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# Wage-Setting Constraints and Firm Responses to Demand Shocks\*

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## Abstract

This paper investigates how institutional wage-setting constraints, such as a national minimum wage or collectively bargained wages, affect firm responses to demand shocks. We develop a framework to interpret heterogeneous shock responses that depend on the constraints firms face, and provide empirical evidence on the relevance of these constraints in shaping firm behavior across three countries with different institutional settings: Portugal, Norway, and Colombia. We discuss the implications of our findings for conventional measures of employer wage-setting power and rent-sharing.

**Keywords:** Demand Shocks, Wage Constraints, Firm Heterogeneity, Rent-Sharing, Monopsony Power, Minimum Wages, Wage Floors, Collective Bargaining.

**JEL:** D22, J31, J42, J51.

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# 1 Introduction

A growing body of literature documents substantial heterogeneity in wages among observably similar workers, a pattern often linked to employer wage-setting power (Card et al. 2018, Manning 2021, Lamadon et al. 2022). To quantify this phenomenon, recent research has focused on estimating two key parameters: firm-level labor supply and rent-sharing elasticities. A common approach exploits firm-level demand shocks and constructs estimates of these elasticities based on adjustments in employment and wages. The labor supply elasticity is then used to quantify measures of employer wage-setting power, such as “wage markdowns”, i.e., the ratio of wages to the marginal revenue product of labor (Sokolova and Sorensen 2021, Azar and Marinescu 2024, Kline 2025). The rent-sharing elasticity, in turn, informs how wages respond to changes in firm profitability (Abowd and Lemieux 1993, Jäger et al. 2020). These parameters are central to policy evaluation, informing analyses of mergers and acquisitions (e.g., Arnold 2019, Prager and Schmitt 2021) and the effects of minimum wage policies (e.g., Ahlfeldt et al. 2023, Berger et al. 2025), among others.

In practice, however, many labor markets operate under wage-setting constraints—such as national minimum wage laws and sectoral wage bargaining agreements—that limit how firms adjust wages and employment in response to demand shocks. These constraints are prevalent across institutional contexts (OECD 2019, Bhuller et al. 2022, Dube and Lindner 2024, Jäger et al. 2025) and can shape the observed pass-through of firm-level demand shocks to wages and employment.<sup>1</sup> When binding, they may lead constrained firms to adjust primarily along the employment margin, leaving wages unchanged or less affected. This has direct implications for the estimation of firm-level labor supply and rent-sharing elasticities and, if unaccounted for, can distort the estimates of the *potential* wage-setting power that firms hold.<sup>2</sup> Although the literature acknowledges that the presence of a binding minimum wage may impact the relationship between firm productivity, wages and employment (e.g., Manning 2003, Kline 2024), we lack a clear understanding of what conventional measures of firm-level labor supply and rent-sharing elasticities capture, as well as systematic evidence on the empirical relevance of wage-setting constraints across different institutional contexts.<sup>3</sup>

This paper fills that gap by developing a theoretical and empirical framework to study

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<sup>1</sup>Besides institutional constraints, firms may also face informal constraints, e.g., due to fairness norms (Dube et al. 2019, Hazell et al. Forthcoming) or pressures against nominal wage cuts (Grigsby et al. 2021).

<sup>2</sup>As noted by Bronfenbrenner (1956), the structural firm-level labor supply elasticity reflects the potential wage-setting power employers may hold, in the absence of constraints (see discussions in Manning 2021).

<sup>3</sup>Kline (2024, p. 121) notes that a binding wage floor can explain the “hockey-stick-like” relationship between firm productivity and wages found in several recent studies; see, e.g., Card et al. (2016) for Portugal, Coudin et al. (2018) for France, Bruns (2019) for Germany, Li et al. (2023) for Canada, Di Addario et al. (2023) for Italy, Boza and Reizer (2024) for Hungary, and Bassier and Budlender (2025) for South Africa.

how institutional wage-setting constraints affect the responses of firms to demand shocks. We lay out a theoretical framework where employers determine wages and employment facing an upward-sloping labor supply curve as well as an institutional wage-setting constraint. The wage constraint allows us to capture distinct institutional features, such as the presence of a national minimum wage, sectoral bargaining agreements with negotiated wage floors, or wage demands from a local union. As in [Manning \(2003\)](#), a firm may operate in one of three possible regimes: it may operate on the labor demand curve where the wage-setting constraint is binding (“demand-constrained”), it may operate on the labor supply curve but still face a binding wage-setting constraint (“supply-constrained”), or it may operate on the labor supply curve without being bound by the wage-setting constraint (“unconstrained”). The framework predicts that conventional approaches to estimate firm-level labor supply and rent-sharing elasticities mismeasure the underlying structural parameters when constraints bind, as some firms adjust wages less and employment more in response to demand shocks. Conventional measures will thus tend to overstate (understate) the structural labor supply (rent-sharing) elasticities that firms face. Crucially, the share of demand-constrained firms serves as a sufficient statistic for the extent of bias in conventional labor supply estimates. We provide empirical evidence from three countries—Portugal, Norway and Colombia—with distinct wage-setting institutions to study the relevance of these constraints by quantifying the shares of demand-constrained firms and the extent of biases in conventional measures.

To empirically test the predictions of our framework, we must circumvent several methodological challenges. First, whether or not a firm is constrained is not directly observable in most employer-employee datasets. We address this by classifying firms based on predetermined indicators of exposure, such as paying wages near the minimum wage, being covered by a sectoral bargaining agreement, or the presence of a strong local union. Second, identifying the effect of product demand shocks requires exogenous variation not linked to firm performance. We follow the internal shock design of [Lamadon et al. \(2022\)](#) and use firm-level balance sheet data to construct plausibly exogenous demand shocks, which are available for a broad range of firms across the productivity distribution and comparable across countries, and an external shock design that exploits export-demand disruptions ([Hummels et al. 2014](#), [Garin and Silvério 2024](#)).<sup>4</sup> Third, cross-country comparisons across economies with varying prevalence of wage-setting institutions could reflect other correlated factors (e.g., macroeconomic conditions). To isolate the role of wage-setting constraints, we compare shock

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<sup>4</sup>The internal shock approach in [Lamadon et al. \(2022\)](#) builds on [Guiso et al. \(2005\)](#), who used changes in value added to capture idiosyncratic demand shocks at the firm level. By comparison, the external shock approach relies on plausibly exogenous changes in observed factors that correlate with firm value added. Prominent examples of the latter approach include [Van Reenen \(1996\)](#) using technological innovations, [Kline et al. \(2019\)](#) using patent allowances, and [Kroft et al. \(2025\)](#) using public procurement winners.

responses across firms with different exposure to constraints within each country. Finally, to address the concern that differences in adjustments across firms may reflect factors other than their exposure to wage-setting constraints, we develop a strategy that exploits both exposure to demand shocks and within-firm changes in exposure to constraints over time.

Our study focuses on three countries—Portugal, Norway and Colombia—that are notable examples of institutional settings prevalent across the globe (OECD and AIAS 2023). In Portugal and Colombia we focus on firms’ exposure to the national minimum wage, which is the most prevalent institutional constraint, though we also consider collectively bargained wage floors in Portugal. For Norway, which does not have a national minimum wage, we focus on firms’ coverage of sectoral collective bargaining agreements as well as the presence of local unions. As in several European countries, local unions engage in firm-level bargaining in Norway, while their role is fairly limited in Portugal and largely absent in Colombia.

We begin by documenting firm-level responses to demand shocks in each country. Using the internal shock design, our estimates of the implied labor supply elasticities (measured as the ratio of employment and wage responses to demand shocks) range from 5.84 in Portugal, 3.29 in Norway and 4.47 in Colombia, whereas rent-sharing elasticities (measured as the ratio of wage to value-added responses) range from 0.090 in Portugal, 0.186 in Norway, and 0.072 in Colombia. The external shock design reveals similar estimates in Portugal and Norway. These results point to meaningful cross-country heterogeneity in firms’ adjustments to demand shocks, which may reflect differences in the extent to which firms face wage-setting constraints or underlying differences in employers’ wage-setting power.

Moving beyond cross-country differences, our results point to a consistent within-country pattern: wage responses are significantly lower among constrained firms. As a result, the implied elasticities differ substantially between firms that are constrained by wage-setting institutions and those that are not, for each of the three countries. Specifically, the implied labor supply elasticities for constrained firms are between 33 and 90 percent larger than those for unconstrained firms, while the implied rent-sharing elasticities are between 32 and 48 percent smaller. Consistent with our theoretical predictions, we find lower rent-sharing and higher labor supply elasticities for Norwegian firms with high local union densities, likely reflecting the strength of wage-setting constraints imposed by strong local unions.

This evidence suggests that constrained firms may appear to face a flatter labor supply curve—seemingly closer to the competitive benchmark—but this may in fact be driven by the institutionally imposed wage-setting constraints. Our calculations reveal that conventional estimands yield an average firm-level labor supply elasticity that is up to 14% upward-biased, resulting in distorted estimates of the potential wage-setting power that firms possess. Our evidence nevertheless indicates that less than 10% of firms are demand-constrained. Notably,

even with small shares of demand-constrained firms, we observe large differences in implied elasticities between the constrained and unconstrained firms, reflecting that conventional measures are quite sensitive to biases resulting from the presence of wage-setting constraints.

Finally, we document that the patterns observed across comparisons of constrained and unconstrained firms also hold in a design that focuses on firms that become constrained following minimum wage increases in Portugal. Specifically, our findings suggest that the responsiveness of wages to demand shocks diminishes significantly when firms become constrained due to minimum wage hikes. These findings reinforce the role of wage-setting constraints in shaping the pass-through of firm-level demand shocks and highlight the importance of accounting for institutional features when measuring employer market power.

This paper contributes to several strands of literature. First, we show both theoretically and empirically that firm-level responses to demand shocks are shaped by wage-setting constraints, thereby contributing to our understanding of the role of labor market institutions (Dube and Lindner 2024, Jäger et al. 2025). A related literature finds that wage responses to demand shocks are stronger for firms bound by firm-level wage bargaining relative to sectoral bargaining (Gürtzgen 2009, Rusinek and Rycx 2013, Card et al. 2014).<sup>5</sup> More recently, Olsson (2024) shows that Swedish firms whose workers were covered by predetermined union wage contracts adjusted more through employment than through wages during the Great Recession.<sup>6</sup> Our study provides evidence on heterogeneity in responsiveness across firms across both wage and employment margins, and complements this line of research in three ways: we use direct, firm-level measures of exposure to wage-setting constraints—the “bite” of the minimum wage, collective bargaining coverage, and the presence of strong local unions—rather than comparing institutional regimes; we provide comparable quasi-experimental evidence across multiple settings; and we exploit within-firm variation in constraints.<sup>7</sup>

More precisely, our work relates to recent studies that quantify labor market power in the presence of wage-setting constraints. Tortarolo and Zarate (2018) estimate higher labor supply elasticities for firms that employ more minimum-wage workers in Colombia. Hermo (2025) shows evidence of heterogeneous wage and employment responses to wage floor changes in Argentina by firms’ exposure to constraints that are consistent with our

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<sup>5</sup>Other articles study rent-sharing by focusing on wages negotiated in union contracts directly (e.g., Holmlund and Zetterberg 1991, Abowd and Lemieux 1993, Card and Cardoso 2022, Hermo 2025).

<sup>6</sup>Similarly, Balsvik et al. (2015) investigate the impacts of increased import competition from China in a regional analysis for the Norwegian manufacturing sector, finding evidence of declining manufacturing employment and no wage adjustments, which they argue reflect the presence of wage-setting constraints.

