_2) and weights on fiscal fertility externalities (x^*_3). Average number of children per woman, welfare changes in % of consumption.