<sup>7</sup>Amodio et al. (2025) show cross-country estimates of (inverse) labor supply elasticities as well, however they do not compare constrained and unconstrained firms within countries. An earlier example showing cross-country evidence is Holmlund and Zetterberg (1991), who estimate industry-level rent-sharing models in Sweden, Norway, Finland, Germany, and the US, and discuss the role of wage-setting practices.

theoretical framework. [Bassier and Budlender \(2025\)](#) develop a cross-sectional kink design to identify constrained and unconstrained firms, and estimate smaller rent-sharing elasticities among constrained firms in South Africa, consistent with our evidence. Related, [Faia et al. \(2026\)](#) test the implications of minimum wage in a monopsony framework, while [Diegmann et al. \(2026\)](#) suggest that departures from a standard monopsony model may better explain the size-wage premium in large German firms. Our paper makes several distinct contributions in this regard. First, we explicitly discuss the implications of wage-setting constraints for the identification of firm-level elasticities, informing a discussion of potential remedies. Second, we provide comparable quasi-experimental evidence from multiple settings and leverage variation in wage-floor constraints. Third, we develop a framework that combines insights from models of monopsony and firm-level bargaining, besides allowing for wage-floor constraints.<sup>8</sup> This framework rationalizes the presence of rent-sharing among constrained firms, unlike stylized models of firm-level bargaining.<sup>9</sup> The model can also rationalize that wages in some firms lie strictly above the marginal revenue product of labor, i.e., the presence of “wage markups”, which is ruled out in static monopsony models.<sup>10</sup>

More broadly, our paper relates to a growing literature on monopsony power in the labor market (e.g., [Card et al. 2018](#), [Manning 2021](#), [Berger et al. 2022](#), [Lamadon et al. 2022](#), [Kline 2025](#)), which typically estimates structural firm-level labor supply elasticities that inform subsequent counterfactual analyses. For example, [Berger et al. \(2025\)](#) studies the efficiency implications of minimum wage policies in the US, relying on estimated elasticities from [Berger et al. \(2022\)](#), and [Ahlfeldt et al. \(2023\)](#) estimates labor supply elasticities and uses them to study the spatial implications of the minimum wage in Germany.<sup>11</sup> Our work suggests that estimating structural elasticities by fitting structural labor supply equations or rent-sharing relationships when the data are generated by an economy that operates under wage-setting constraints may result in biased estimates of the structural parameters. Furthermore, we

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<sup>8</sup>Examples of recent studies that explicitly incorporate unions and firm-level bargaining in monopsony settings include [Azkarate-Askasua and Zerecero \(2025\)](#), who show that unions partially offset the impacts of employer wage-setting power, and [Wong \(2026\)](#), who explores how union bargaining power interacts with firm labor and product market power, both in the context of French manufacturing.

<sup>9</sup>In stylized models of firm-union Nash bargaining, wages depend on the average revenue product of labor and an outside option. However, as first pointed out by [McDonald and Solow \(1981\)](#), the equilibrium level of the average revenue product of labor does not depend on demand shifters when the production function is homogeneous, implying the absence of rent sharing. By contrast, our framework is able to generate non-zero rent-sharing elasticities also among constrained firms, even when the production function is homogeneous.

<sup>10</sup>Recent evidence by [Yeh et al. \(2022\)](#) and [Chan et al. \(2025\)](#) shows that a significant fraction of firms have markdowns above one, a feature that can be rationalized in models of firm-level bargaining.

<sup>11</sup>Several recent articles use firm-level data to fit structural models that later deliver policy-relevant implications on labor market power. For example, [Lamadon et al. \(2022\)](#) relies on the pass-through of firm-level shocks to estimate the size of labor market rents earned by workers and firms, and [Deb et al. \(2024\)](#) measures the implications for wage inequality of product and labor market power, both in the US.

show that relying on instruments will not solve the problem. This insight complements recent work by [Dhyne et al. \(2025\)](#) and [Chan et al. \(2025\)](#), who show that labor adjustment costs imply that the observed extent of labor market power may not reflect the structural elasticities. In a similar spirit, [Balke and Lamadon \(2022\)](#) and [Agostinelli et al. \(2025\)](#) consider firms’ engagement in long-term employment relationships.

This paper is structured as follows. Section 2 introduces the framework we use to interpret firm responses to demand shocks under wage-setting constraints. Section 3 describes the institutional context and data sources for Portugal, Norway, and Colombia. Section 4 describes our empirical strategy. Section 5 presents our findings, and Section 6 concludes.

## 2 Framework

This section proceeds in three steps. First, we lay out an economic model in which firms maximize profits facing a labor supply constraint and a wage-setting constraint and discuss how the wage constraint affects the responses to demand shocks. Second, we discuss what is identified by usual approaches to estimate labor supply and rent-sharing elasticities, relying on the potential outcomes from the model. Finally, we introduce a novel strategy for causally testing the role of constraints in shaping firms’ responses to demand shocks.

### 2.1 Theoretical Model

Firms  $j \in \mathcal{J}$  produce a homogeneous good but differ in their total factor productivity  $\Phi_j$ . Workers’ preferences result in upward-sloping labor supply curves to the firm. Additionally, firms face an institutional wage-setting constraint. Labor market clearing determines the overall level of wages and, as a result, the constrained status of different firms.

#### 2.1.1 Labor Supply

We let the measure of workers willing to work for firm  $j$  at wage  $W_j$  be

$$H(\psi_j W_j, \mathbb{W}; A_j) = A_j \left( \frac{\psi_j W_j}{\mathbb{W}} \right)^\eta, \quad (1)$$

where  $A_j$  is the amenity value of working for the firm,  $\mathbb{W}$  is an aggregate wage index that will be determined in equilibrium,  $\psi_j$  is the firm’s hiring probability (an endogenous quantity that allows us to generate labor rationing), and  $\eta > 0$  is the elasticity of labor supply. Appendix A.1 provides a micro-foundation of this labor supply equation assuming that

workers have idiosyncratic tastes for different firms (Card et al. 2018). The quantity  $\psi_j W_j$  can be interpreted as the expected wage workers receive when choosing to work for firm  $j$ .

The structure in Equation (1) imposes several assumptions for notational convenience that we later relax. First, the assumption that the labor supply elasticity  $\eta$  is homogeneous across firms simplifies the interpretation of our empirical analysis of demand shocks. We explicitly relax this assumption in Section 2.3 when we discuss our design exploiting within-firm variation in wage-setting constraints over time. Second, for notational simplicity, we also abstract from market-level adjustments. However, our empirical analysis exploits within-market variation in firm-level shocks by flexibly controlling for market-by-year effects.

### 2.1.2 Firm Optimization Problem

We assume that firm  $j$ 's revenue is  $P\Phi_j f_j(L_j)$ , where  $P$  is the output price,  $\Phi_j$  is total factor productivity,  $L_j$  is the number of workers employed by the firm, and  $f_j(\cdot)$  is a standard production function with  $f_j'(\cdot) > 0$  and  $f_j''(\cdot) < 0$ .<sup>12</sup> Additionally, we assume that the firm takes  $P$  as exogenous, and set  $P = 1$  as the numeraire of the economy.

Firm  $j$  chooses the wage and employment level to maximize profits  $\Pi_j = \Phi_j f_j(L_j) - W_j L_j$ . Following the recent monopsony literature, we let the firm face the labor supply constraint

$$L_j \leq H(\psi_j W_j, \mathbb{W}; A_j). \quad (2)$$

We assume that the firm is atomistic, so it ignores its influence on the aggregate wage index  $\mathbb{W}$ .<sup>13</sup> Additionally, given a wage floor  $\underline{W} > 0$ , the firm faces a wage-setting constraint

$$W_j \geq g_j(\Upsilon_j, \underline{W}), \quad (3)$$

where  $\Upsilon_j(L_j) = \Phi_j f_j(L_j)/L_j$  is the average revenue product of labor, and the wage-setting constraint  $g_j(\Upsilon_j, \underline{W}) \in [\underline{W}, \Upsilon_j]$  is weakly increasing in both arguments.<sup>14</sup>

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<sup>12</sup>While we exclude inputs other than labor from the production process, flexible inputs such as capital or materials can be added. For instance, let  $M_j$  be a flexible input with exogenous price  $P_M$  and let  $\tilde{f}_j(M_j, L_j)$  be the relevant production function. The optimal level of the input for any  $L_j$  is  $M_j^* = M_j(P_M, L_j)$ . Then, we can study the optimal choice of labor using the production function  $f_j(L_j) = \tilde{f}_j(M_j(P_M, L_j), L_j)$ .

<sup>13</sup>Berger et al. (2022, Section II.A) discuss how strategic interactions between firms in an oligopsonistic setting pose challenges to the identification of firm-level labor supply elasticities using the conventional approach that relies on firm-level demand shocks. They instead propose an alternative indirect inference approach for identification. Notably, the identification problem that we discuss in Section 2.2 is distinct and carries over in an oligopsonistic setting, but arises even in settings without such strategic interactions.

<sup>14</sup>We note that  $g_j(\Upsilon_j, \underline{W}) \in [\underline{W}, \Upsilon_j]$  is a natural restriction in this setting. A negotiated wage above the average product  $\Upsilon_j$  would imply that the firm operates at a loss, whereas a wage below  $\underline{W}$  would violate the institutional constraint. In this spirit, Moene and Wallerstein (1997) present a model of local bargaining where they impose that negotiated wage is at least as large as the market wage faced by the firm.

**The unconstrained solution.** We first consider the stylized case when the wage-setting constraint does not bind. The profit-maximizing wage is given by  $W_j^* = \mu \Phi_j f'_j(L_j^*)$ , where  $\Phi_j f'_j(L_j^*)$  is the equilibrium marginal revenue product of labor (MRPL) and  $\mu = \eta/(\eta + 1)$  is the markdown factor (or what [Robinson 1933](#) defined as the “exploitation index”). For any wage, it is optimal to hire all workers supplying labor, i.e.,  $\psi^* = 1$  and  $L_j^* = H_j(W_j^*, \mathbb{W}; A_j)$ . Panel (a) of [Figure 1](#) shows the unconstrained equilibrium, relying on the inverse labor supply curve  $W(H)$ . The firm picks an employment level that equates the MRPL and the marginal cost of labor (MCL), and the wage is determined by the labor supply equation.

**Production technology.** It will be informative to discuss special cases for  $f_j(\cdot)$  that will allow us to draw sharper comparative statics from the framework and compare with existing work. To do so, we rely on the output elasticity of labor  $\alpha_j(L) \equiv Lf'(L)/f(L) > 0$ .

**Assumption 1** (Production Technologies). **(a)** *The output elasticity of labor is constant,  $\alpha_j(L) = \alpha$  for  $\alpha \in (0, 1)$ . Equivalently, the production function takes the Cobb-Douglas form  $f_j(L_j) = L_j^\alpha$ .* **(b)** *The output elasticity of labor is decreasing,  $\alpha'_j(L_j) < 0$  for all  $L_j$ .*

Assumption [1\(a\)](#) corresponds to the Cobb-Douglas case with a single input, which implies that the output elasticity is constant. Assumption [1\(b\)](#) requires that the output elasticity of labor declines with employment. This property naturally arises in the presence of fixed factors or organizational constraints. As we will discuss below, these alternative assumptions have implications for quantifying the responses of firms to a change in productivity  $\Phi$ .

In the following, we derive the theoretical solutions for cases where wage-setting constraints bind. First, we discuss pure wage floor constraints, where  $g_j(\Upsilon_j, \underline{W}) = \underline{W}$ . Second, we consider firm-level bargaining between a local union and the firm, imposing the wage constraint  $g_j(\Upsilon_j, \underline{W}) = \kappa_j \Upsilon_j + (1 - \kappa_j) \underline{W}$ , where  $\kappa_j \in (0, 1)$  is a parameter reflecting union bargaining power. We also discuss predictions for general forms of  $g_j(\Upsilon_j, \underline{W})$ .

### 2.1.3 Wage Floor Constraint

We consider the behavior of constrained firms when the constraint takes the form  $W_j \geq \underline{W}$ . This case is motivated by the global prevalence of minimum wage policies, but can also be motivated by wage-setting institutions prevalent in many countries, whereby sectoral wage floors are negotiated between employer associations and sectoral unions, while local bargaining is limited (e.g., [Portugal](#)). [Appendix A.2](#) provides derivations and details.

**Solution to firm problem.** The solution  $(W_j^*, L_j^*)$  can be characterized by three distinct regimes ([Manning 2003](#), [Ahlfeldt et al. 2023](#), [Berger et al. 2025](#)). A high-productivity firm

will be in the *unconstrained* regime, choosing an optimal wage above the wage floor, as shown in Panel (a) of Figure 1. Low-productivity firms will be wage-constrained, in which case  $W_j^* = \underline{W}$ , and may operate under two regimes.

First, *supply-constrained* firms are those that choose to hire all workers that are willing to work at the wage floor, i.e.,  $L_j^* = H_j(\underline{W}, \mathbb{W}; A_j)$ , so employment is determined by labor *supply*. As shown in Panel (b) of Figure 1, these firms would prefer to pay a wage below  $\underline{W}$  and hire fewer workers, but because of the constraint, they choose a higher wage-employment pair. The binding constraint creates a wedge between the MRPL and the wage floor  $\underline{W}$  (the relevant MCL) at the equilibrium employment level.

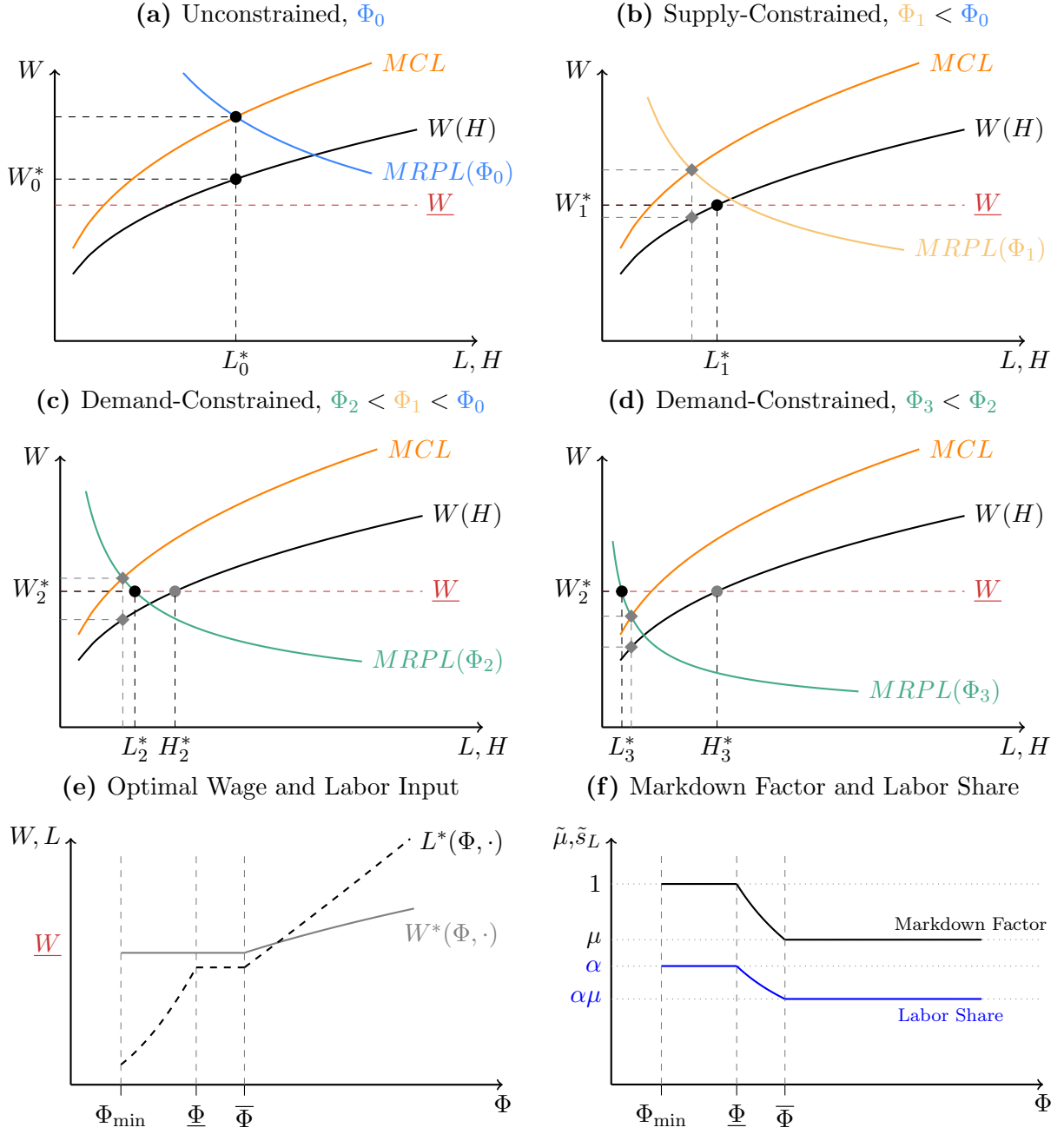
Second, *demand-constrained* firms choose employment  $L_j^*$  to equate the MRPL to the wage floor, i.e.,  $\Phi_j f_j'(L_j^*) = \underline{W}$ , so employment is determined by labor *demand*. Panels (c) and (d) of Figure 1 illustrate the equilibrium for firms that differ in the level at which the MRPL intersects with the wage floor  $\underline{W}$ , which determines employment in the latent unconstrained equilibrium. In Panel (c) the intersection occurs above  $\underline{W}$  and employment is larger than the latent unconstrained level, whereas Panel (d) shows the alternative case. While this distinction matters for the response of the labor market to the introduction of a wage floor (e.g., Ahlfeldt et al. 2023, Faia et al. 2026), we note that it does not affect firm responses to changes in productivity  $\Phi$ , which is the focus of our analysis.

Demand-constrained firms ration employment, i.e.,  $L_j^* < H_j(\psi_j \underline{W}, \mathbb{W}; A_j)$ . This is illustrated by the difference between  $L^*$  and  $H^*$  in Panels (c) and (d) of Figure 1. To determine the value of  $\psi_j$  we assume that workers correctly anticipate the firm's choice, and so the equilibrium hiring probability  $\psi_j^*$  will be implicitly determined by  $\psi_j^* = L_j^*/H_j(\psi_j^* \underline{W}, \mathbb{W}; A_j)$ .

**Threshold productivity levels.** The solution is characterized by two threshold productivity levels, denoted by  $\bar{\Phi}_j$  and  $\underline{\Phi}_j$ , where firms with  $\Phi_j > \bar{\Phi}_j$  are unconstrained,  $\Phi_j \in (\underline{\Phi}_j, \bar{\Phi}_j]$  are supply-constrained, and  $\Phi_j \leq \underline{\Phi}_j$  are demand-constrained. The upper threshold can be derived by equating the unconstrained wage to the wage floor, resulting in  $\bar{\Phi}_j = \underline{W}/[\mu f_j'(H(\underline{W}, \mathbb{W}; A_j))]$ . The lower threshold is given by the productivity that makes the demand-constrained employment equal to the supply-constrained employment  $H(\underline{W}, \mathbb{W}; A_j)$ , so that  $\underline{\Phi}_j = \underline{W}/[f_j'(H(\underline{W}, \mathbb{W}; A_j))]$ . Note that the thresholds depend on the wage floor  $\underline{W}$ , firm  $j$ 's production function  $f_j(\cdot)$ , and the amenity value  $A_j$ .

**Potential outcomes.** The theoretical framework delivers potential outcomes for firm  $j$ , depending on its productivity, its amenity value, the wage floor, and the aggregate wage index, which we denote by  $L_j^*(\Phi_j, A_j, \underline{W}, \mathbb{W})$  and  $W_j^*(\Phi_j, A_j, \underline{W}, \mathbb{W})$ . Panel (e) of Figure 1 plots the optimal wage  $W^*$  and employment  $L^*$  as productivity  $\Phi$  changes, holding constant

**Figure 1:** Solution to Firm Optimization Problem with Wage Floor Constraint.



Notes: The figure illustrates the equilibrium of the model of firm maximization subject to a labor supply and a wage floor constraint introduced in Section 2.1. Panel (a) shows an unconstrained firm for which the wage floor is not binding. Panel (b) shows a supply-constrained firm for which both constraints are binding. Panels (c) and (d) show demand-constrained firms for which only the wage floor is binding, and which differ in their latent monopsony equilibrium, indicated by gray-filled diamonds.  $W(H)$  stands for the inverse labor supply curve,  $MCL$  for the marginal cost of labor, and  $MRPL$  for the marginal revenue product of labor. Panel (e) shows the optimal wage (solid line) and labor input (dashed line) as functions of productivity  $\Phi$ , whereas Panel (f) shows the observed markdown factor  $\tilde{\mu}$  and labor share  $\tilde{s}_L$ .  $\Phi_{\min}$  indicates the minimum productivity,  $\mu = \eta/(\eta + 1)$  denotes the unconstrained markdown factor, and  $\alpha$  is the output elasticity of labor with a homogeneous production technology.

the remaining parameters. When  $\Phi_j > \bar{\Phi}_j$ , the firm is unconstrained, and both employment and wages increase with productivity. When  $\Phi_j \in (\underline{\Phi}_j, \bar{\Phi}_j]$ , the firm is supply-constrained, and thus employment and wages are determined by the wage floor and the labor supply curve. Finally, for  $\Phi_j \leq \underline{\Phi}_j$ , the firm is demand-constrained, so potential outcomes are determined by the intersection of the MRPL and the wage floor.

The wage-setting constraints also affect the observed markdown factor and labor share. Panel (f) of Figure 1 shows the potential outcomes for these objects, assuming a homogeneous production technology (Assumption 1(a) holds) so that the labor share of unconstrained firms is constant. We highlight two results. First, we observe a markdown factor below 1 for supply-constrained firms. Second, exposure to the wage floor increases the labor share.

#### 2.1.4 Beyond Pure Wage Floor Constraints

**Local bargaining constraint.** Motivated by settings with firm-level bargaining, where  $\underline{W}$  can be interpreted as an outside wage, or two-tier bargaining, where  $\underline{W}$  is a floor determined in sectoral negotiations, we consider the constraint  $g_j(\Upsilon_j, \underline{W}) = \kappa_j \Upsilon_j + (1 - \kappa_j) \underline{W}$ .<sup>15</sup> This constraint can be derived from a Nash bargaining problem in which the firm first commits to an employment level, and the local union then negotiates over the wage, treating employment as given.<sup>16</sup> We interpret  $\kappa_j \in (0, 1)$  as the bargaining power of the local union.

This setting yields unconstrained, supply-constrained, and demand-constrained firms as well. Panels (a), (b), and (c) of Appendix Figure A.1 illustrate the regimes, and Panel (d) shows the potential outcomes, under a homogeneous production function (Assumption 1(a)). See Appendix A.3 for derivations. We show that unconstrained firms only exist in this framework when the unconstrained markdown factor  $\mu$  is large relative to the parameter  $\kappa$ . Strong union power ( $\kappa \rightarrow 1$ ) or employer power ( $\mu \rightarrow 0$ ) result in all firms being constrained.

Our formulation of a local bargaining constraint delivers several novel insights. First, unlike stylized firm-union bargaining models, our model features unconstrained firms—high-productivity firms facing a relatively weak union—who find it optimal to set a wage strictly above  $g_j(\Upsilon_j, \underline{W})$  in order to expand. Second, supply-constrained firms adjust both wages and employment in response to changes in  $\Phi_j$  (see Panel (d) of Appendix Figure A.1). This framework thus captures insights from both monopsony and bargaining models. Third, as

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<sup>15</sup>In many settings, wage floors are determined in sectoral negotiations between employer associations and sectoral unions. For firm  $j$ , we assume the wage floor is unrelated to  $j$ 's productivity and only depends on sectoral factors. As firms may be covered by different agreements, we can allow the wage floors to vary across firms, i.e.,  $\underline{W}_j$ , without losing any of the insights developed here. In the empirical analysis, we test whether firm-level demand shocks affect firm-level average wage floors,  $\underline{W}_j$ , and find no such effects.

<sup>16</sup>More precisely, the Nash bargaining problem is  $\max_{\{W\}} [(W - \underline{W})L]^{\kappa_j} [\Phi f(L) - WL]^{1-\kappa_j}$ . See Holden (1988) and Moene (1988) for models with similar structure. These papers, however, do not incorporate a firm-level labor supply curve nor allow for unconstrained firms for whom the wage constraint does not bind.

shown in Appendix Figure A.2, the wage in demand-constrained firms is above the marginal revenue product of labor, where the “wage markup” depends on the union bargaining power  $\kappa_j$ . Notably, we also observe some supply-constrained firms with a wage above the MRPL.

**General wage-setting constraint.** Our discussion thus far assumed a constant labor supply elasticity  $\eta$ . Appendix A.4 discusses the solution to the problem with the general wage-setting constraint  $W_j \geq g_j(\Upsilon_j, \underline{W})$ . We compare the optimal firm wage  $W_j^*$  with the equilibrium level of the MRPL, generalizing the results from the local bargaining case. We note that we expect observed markdown factors above one for some firms, a fact that has been documented recently (Yeh et al. 2022, Chan et al. 2025).

**General equilibrium.** The general equilibrium is characterized by a wage index  $\mathbb{W}^*$  that equates aggregate labor supply to firms to the working-age population. Appendix A.5 defines the equilibrium and shows that it exists and is unique. The wage index  $\mathbb{W}^*$  depends on the wage floor  $\underline{W}$  and the productivities  $\Phi_j$ , underscoring the importance of accounting for general equilibrium effects in the empirical analyses later when we shift these quantities.

### 2.1.5 Comparative Statics and Testable Predictions

We now study the wage, employment, and value-added responses to a marginal productivity shock  $d \ln \Phi_j$  under the constrained and unconstrained regimes, for each type of wage-setting constraint. In our framework, the implications of a marginal productivity shock are identical to those of a marginal product demand shock.<sup>17</sup> We derive the ratio of employment to wage responses, typically used to measure labor supply elasticities (e.g., Sokolova and Sorensen 2021), and the ratio of wage to value-added responses, used as a measure of rent-sharing elasticities (e.g., Card et al. 2018). Note that, as we abstract from inputs, value added equals total revenue, i.e.,  $VA_j = \Phi_j f(L_j)$ . See Appendix A.6 for formal derivations.

We start with unconstrained firms since their solution does not depend on the particular form of wage-setting constraints. Our results depend on the curvature of the production function, which we summarize by the output elasticity of labor  $\alpha_j(L)$  and the absolute value of the employment elasticity of the marginal revenue product of labor  $\gamma_j(L) \equiv -L f''(L)/f'(L)$ .

**Result 1** (Response to Shocks among Unconstrained Firms). **(a)** *The ratio of the employment response to wage response to a shock is  $\frac{d \ln L_j / d \ln \Phi_j}{d \ln W_j / d \ln \Phi_j} = \eta$ .* **(b)** *The ratio of the wage response to value-added response to a shock is  $\frac{d \ln W_j / d \ln \Phi_j}{d \ln VA_j / d \ln \Phi_j} = \frac{1}{1 + \eta(\alpha_j(L_j^*) + \gamma_j(L_j^*))} \equiv \theta_j$ .*

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<sup>17</sup>Alternatively, one could allow for price heterogeneity and analogously consider changes in output prices.

We see that, for unconstrained firms, the ratio of employment to wage responses reveals the structural labor supply elasticity, which is assumed to be homogeneous across firms. The ratio of wage to value added responses, which we denote by  $\theta_j$ , depends on  $\eta$  and the sum of elasticities  $\alpha_j(L^*) + \gamma_j(L^*)$ , and is thus heterogeneous across firms. We refer to  $\theta_j$  as the “structural” rent-sharing elasticity, as it reveals the wage pass-through without constraints.<sup>18</sup> Interestingly, the rent-sharing elasticity is inversely related to  $\eta$ . As  $\eta \rightarrow \infty$ , the model converges to a perfectly competitive benchmark where workers are paid their marginal revenue product of labor and the rent-sharing elasticity is zero.

Imposing further assumptions on the production technology allows deriving sharper predictions for the rent-sharing elasticity. Under homogeneous production functions, as in Assumption 1(a), one has that  $\alpha_j(L) + \gamma_j(L) = 1$  implying  $\theta_j = 1/(1 + \eta)$  for all  $j$ . Under Assumption 1(b) one instead has  $\alpha_j(L) + \gamma_j(L) > 1$ , implying  $\theta_j < 1/(1 + \eta)$ .<sup>19</sup>

We now discuss the behavior of constrained firms, starting with the ratio of employment to wage responses to the shock.

**Result 2** (Employment to Wages). *Let the ratio of employment to wage responses be  $\tilde{\eta}_j = (d \ln L_j / d \ln \Phi_j) / (d \ln W_j / d \ln \Phi_j)$ . (a) Under a pure wage floor constraint,  $\tilde{\eta}_j$  is undefined for supply-constrained firms, and  $\tilde{\eta}_j \rightarrow \infty$  for demand-constrained firms. (b) Under a local bargaining constraint,  $\tilde{\eta}_j = \eta$  for supply-constrained firms and  $\tilde{\eta}_j = [\lambda_j(L)(\alpha_j(L) + \gamma_j(L) - 1)]^{-1}$  for demand-constrained firms, where  $\lambda_j(L) = \kappa_j \Upsilon_j / (\kappa_j \Upsilon_j + (1 - \kappa_j) \underline{W}) \in (0, 1)$ .*

The ratio of employment to wage responses may differ from the structural labor supply elasticity. In particular, for demand-constrained firms, we expect  $\tilde{\eta}_j$  to be large. To see this for the local bargaining constraint note that, under homogeneous production functions (Assumption 1(a)) we have  $\alpha_j(L) + \gamma_j(L) = 1$  and thus  $\tilde{\eta}_j$  diverges as well. More generally, with “weak curvature” ( $\alpha_j(L) + \gamma_j(L)$  close to 1), we expect a large  $\tilde{\eta}_j$  satisfying  $\tilde{\eta}_j > \eta$ .

Finally, we discuss the ratio of wage to value-added responses to shocks.

**Result 3** (Wages to Value Added). *Let the ratio of wage to value-added responses be  $\tilde{\theta}_j = (d \ln W_j / d \ln \Phi_j) / (d \ln VA_j / d \ln \Phi_j)$ . (a) Under a wage floor constraint,  $\tilde{\theta}_j = 0$  for constrained firms. (b) Under a local bargaining constraint,  $\tilde{\theta}_j = \lambda_j(L) / (1 + \eta \lambda_j(L))$  for supply-constrained firms and  $\tilde{\theta}_j = \lambda_j(L) [1 - (\alpha_j(L) + \gamma_j(L))^{-1}]$  for demand-constrained firms.*

<sup>18</sup>Note that we defined the rent-sharing elasticity as the elasticity of wages to value added. Alternatively, one can define it as the elasticity with respect to productivity  $\Phi$  or value added per worker  $VA/L$ .

<sup>19</sup>Let us show more formally how Assumption 1 relates to the elasticity sum  $\alpha_j(L) + \gamma_j(L)$ . First, for  $f_j(L) = L^\alpha$  we have  $\alpha_j(L) = \alpha$  and  $\gamma_j(L) = 1 - \alpha$ , thus  $\alpha_j(L) + \gamma_j(L) = 1$ . Second, we show that  $\alpha_j(L) + \gamma_j(L) > 1$  for  $\alpha'_j(L) < 0$ . Differentiating  $\alpha_j(L) \equiv L f'_j(L) / f_j(L)$  with respect to  $L$  yields  $\alpha'_j(L) = \frac{\alpha_j(L)}{L} [1 - (\alpha_j(L) + \gamma_j(L))]$ . Since  $\alpha_j(L) > 0$  and  $L > 0$ , the condition  $\alpha'_j(L) < 0$  implies  $\alpha_j(L) + \gamma_j(L) > 1$ .

The rent-sharing elasticity is zero under a pure wage floor constraint. Under local bargaining we also expect low rent-sharing elasticities (as  $\lambda_j(L) \in (0, 1)$  and, under Cobb-Douglas,  $\alpha_j(L) + \gamma_j(L) = 1$ ). Our model generates positive rent-sharing for supply-constrained firms even under homogeneous production functions, a result that has eluded previous literature.<sup>20</sup> Note that, whereas wage-setting constraints reduce the observed rent-sharing *elasticity*, they actually increase the observed labor share or “rent-sharing *level*”, as shown in Panel (f) of Figure 1 for the wage floor case and Appendix Figure A.2 for the local bargaining case.

**Heterogeneous labor supply elasticities.** Our derivations based on Equation (1) maintain the assumption of a homogeneous labor supply elasticity  $\eta$ . This assumption, however, is not necessary to obtain our key predictions. We can instead allow for a firm-specific labor supply elasticity  $\eta_j$ , and the Results 1–3 above would hold re-interpreting  $\eta$  as a firm-specific parameter.<sup>21</sup> Similarly, our framework can accommodate multiple worker types with iso-elastic labor supply elasticities for each type.<sup>22</sup> We assume, however, that such firm- or type-specific labor supply elasticities are invariant to the demand shock. This rules out, for example, non-iso-elastic preference structures in which the elasticity changes with the wage (e.g., Kline 2025) or the firm’s workforce composition (e.g., Volpe 2024). While our derivations do not apply directly to such cases, we expect similar effects of wage-setting constraints, i.e., the observed responsiveness to shocks would depend on the bite of the constraints.

**Testable implications.** Given these comparative statics, consider studying the average responses across firms that are plausibly constrained and those that are plausibly unconstrained, as we do in our empirical analysis. First, consider the case of a wage floor constraint. For plausibly constrained firms, we expect to have small wage responses and positive employment responses. We thus expect to find larger estimates of labor supply elasticities and smaller estimates of rent-sharing elasticities than among plausibly unconstrained firms. Second, consider firms that face strong unions, captured by a high local union density. Firms with strong unions are more likely to be demand-constrained, which means that we also ex-

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<sup>20</sup> Prior work has noticed that, under a homogeneous production function and no labor supply constraint, the equilibrium level of the average revenue product of labor is constant, which results in a constant wage (see, e.g., McDonald and Solow 1981, p. 899, Manning 1987, and Abowd and Lemieux 1993, p. 987). This holds in our model for demand-constrained firms only, i.e., when the labor supply constraint does not bind.

<sup>21</sup>Under the micro-foundation of Appendix A.1, a reason for differences in labor supply elasticities may be differences in dispersion of idiosyncratic worker preferences across sectors (e.g., Lamadon et al. 2022).

<sup>22</sup>For instance, we can interpret  $j$  as the combination of firm and occupation. Then, a firm-occupation cell may be unconstrained, supply-constrained, or demand-constrained. Such a model extension would need to consider that there may be spillovers across occupations within the firm (Adamopoulou et al. forthcoming).

pect these firms to exhibit a higher ratio of employment to wage responses.<sup>23</sup>

## 2.2 Identification

We consider the identification of labor supply and rent-sharing elasticities using wage, employment, and value added data before ( $t = 0$ ) and after ( $t = 1$ ) a demand shock. For convenience, we denote our data using first differences, denoted  $\Delta \ln X = \ln X_1 - \ln X_0$  for some  $X$ . We assume that the demand shock is simply a binary revenue shifter  $Z_j$ , so the data are  $\{\Delta \ln L_j, \Delta \ln W_j, \Delta \ln VA_j, Z_j\}_{j \in \mathcal{J}}$ . Later, we use  $Z_j$  as an instrument in our identification discussion. We focus on the case of wage floor constraints, however, similar results would hold for other types of constraints allowed by our framework.

**Target parameters.** We define two parameters that we believe are of interest to researchers and policy-makers. We define these parameters in terms of counterfactual outcomes without constraints,  $\{L_{jt}^*(\Phi, \underline{W} = 0), W_{jt}^*(\Phi, \underline{W} = 0)\}_{j \in \mathcal{J}}$  for  $t \in \{0, 1\}$ . Then, we denote our counterfactual data as  $\{\Delta \ln W_j^*, \Delta \ln L_j^*, \Delta \ln VA_j^*, Z_j\}_{j \in \mathcal{J}}$ .

**Definition 1** (Target Parameters). *The target parameters are*

$$\bar{\eta} = \frac{\mathbb{E} [\Delta \ln L_j^* | Z_j = 1] - \mathbb{E} [\Delta \ln L_j^* | Z_j = 0]}{\mathbb{E} [\Delta \ln W_j^* | Z_j = 1] - \mathbb{E} [\Delta \ln W_j^* | Z_j = 0]}$$

and

$$\bar{\theta} = \frac{\mathbb{E} [\Delta \ln W_j^* | Z_j = 1] - \mathbb{E} [\Delta \ln W_j^* | Z_j = 0]}{\mathbb{E} [\Delta \ln VA_j^* | Z_j = 1] - \mathbb{E} [\Delta \ln VA_j^* | Z_j = 0]}.$$

The parameter  $\bar{\eta}$  indicates the average firm-level responsiveness of employment to wage changes in a hypothetical economy without wage constraints. This parameter reveals the average potential monopsony power that firms hold and is informative, for example, of the extent to which minimum wage policies may result in employment losses. Similarly,  $\bar{\theta}$  indicates the average firm-level responsiveness of wages to value-added changes. This parameter, for example, informs how government policies that influence a firm's demand are transmitted to workers' wages in a setting without wage constraints.

**Connection of target parameters with theoretical model.** We assume that the shock  $Z_j$  affects the firm productivity  $\Phi_j$ , and denote the (unobserved) vector of productivity changes as  $\{\Delta \ln \Phi_j\}_{j \in \mathcal{J}}$ . The following lemma establishes the relationship between the target parameters defined in Definition 1 and the model's structural parameters.

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<sup>23</sup>To see this, note that a larger  $\kappa_j$  shifts the lower threshold  $\underline{\Phi}_j$  to the right in Panel (d) of Appendix Figure A.1, increasing the range of productivities  $\Phi_j$  that are demand-constrained (see Appendix A.3).

**Lemma 1** (Identification of Structural Parameters). *The target parameter  $\bar{\eta}$  identifies the structural labor supply elasticity, i.e.,  $\bar{\eta} = \eta$ . The target parameter  $\bar{\theta}$  identifies a weighted average of the firm-specific rent-sharing elasticities  $\theta(\alpha_j, \gamma_j)$ :*

$$\bar{\theta} = \mathbb{E}[\omega(\alpha_j, \gamma_j) \theta(\alpha_j, \gamma_j)],$$

where  $(\alpha_j, \gamma_j)$  summarize firm  $j$ 's technology,  $\theta(\alpha_j, \gamma_j) = (1 + \eta[\alpha_j + \gamma_j])^{-1}$  is the structural rent-sharing elasticity, and the weight  $\omega(\alpha_j, \gamma_j) = \delta(\alpha_j, \gamma_j) \pi(\alpha_j, \gamma_j) / \mathbb{E}[\delta(\alpha_j, \gamma_j) \pi(\alpha_j, \gamma_j)]$  is proportional to  $\delta(\alpha_j, \gamma_j) = (1 + \eta[\alpha_j + \gamma_j]) / (1 + \eta\gamma_j)$ , which is the coefficient on  $\Delta \ln \Phi_j$  in the unconstrained value-added response, and  $\pi(\alpha_j, \gamma_j) = \mathbb{E}[\Delta \ln \Phi_j | Z_j = 1, \alpha_j, \gamma_j] - \mathbb{E}[\Delta \ln \Phi_j | Z_j = 0, \alpha_j, \gamma_j]$ , which is the technology-specific first stage. Under Assumption 1(a),  $\theta(\alpha_j, \gamma_j)$  is constant across firms and  $\bar{\theta} = \frac{1}{1+\eta}$ .

Proofs are available in Appendix B.

The result that  $\bar{\eta} = \eta$  is general and holds regardless of the production function form. However, to obtain a constant rent-sharing elasticity, we need a stronger assumption on the production technology (as in Kline 2025, Section 6.1). For simplicity, in what follows we use  $\eta$  and  $\theta$  to denote the target parameters, with the caveat that the interpretation of the rent-sharing elasticity parameter depends on the assumptions on the production technologies.

**Conventional estimands.** Conventional estimands of these elasticities divide the ratio of outcome responses to the revenue shock:

$$\eta^{CE} = \frac{\mathbb{E}[\Delta \ln L_j | Z_j = 1] - \mathbb{E}[\Delta \ln L_j | Z_j = 0]}{\mathbb{E}[\Delta \ln W_j | Z_j = 1] - \mathbb{E}[\Delta \ln W_j | Z_j = 0]},$$

$$\theta^{CE} = \frac{\mathbb{E}[\Delta \ln W_j | Z_j = 1] - \mathbb{E}[\Delta \ln W_j | Z_j = 0]}{\mathbb{E}[\Delta \ln VA_j | Z_j = 1] - \mathbb{E}[\Delta \ln VA_j | Z_j = 0]}.$$

These estimands can be motivated by 2SLS systems. For the labor supply elasticity, we treat  $Z_j$  as an instrument for wages in a regression of employment changes on wage changes. Similarly, for the rent-sharing elasticity, we treat  $Z_j$  as an instrument for value-added in a regression of wage changes on value-added changes. Our goal is to clarify the behavior of these estimands in a world with wage-setting constraints.

**Transitions across constrained states.** As the productivity shock is discrete, we may observe firms changing their constrained status in the theoretical model. It will be useful to introduce notation to reflect those cases. We let  $\rho_{xy} \in [0, 1]$  denote the share of firms that had status  $x$  at  $t = 0$  and  $y$  at  $t = 1$ . For instance, starting from being unconstrained

( $u$ ) in  $t = 0$ ,  $\rho_{uu}$  denotes the share of firms that continue being unconstrained,  $\rho_{us}$  the share that move to being supply-constrained ( $s$ ) in  $t = 1$ , and  $\rho_{ud}$  the share that change to demand-constrained ( $d$ ). By definition,  $\sum_{x \in \{u,s,d\}} \sum_{y \in \{u,s,d\}} \rho_{xy} = 1$ .

### 2.2.1 Absence of Wage-Setting Constraints

We study firm responses allowing for shocks to both productivity,  $\Delta \ln \Phi_j$ , and labor supply,  $\Delta \ln A_j$ . We formalize the absence of binding wage-setting constraints as follows.

**Assumption 2** (Absence of Wage-Setting Constraints). **(a)** *All firms in the economy are unconstrained, before and after the shock. Formally,  $\rho_{uu} = 1$ .* **(b)** *There are no demand-constrained firms in the economy. Formally,  $\rho_{du} + \rho_{ds} + \rho_{dd} + \rho_{ud} + \rho_{sd} = 0$ .*

Assumption 2(a) is stronger in the sense that it imposes that all firms in the economy are unconstrained. Assumption 2(b) is weaker as it also allows for the presence of supply-constrained firms, which is consistent with moderate wage floor levels.<sup>24</sup> This distinction is useful since we can allow for supply-constrained firms in some of our identification results.

The presence of labor supply shifters creates a well-known simultaneity problem: because equilibrium wages and employment are determined by these shifters, naive regressions of employment changes on wage changes, or of wage changes on value-added changes, will yield biased estimates of labor supply and rent-sharing elasticities. To address this, we introduce the key assumptions required for identification using the demand shock  $Z_j$ .

**Assumption 3** (Relevance). *The shock  $Z_j$  is correlated with changes in productivity  $\Phi_j$ , i.e.,  $\mathbb{E}[\Delta \ln \Phi_j | Z_j = 1] \neq \mathbb{E}[\Delta \ln \Phi_j | Z_j = 0]$ .*

**Assumption 4** (Exogeneity). *The shock  $Z_j$  is independent of changes in the labor supply shifters  $A_j$ , i.e.,  $Z_j \perp \Delta \ln A_j$ .*

We require the shock  $Z_j$  to predict changes in productivity while remaining unrelated to changes in workers' preferences. We can then establish the following result.

**Proposition 1** (Identification without Wage-Setting Constraints). *Consider an instrument  $Z_j$  that satisfies Assumptions 3 and 4. (i) Under Assumption 2(b) (no demand-constrained firms),  $\eta^{CE}$  identifies  $\eta$ . (ii) Under Assumption 2(a) (no constrained firms)  $\eta^{CE}$  identifies  $\eta$  and  $\theta^{CE}$  identifies  $\theta$ .*

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<sup>24</sup>More precisely, Assumption 2(b) allows for a wage floor level  $\underline{W}$  such that  $\Phi_j < \bar{\Phi}_j$  for some  $j \in \mathcal{J}$  but  $\Phi_j > \underline{\Phi}_j$  for all  $j \in \mathcal{J}$ , both before and after the shock. Notably, Assumption 2(a) implies Assumption 2(b).

Proposition 1 establishes a crucial benchmark for our analysis. It confirms that the conventional approach recovers the structural labor supply elasticity and a weighted average of rent-sharing elasticities. A failure of the conventional approach to identify  $\eta$  and  $\theta$  must therefore arise from the presence of the wage-setting constraints, as we discuss next.

### 2.2.2 Binding Wage-Setting Constraints

We now turn to the case where wage-setting constraints are present. Two main differences from the previous case arise. First, the responses of demand-constrained firms to the shock will not reflect movements along the labor supply curve. Second, the wage responses for constrained firms will be muted, as they are bound by the wage floor. We formalize two assumptions on the presence of constrained firms.

**Assumption -2** (Presence of Wage-Setting Constraints). *(a) (Weak Constraints) There are constrained firms in the economy. Formally,  $\rho_{uu} < 1$ . (b) (Strong Constraints) There are demand-constrained firms in the economy. Formally,  $\rho_{du} + \rho_{ds} + \rho_{dd} + \rho_{ud} + \rho_{sd} > 0$ .*

Assumption -2 defines the regimes where wage-setting constraints are binding, indicating the negation of Assumption 2. We can now establish the following identification result.

**Proposition 2** (Identification Failure with Wage-Setting Constraints). *Let Assumptions 3 and 4 hold. Then: (i) Under Assumption -2(b) (strong constraints), the conventional estimator  $\eta^{CE}$  does not identify  $\eta$ ; (ii) Under Assumption -2(a) (weak constraints), the conventional estimator  $\theta^{CE}$  does not identify  $\theta$ .*

This result shows that, even in the presence of the usual relevance and exogeneity assumptions, identification fails when constraints are present. Consider the labor supply case to develop some intuition for the result. The reason for the failure is that demand-constrained firms respond with employment but not wages, and thus the numerator in  $\eta^{CE}$  is “too large”. We can see this as an *exclusion restriction failure*. The employment response of demand-constrained firms is not driven by changes in the wage, but rather the demand shock directly affects the employment choice of the firm as it takes the wage as given.<sup>25</sup>

Next, we derive an explicit expression for the bias of the conventional estimand of the labor supply elasticity. We focus on the labor supply elasticity as it is directly related to the wage markdown, which is the most commonly used measure of labor market power (see, e.g., Yeh et al. 2022, Berger et al. 2022, Lamadon et al. 2022, Azar and Marinescu 2024).

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<sup>25</sup>Some studies focus instead on estimating the *inverse* labor supply elasticity by regressing wages on employment and instrumenting employment with a demand shifter (see, e.g., Amodio et al. 2025). This approach faces a similar identification challenge: as the wage is determined by the wage constraint, the estimate of the inverse labor supply elasticity will be “too small”. Formally, the identification challenge can be re-stated as a censoring problem—the potential unconstrained wage is not always observed.

**Corollary 1** (Bias Decomposition Formula). *Let Assumptions 1(a), 3, and 4 hold for all firm types. Furthermore, assume no transition across groups as a result of the shock,  $\rho_{uu} + \rho_{ss} + \rho_{dd} = 1$ . Denote the  $uu$ -,  $ss$ -, and  $dd$ -specific average changes in productivity  $\ln \Phi$  between treated ( $Z_j = 1$ ) and control ( $Z_j = 0$ ) firms by  $\zeta_{uu}$ ,  $\zeta_{ss}$ , and  $\zeta_{dd}$ , respectively, and further assume that  $\text{sgn}(\zeta_{uu}) = \text{sgn}(\zeta_{ss}) = \text{sgn}(\zeta_{dd})$ . Then,*

$$\eta^{CE} - \eta = \frac{\rho_{dd}}{\rho_{uu}} \left( \frac{1 + \eta(1 - \alpha)}{1 - \alpha} \right) \frac{\zeta_{dd}}{\zeta_{uu}} \geq 0. \quad (4)$$

This formula shows more explicitly that the conventional estimand  $\eta^{CE}$  may be upward-biased, where the bias vanishes in the absence of demand-constrained firms. More complicated formulas can be derived for cases with transitions, though with a common theme: demand-constrained firms before or after the shock will respond proportionally more via employment relative to wages, resulting in “excess” employment responses.

Corollary 1 provides insights into the drivers of the bias. First, the bias is increasing in the share  $\rho_{dd}$ , which is expected as demand-constrained firms respond to shocks with employment and not wages. Second, the bias increases in  $\eta$ . The reason is that a large  $\eta$  makes the wage of unconstrained firms less responsive relative to employment. Hence, the average employment response across all firms is divided by a smaller wage response, leading to a larger estimate. Finally, the bias is increasing in the ratio  $\zeta_{dd}/\zeta_{uu}$ . If the shock is stronger for demand-constrained firms, we will see more employment responses but still no wage responses, resulting once again in a larger estimate.

We can also use Corollary 1 to derive an expression for the share of demand-constrained firms as a function of  $\eta^{CE}$  and  $\eta$ . Assuming  $\zeta_{dd}/\zeta_{uu} = 1$  and solving for  $\rho_{dd}$  in Equation (4),

$$\rho_{dd} = \rho_{uu}(\eta^{CE} - \eta) \left( \frac{1 - \alpha}{1 + \eta(1 - \alpha)} \right). \quad (5)$$

Using this equation we can plug in values for  $\eta^{CE}$ ,  $\eta$ , and  $\alpha$  to quantify the presence of demand-constrained firms, given the share of unconstrained firms  $\rho_{uu}$ . Appendix C derives an analogous corollary and formula for the case of local bargaining constraints.

### 2.2.3 Heterogeneity Analysis

Our discussion implies that observed responses to demand shocks will systematically differ based on whether firms are affected by wage-setting constraints. In our empirical analysis, we will split the sample into plausibly constrained and plausibly unconstrained firms and estimate labor supply and rent-sharing elasticities separately, exploring these predictions.

## 2.3 Exploiting Variation in Exposure to Wage-Setting Constraints

While our framework suggests that firms’ exposure to wage-setting constraints affects their responses to demand shocks, other differences across firms may also lead to heterogeneous shock responses. For instance, a model with heterogeneous labor supply elasticities  $\eta_j$  that are negatively correlated with productivity  $\Phi_j$  would also result in the estimator  $\eta^{\text{CE}}$  being larger for constrained firms.<sup>26</sup> Such underlying differences in firm-level labor supply elasticities could reflect, e.g., differences in workers’ outside options (e.g., [Kline 2025](#)). We now propose a strategy to causally test for the role of constraints in affecting responses to shocks that allows us to relax the assumption of homogeneous  $\eta$  that was implicit in [Section 2.1](#).

Our identification strategy relies on within-firm variation in exposure to constraints to compare the responsiveness of firms for which the constraint tightens relative to similar firms without changes in the constraints. Specifically, we rely on two sources of variation occurring at time  $t^*$ : a tightening in the wage floor affecting the constrained status of some firms and an orthogonal demand shock  $Z_j$ . Denote by  $\underline{W}^{\text{pre}}$  and  $\underline{W}^{\text{post}}$  the wage floors before and after  $t^*$ , where  $\underline{W}^{\text{post}} > \underline{W}^{\text{pre}}$ . Relying on pre-event firm wages  $W_{j,t < t^*}$ , we define for this exercise three groups of firms. We denote by “CC” firms that are constrained both at the old and new wage floors ( $W_{j,t < t^*} < \underline{W}^{\text{pre}} < \underline{W}^{\text{post}}$ ). Similarly, “UC” firms are not constrained but would be under the new floor ( $\underline{W}^{\text{pre}} < W_{j,t < t^*} < \underline{W}^{\text{post}}$ ) and “UU” are firms “just” unconstrained at both wage floor levels ( $\underline{W}^{\text{pre}} < \underline{W}^{\text{post}} < W_{j,t < t^*} < (1 + c)\underline{W}^{\text{post}}$  for a small  $c$ ).

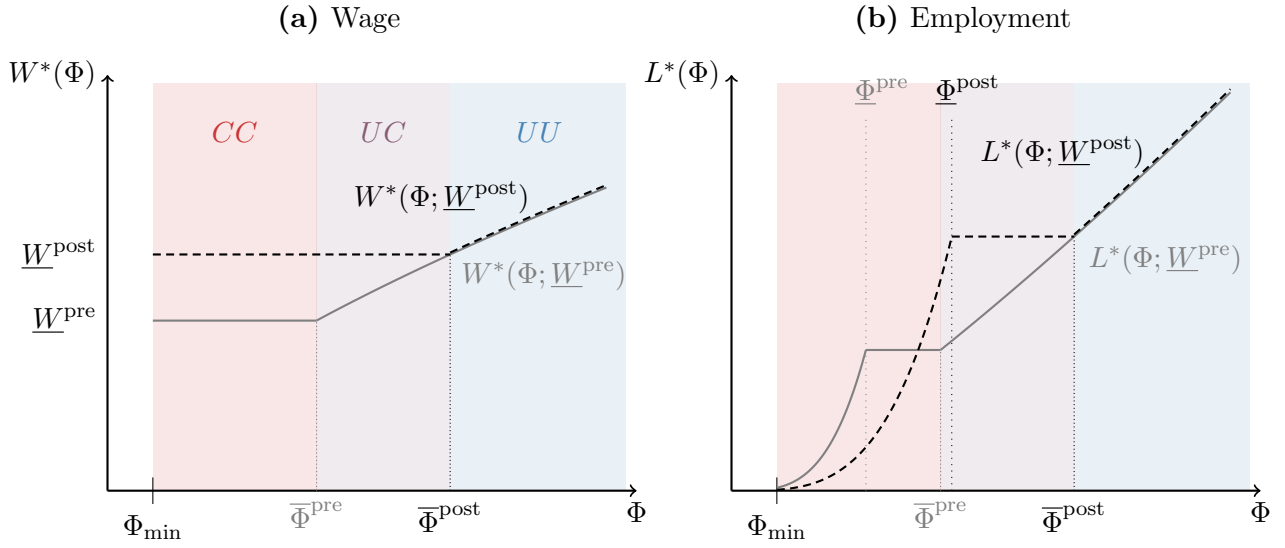
Before presenting this new identification strategy, let us clarify the predictions of the model with variation in both wage floors and demand shocks in the case of homogeneous  $\eta$ . Start with wages. Panel (a) of [Figure 2](#) illustrates the potential outcomes, which now depend on both firm productivity  $\Phi$  and the wage floor  $\underline{W}$ .<sup>27</sup> Productivity  $\Phi$  will not change on average for firms that do not receive a shock ( $Z_j = 0$ ), and thus wages will change only for constrained firms due to the wage floor hike. Productivity  $\Phi$  will increase for firms that receive a shock ( $Z_j = 1$ ), and thus observed wage changes will reflect the demand shock

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<sup>26</sup>Another possibility is that there exist differences in product market power, which correlate with productivity (e.g., [De Loecker et al. 2020](#), [Vera 2022](#)) and thus the bite of wage-setting constraints. If so, firms with greater product-market power would exhibit lower rent-sharing elasticities, as they restrict output expansion to maintain high prices, thereby dampening increases in employment and wages. This implies that observed differences in rent-sharing elasticities across constrained and unconstrained firms would be an under-estimate of the true role of wage-setting constraints. Notably, however, our labor supply elasticity estimates would not be affected, as unconstrained firms still operate on the labor supply curve. Yet another possibility is that firms face different labor adjustment costs (e.g., [Dhyne et al. 2025](#), [Chan et al. 2025](#)), which limit the extent to which firms can flexibly adjust labor inputs. Notably, if firms facing wage-setting constraints also face higher labor adjustment costs, then the latter is expected to counteract the role of the former, which may again yield an under-estimate of the true role of wage-setting constraints.

<sup>27</sup>For simplicity, we hold  $\mathbb{W}$  constant. In practice, we would expect a wage floor hike to affect  $\mathbb{W}$ . However, as this effect is common to all firms it will be netted out when differencing  $Z_j = 1$  and  $Z_j = 0$  firms.

**Figure 2:** Potential Outcomes Following an Increase in the Wage Floor.



Notes: The figure illustrates the potential outcomes following an increase in the wage floor from  $\underline{W}^{\text{pre}}$  to  $\underline{W}^{\text{post}}$  at time  $t^*$  for the model with a wage floor constraint introduced in Section 2.1. Panel (a) shows the potential outcomes for the wage. Panel (b) shows the potential outcomes for employment. Three types of firms are highlighted in both panels, which are defined by comparing the firm’s pre-shock wage to the wage floors  $\underline{W}^{\text{pre}}$  and  $\underline{W}^{\text{post}}$ : those constrained at both wage floors (CC), those unconstrained before but constrained after the increase (UC), and those unconstrained at either wage floor (UU).

and, for constrained firms, the wage floor hike. By comparing firms with  $Z_j = 0$  and  $Z_j = 1$  within each group (CC, UC, UU), we can evaluate the group-specific responses to the demand shock controlling for the wage-floor hike. Appendix Figure F.1 illustrates this procedure in an event study design. The left column in Panels (a) and (b) shows the evolution of wages for firms with  $Z_j = 0$  and  $Z_j = 1$ , respectively, whereas Panel (c) shows their difference. Similarly, we can construct an estimate of the employment response to the shock for each group. Panel (b) of Figure 2 shows the potential outcomes for employment and the right column in Appendix Figure F.1 illustrates these in an event study design.

Let us now describe our new strategy comparing firms for which constraints tightened (UC) to firms that remained “just” unconstrained (UU), again starting with wage responses. We only assume homogeneous  $\eta$  between these groups. Firms that are always unconstrained respond the most, as the full demand shock is passed through to wages. Firms that are unconstrained at  $\underline{W}^{\text{pre}}$  but constrained at  $\underline{W}^{\text{post}}$  respond less, as the increase in the wage floor increases the wage of control firms ( $Z_j = 0$ ) in the post-period. The assumption on  $\eta$  implies that UU firms reveal the counterfactual response of UC firms had they not received the constraint, thus the differential response between UU and UC firms is the causal effect of the constraint on the wage responsiveness to shocks. Then, if the demand shock is similarly

strong across groups, we expect to find a smaller rent-sharing elasticity among UC firms.<sup>28</sup>

Employment responses depend on the supply- and demand-constrained status of different firms. Note that, while all firms in the UC group are unconstrained before the wage floor hike, there will be a mix of supply- and demand-constrained firms after the hike ( $\underline{\Phi}^{\text{post}} > \overline{\Phi}^{\text{pre}}$  is possible, as illustrated in Panel (b) of Figure 2). We might then find strong employment responsiveness in the UC group.<sup>29</sup> Given this ambiguity, we let the data inform us about the strength of employment responses across groups. To the extent that employment responses are not too different, we would expect the labor supply elasticity obtained from UC responses to be larger than the one obtained from UU responses.

The interpretation that this experiment reveals the causal effect of wage-setting constraints relies on two variations on assumptions we used before—homogeneity and exogeneity. First, and most importantly, we now weaken the assumption of homogeneous labor supply elasticity. Instead, we only require that the firms being compared face the same elasticity, which ensures that the UU group is a good counterfactual for the UC group. To discuss the second assumption, note that this design is tantamount to a triple-differences; in particular, the difference-in-differences compares shocked firms within each group over time, while the triple difference compares the groups. As is well-known in triple-differences designs, we can allow for a weaker exogeneity assumption requiring only that the relative expected  $\Delta \ln A_j$  between  $Z_j = 1$  and  $Z_j = 0$  firms is equal between UU and UC firms (Olden and Møen 2022). That is, we can allow the treated and control firms to be trending differently, though such “differential trends” should be similar across UU and UC firms. While it is important to allow for weaker assumptions, we find no evidence of differential pre-trends in any group, which suggests that the shocks we use are indeed orthogonal to changes in supply shifters.

To summarize, we discussed how to exploit variation in exposure to constraints to test the main mechanism behind the model in Section 2.1. The idea is to compare the responsiveness to shocks of firms that experience an increase in the wage floor (UC) to firms that do not (UU) in two steps. First, we compare treated ( $Z_j = 1$ ) and control ( $Z_j = 0$ ) firms *within* each group, which allows us to difference out the effect of the wage floor increase in UC firms. Second, we compare the responsiveness to shocks among UC firms relative to UU firms, which represent the counterfactual responsiveness of UC firms had they not received the constraint. This approach relies on weaker assumptions than a heterogeneity analysis across constrained and unconstrained firms, and thus provides a stronger test of our model.

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<sup>28</sup>One could also compare the wage paths of UC and CC firms, maintaining the assumption of homogeneous  $\eta$  between them. In this case, we expect larger wage responses in the UC group.

<sup>29</sup>Appendix Figure F.1 illustrates an event study design with an example where we assume that the employment response for UU firms is in between the responses of CC and UC firms.

## 3 Institutional Settings and Data

### 3.1 Institutional Settings

We study the following three countries that differ substantially in their wage-setting institutions: Portugal, Norway, and Colombia. Portugal features a national minimum wage (MW) combined with a widespread coverage of collective bargaining agreements (CBAs). Norway does not have a national MW, while around half of the private sector workers are covered by CBAs. Colombia has a national MW, but almost nonexistent CBA coverage. Employers and local unions engage in firm-level bargaining in Norway, while the role of labor unions is limited in Portugal and absent in Colombia. Together, the three countries provide notable examples of institutional settings prevalent across the globe (OECD and AIAS 2023).

**Portugal.** Portugal has a long-standing system of sectoral bargaining and a binding MW. Although the labor union membership rate is only around 10%, CBA coverage reached roughly 90% of workers between 2010 and 2013 (Addison et al. 2023). Wage floors are negotiated between employer associations and sectoral unions, and automatically extended to non-covered firms. The national MW sets a floor for negotiated CBA wage floors, whereas workers' wages are typically set around 20% to 25% above CBA wage floors (Card and Cardoso 2022). In practice, negotiated wage floors are set close to the MW in many industries; see Appendix Figure F.4. The MW has become increasingly binding over time: the share of minimum wage workers rose from 8% in 2006 to 24% in 2017 (see Appendix Figure F.2).

**Norway.** In Norway, approximately half of private-sector workers are covered by CBAs, and, unlike Portugal, there is no national MW policy. The system is two-tiered, featuring centralized sectoral negotiations that determine CBA wage floors and general wage increments, followed by firm-level bargaining between employers and local unions (Bhuller et al. 2022). The sectoral bargaining agreements are not automatically extended to all firms, thus both covered and non-covered firms are present within narrowly defined industries. However, since 2004, the national labor board has allowed for limited extensions of collective agreements to non-covered firms, reaching around 10% of private-sector workers in 2023.

**Colombia.** In Colombia the national MW is highly binding, whereas the role of collective bargaining and unionization is limited. In 2023, the MW was equivalent to approximately 90% of the average wage, being the highest ratio among OECD countries (OECD 2025), compared to around 60% for Portugal or 20% for the United States. High levels of informality

accompany the comparatively high MW in the formal sector: around half of the workforce is employed in the informal sector (Delgado-Prieto 2024), where the MW is not enforced.

**Globe.** The wage-setting practices prevalent in these countries represent institutional settings that are widespread across the globe. As shown in Appendix Figure F.3, most OECD and non-OECD countries in the ICTWSS database (OECD and AIAS 2023) can be categorized into one of the three wage-setting systems represented by the countries we consider. Despite such broad similarities, the ICTWSS database includes countries that differ along other dimensions, importantly, their level of economic development. Although we emphasize that the examples of wage-setting constraints that we consider are prevalent around the globe, we do not argue that our evidence is directly applicable to other contexts.

## 3.2 Data Sources

We exploit multiple administrative and survey datasets. In Portugal and Norway, we link the matched employee-employer datasets with firms’ balance sheets and trade data. In Colombia, we rely on a longitudinal survey of manufacturing firms. We briefly discuss the data sources here, with Appendix D providing further details on the data and sample restrictions.

**Portugal.** The *Quadros de Pessoal* (QP) is a matched employer-employee dataset that captures information on all private-sector employers in Portugal each October. We link the QP with the firm’s balance sheet data to obtain information on value added for operating firms using the *Sistema de Contas Integradas das Empresas* (SCIE) dataset. Next, we link it to the disaggregated annual imports and exports data at the 6-digit product code (HS6) and destination country to obtain information on international trade exposure. The resulting dataset covers the years 2004 through 2017.

**Norway.** We combine several sources of administrative data. First, we use the population-wide matched employer-employee registers (Amelding/ATMLTO), which cover all employers, both in the private and public sectors. Second, we link the employer-employee registers to firms’ balance sheet data. Third, we use firm-level data on CBA coverage and labor union density.<sup>30</sup> Fourth, we link the matched employer-employee data to customs data to obtain information on international trade, containing exported values at the 8-digit HS product code and destination country level. We study private sector firms from 1997 through 2019.

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<sup>30</sup>We define a firm as being covered by a CBA if the firm or one of its establishments has adopted a sectoral CBA, which was negotiated between a union confederation and an employer association, or if any of the workers in the firm were covered by a policy-based extension of wage floors in sectoral CBAs. We define labor union density as the share of a firm’s workers that are formally members of a labor union.

**Colombia.** The *Encuesta Anual Manufacturera* (EAM) is a longitudinal survey providing annual information on firms in Colombia’s manufacturing sector. The survey covers medium to large firms, typically above 10 employees.<sup>31</sup> Firms report information on employment (both temporary and permanent), labor costs, gross production, intermediate consumption, value added, among other items. Importantly, unlike in Portugal and Norway, we cannot track workers’ information over time. Our analysis uses EAM data from 2000 to 2017.

**Key variables.** In Portugal and Norway, we define the firm wage as the mean hourly wage of “stayers”, i.e., workers that remain in the firm before and after the shock, and we define employment as the count of wage workers. We also provide evidence for all incumbent workers, including “movers”. Due to differences in the data sources, in Colombia we use the mean wage of permanent workers and count the number of permanent workers. For the three countries, we calculate value added as total revenues minus total costs of materials and services purchased from other firms, and it includes profits, labor costs and taxes.

**International trade.** We use publicly available bilateral trade flow data from the BACI-CEPII dataset to measure import demand worldwide (Gaulier and Zignago 2010). This provides annual traded values between country pairs using the HS6 classification of products.

### 3.3 Measuring Demand Shocks

We construct firm-level measures of demand shocks. We use two demand shocks at the firm level: an internal instrument, capturing large changes in firm value added, and an external instrument, using changes in export demand that are externally determined.

#### 3.3.1 Internal Shock

We implement the internal shock design proposed by Lamadon et al. (2022), which is commonly used in the literature. The intuition is to exploit transitory shocks to firm-level value-added growth that permanently shift the level of value added, while being orthogonal to past and future value-added trends beyond a short transitory window (typically two years). Assuming that firm-level value added can be approximated by a time-series process with permanent and transitory components, and that shocks to the transitory component are independent of firm-level labor-supply shifts, we can exploit this variation as an exogenous

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<sup>31</sup>The survey is required by law for establishments with 10 or more employees, or those that meet an annual inflation-adjusted production-value threshold.

change in labor demand. While the internal shock is not without limitations, it provides a reliable source of firm-level variation in labor demand across the three countries, and captures variations in labor demand across the distribution of firms in each country.

We define an internal demand shock indicator  $Z_j^{\text{Int}}$  equal to one for firms that experience a value-added growth exceeding the median within their 2-digit industry in each country in the same year, i.e., the assignment is within the same “cohort” of firms. This allows us to compare firms that experience positive demand shocks with those that do not within the same time window. Our main findings are robust to alternative shock definitions, such as defining value-added growth within groups of firms depending on whether they face wage constraints or when computing the median weighted by firm employment size.

### 3.3.2 External Shock

We implement the export shock design pioneered by [Hummels et al. \(2014\)](#) and adopted by several recent papers (e.g., [Berman et al. 2015](#), [Garin and Silvério 2024](#), [Hermo 2025](#)). Specifically, we leverage changes in international demand across country-products following the 2007–2009 Great Recession interacted with firms’ exposure to those country-products in a shift-share strategy. Formally, the external export shock is defined as

$$Z_j^{\text{Ext}} = \sum_{p \in \mathcal{P}} s_{jp} \Delta_p,$$

where  $p$  denotes country-product pairs,  $\mathcal{P}$  is the set of country-products,  $s_{jp}$  denotes the exposure shares, and  $\Delta_p$  denotes changes in international demand for country-product  $p$ . The exposure shares are defined as  $s_{jp} = \text{Exp}_{jp}^{2005-07} / \left( \sum_{p \in \mathcal{P}} \text{Exp}_{jp}^{2005-07} \right)$ , where  $\text{Exp}_{jp}^{2005-07}$  is the sum of real exported value from firm  $j$  to country-product  $p$  over 2005 through 2007.<sup>32</sup>

We leverage bilateral trade flow data from BACI to measure the country-product shocks ([Gaulier and Zignago 2010](#)). Following [Garin and Silvério \(2024\)](#), we compute  $\Delta_p$  as the symmetric growth rate of imports of country-product combination  $p$  from all other countries, excluding imports from domestic firms, between 2006–07 (“pre”) and 2009–10 (“post”),

$$\Delta_p = \frac{NNI_p^{\text{post}} - NNI_p^{\text{pre}}}{\frac{1}{2} \cdot (NNI_p^{\text{post}} + NNI_p^{\text{pre}})}, \quad (6)$$

where  $NNI$  is total non-domestic imports in real US dollars. As in [Garin and Silvério \(2024\)](#), to limit the influence of outliers, we winsorize  $Z_j^{\text{Ext}}$  at the 5th and 95th percentiles.

Our export shock has a shift-share structure, with its exogenous variation arising from

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<sup>32</sup>We deflate nominal export values with the US Annual CPI for all Urban Consumers (CPI-U).

multiple country-product shocks (Borusyak et al. 2022), rather than from the exposure shares themselves (Goldsmith-Pinkham et al. 2020). Thus, the key identification assumption is that the changes in international demand are uncorrelated with underlying trends in mean firm performance across firms exposed to each country-product. Since the disruptions in international demand arose from the Great Recession, we argue that they are exogenous to each firm, making this assumption plausible (see discussion in Garin and Silvério 2024).

### 3.4 Classifying Firm Exposure to Wage-Setting Constraints

This section describes how we classify firms as plausibly constrained, allowing us to test the predictions of the framework in Section 2.

#### 3.4.1 Baseline Definition for Heterogeneity Analyses

For Portugal and Colombia, we focus on the firm-level bite of the national MW, classifying firms as plausibly constrained if their average wage was within 15 percent of the national MW prior to the shock, and unconstrained otherwise.<sup>33</sup> In line with our theoretical framework, we imagine firms in the former group as having on average lower productivity and thus being wage-constrained, whereas firms in the latter group as having higher productivity and thus operating in the unconstrained regime. We assess the robustness of our findings by varying the 15% threshold used to classify firms as “constrained”. Additionally, for Portugal, we approximate CBA wage floors following Cardoso and Portugal (2005) and explore a constrained classification relative to the CBA wage floor, yielding similar results.<sup>34</sup>

In Norway, our baseline analysis classifies firms that prior to the shock were exposed to a sectoral CBA—either through formal adoption or policy-based extensions—as plausibly constrained and the remainder as unconstrained. The reason is that CBA-covered firms face a wage constraint, and the rest do not. In line with our theoretical framework, we imagine the former set of firms as facing a wage floor that is absent by construction for the latter group. To study the role of local bargaining constraints, we split CBA-covered firms—which are more likely to face local unions—into above- and below-median union density, always before the shock. In line with our theoretical framework, we assume that both types of firms

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<sup>33</sup>In Portugal, we use firms’ average base wages of all incumbent workers, while in Colombia, we rely on the average wages of permanent workers.

<sup>34</sup>The reason our results do not change much when we use distance to CBA wage floors is that the distribution of wage floors is tightly concentrated around the MW. Panel (a) of Appendix Figure F.4 shows the distribution of CBA wage floors, revealing that around 50% of wage floors are almost identical to the MW. Panel (b) shows the distribution of wage cushions, suggesting a strong bite of wage floors, as in Card and Cardoso (2022). Appendix D discusses our approach to approximate CBA wage floors.

optimize under a local bargaining constraint, with high-union-density firms more likely to be demand-constrained because they face a union with stronger bargaining power.

### 3.4.2 Variation in Exposure to Wage-Setting Constraints

We present a dynamic definition of exposure to constraints that allows us to study the effects of imposing constraints on firms’ responsiveness to shocks. To do so, we exploit changes in Portugal’s national MW between 2015 and 2017. We implement the design outlined in Section 2.3 and now classify firms more tightly into five groups based on their mean base wage in 2015 relative to the 2015, 2016, and 2017 MW levels. Appendix Figure F.5 plots the distribution of mean firm base wages from 2015 to 2017 with the different MWs over time.

The first group of firms, which we denote as SC, consists of strongly constrained firms paying at or below the 2015 MW ( $\underline{W}_{j,2015} \leq \delta \times MW_{2015}$ ).<sup>35</sup> They comprise 9.9% of firms in 2015. The second group, denoted UCC, includes firms paying above the 2015 MW but at or below the 2016 MW ( $\delta \times MW_{2015} < \underline{W}_{j,2015} \leq \delta \times MW_{2016}$ ) and represents 7% of firms. The third group, denoted UUC, contains firms paying above the 2016 MW but at or below the 2017 MW ( $\delta \times MW_{2016} < \underline{W}_{j,2015} \leq \delta \times MW_{2017}$ ) and represents 6% of firms. The fourth group, denoted UUU, comprises firms paying more than the 2017 MW but no more than 15% above it ( $\delta \times MW_{2017} < \underline{W}_{j,2015} \leq 1.15 \times MW_{2017}$ ). This group of “just” unconstrained firms accounts for 16.8% of the sample. The remaining 60.2% of firms, denoted SU, are strongly unconstrained firms that paid more than 15% above the 2017 MW ( $\underline{W}_{j,2015} > 1.15 \times MW_{2017}$ ).

## 3.5 Descriptive Statistics

Table 1 presents pre-shock descriptive statistics for firm-cohorts for which we define the internal instrument, separately for all firms and by constrained status using our baseline classifications. Appendix Table F.1 shows the number of firms we use in each cohort. In Portugal and Colombia, constrained firms have lower mean wages, employment, and value added, which is as expected, given that constrained status is defined relative to the MW. In Norway, the pattern is reversed: constrained firms are larger and with higher value added, while mean wages are very similar across groups. This aligns with the institutional setting, where CBAs are more common among more productive firms.

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<sup>35</sup>We set  $\delta = 1.025$ . This 2.5% buffer corresponds to half the size of the yearly MW increases.

**Table 1:** Descriptive Statistics in the Pre-Period by Country and Constrained Status.

	Portugal			Norway			Colombia		
	A	C	U	A	C	U	A	C	U
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
<i>Number of firms</i>	74,194	30,437	57,883	68,436	23,254	54,805	9,569	2,685	8,294
<i>Number of firm-cohorts</i>	356,789	103,762	252,997	517,250	149,076	368,073	63,660	7,249	49,966
(Share constrained)		(0.29)	(0.71)		(0.29)	(0.71)		(0.13)	(0.87)
<i>Log value added</i>									
Mean	12.14	11.38	12.46	15.42	16.09	15.14	13.89	12.46	14.24
SD	1.30	0.92	1.30	1.23	1.39	1.04	1.73	1.13	1.69
N	356,784	103,759	252,995	517,250	149,076	368,073	63,660	7,249	49,966
<i>Log Employment</i>									
Mean	2.16	1.82	2.30	2.31	2.98	2.03	3.14	2.39	3.30
SD	1.06	0.85	1.10	1.09	1.25	0.88	1.27	0.94	1.25
N	356,789	103,762	252,997	517,250	149,076	368,073	59,298	7,034	49,492
<i>Log Wages</i>									
Mean	1.65	1.30	1.79	5.37	5.38	5.37	13.63	13.14	13.71
SD	0.42	0.21	0.39	0.35	0.27	0.37	0.53	0.31	0.51
N	356,711	103,753	252,928	517,250	149,076	368,073	59,296	7,033	49,491

Notes: The table shows descriptive statistics for key firm-level variables in the pre-period by country and constrained status. “A” stands for All firms, “C” for constrained firms, and “U” for unconstrained firms. Constrained refers to firms with an average wage within 15% of the national MW for Portugal and Colombia, or covered by a CBA for Norway, whereas unconstrained refers to the rest of the firms. The statistics are computed as the average over the periods  $-3$  and  $-2$ , weighted by the number of firms in the cohort. The sum of the number of firm cohorts in constrained (C) and unconstrained (U) may differ from the total number of firms in the sample (A), as the constrained definition is missing for a small share of firms. For Portugal and Norway, we use the mean hourly wage of stayers (workers who stayed in the firm for at least 7 years around the shock), measured in Euros and Norwegian Kroner, respectively. For Colombia, we use the mean monthly wage of permanent workers, measured in Colombian Pesos.

To add context for our external shock we discuss firm-level descriptive statistics as well as aggregate exports. Appendix Table F.2 compares exporting and non-exporting firms in 2007. Exporting firms account for 10 percent and 9 percent of firms in Portugal and Norway, respectively. Value added, employment, and wages are higher in exporting firms, as they are positively selected (Bernard et al. 2007). Appendix Figure F.6 shows the evolution of aggregate exports around the Great Recession. Portugal experienced a decline in 2009, followed by a relatively rapid recovery, similar to the EU and US. By comparison, Norway faced a smaller though more persistent decline.

## 4 Empirical Strategy

This section discusses the empirical strategy we use to study the responsiveness to demand shocks and the role of wage-setting constraints, building on the discussion in Section 2.

### 4.1 Baseline Specification Exploiting Demand Shocks

We start with an overview of our empirical approach. As before, let  $L_{jt}$ ,  $W_{jt}$ , and  $VA_{jt}$  denote the level of employment, wages, and value added, respectively, of firm  $j$  in year  $t$ . We consider the following set of reduced-form equations:

$$\ln L_{jt} = \beta_L Z_j \text{Post}_t + \delta_{k(j)t}^L + \omega_j^L + v_{jt}^L, \quad (7a)$$

$$\ln W_{jt} = \beta_W Z_j \text{Post}_t + \delta_{k(j)t}^W + \omega_j^W + v_{jt}^W, \quad (7b)$$

$$\ln VA_{jt} = \beta_{VA} Z_j \text{Post}_t + \delta_{k(j)t}^{VA} + \omega_j^{VA} + v_{jt}^{VA}, \quad (7c)$$

where  $Z_j$  is an indicator for a firm receiving a demand shock,  $\text{Post}_t$  is an indicator for the post-shock period,  $k(j)$  is the local labor market of  $j$ , and  $v_{jt}^L$ ,  $v_{jt}^W$ , and  $v_{jt}^{VA}$  are the residuals. The parameters  $\beta_L$ ,  $\beta_W$ , and  $\beta_{VA}$  are reduced-form coefficients that capture the effects of the shock on employment, wages, and value added. The inclusion of  $\delta_{k(j)t}^Y$ , for  $Y \in \{W, L, VA\}$ , controls for common changes for all firms in a local labor market, such as those driven by the wage index  $\mathbb{W}$  in the theoretical model. The firm fixed effects  $\omega_j^Y$  control for time-invariant differences across firms, such as those driven by different baseline amenity values  $A_j$  in the labor supply case, or productivity levels  $\Phi_j$  in the wage case.

The conventional estimands of labor supply and rent-sharing elasticities are constructed as the ratios of the reduced-form coefficients. Specifically, we estimate  $\eta^{CE} = \beta_L/\beta_W$  and  $\theta^{CE} = \beta_W/\beta_{VA}$ . This is equivalent to an instrumental variables (IV) approach, where the shock  $Z_j$  is an instrument for the wage change  $\Delta \ln L_j$ , in the labor supply case, or the value

added change  $\Delta \ln VA_j$ , in the rent-sharing case. For this approach to be valid, the standard IV assumptions must hold. Relevance requires that  $Z_j$  has a non-zero first stage effect (either  $\beta_W \neq 0$  or  $\beta_{VA} \neq 0$ ), which maps to Assumption 3. The shock  $Z_j$  must be uncorrelated with unobserved determinants of employment, when we estimate the labor supply elasticity, or wages, when we estimate the rent-sharing elasticity. In our framework, this relates to Assumption 4 on shock exogeneity. As discussed in Section 2.2, however, exclusion may not hold due to the presence of demand-constrained firms, leading to an identification failure.

**Estimating the reduced-form effects of shocks.** To construct the elasticities, we require estimates of the reduced-form effects of demand shocks. We obtain those estimates from dynamic models of the form

$$\ln Y_{jt} = \sum_{\tau \in \mathcal{T}} \beta_{Y,\tau} Z_j \mathbf{1}\{\tau = t - t^*\} + \delta_{k(j)t}^Y + \omega_j^Y + v_{jt}^Y, \quad (8)$$

where  $Y_{jt}$  represents either employment  $L_{jt}$ , mean wages  $W_{jt}$ , or value added  $VA_{jt}$ ,  $Z_j$  is either the internal or external instrument,  $\tau$  indexes relative years and  $\mathcal{T}$  is the set of relative years included,  $\mathbf{1}\{\tau = t - t^*\}$  is an indicator for years relative to the event year  $t^*$ ,  $v_{jt}^Y$  is the residual, and  $\omega_j^Y$  and  $\delta_{k(j)t}^Y$  are defined as before, for  $Y \in \{W, L, VA\}$ . In practice, we define the local labor market  $k$  as the 2-digit sector reported by the firm in year  $t^* - 1$ .

The interpretation of the coefficients  $\{\beta_{Y,\tau}\}_{Y \in \{W,L,VA\}, \tau \in \mathcal{T}}$  depends on the shock that we use. In the case of the binary internal shock  $Z_j^{\text{Int}}$ , the coefficients reflect the evolution of the outcomes for firms that receive a value-added increase relative to those that do not. In the case of the continuous external export shock  $Z_j^{\text{Ext}}$ , they reflect the effect of a marginal one-unit increase in the demand for their exports. The coefficients for  $t < t^*$  allow us to assess the plausibility of the parallel trends assumption underlying our design.

The reduced-form parameters  $\{\beta_L, \beta_W, \beta_{VA}\}$  are defined as the mean post-period coefficient from estimates of (8), and the elasticities are computed as  $\eta^{CE} = \beta_L/\beta_W$  and  $\theta^{CE} = \beta_W/\beta_{VA}$ . Standard errors for the elasticities are estimated with the delta method.

**Internal shock.** We implement a cohort-based estimation strategy for the internal shock (Cengiz et al. 2019). We construct a dataset at the cohort  $c$ , firm  $j$ , year  $t$  level, in which  $Z_j^{\text{Int}}$  is equal to one for firms with above-median value-added growth between years 0 and  $-1$  within their sector  $k$ . Then, we estimate a version of Equation (8) where we control for firm-by-cohort and cohort-by-sector-by-year fixed effects. For each cohort  $c$ , we keep years from  $c - 4$  to  $c + 3$  so that  $\mathcal{T} = \{-4, \dots, 3\}$ . We omit the category  $-2$  (following Lamadon et al. 2022). Reduced-form effects are defined as the mean of coefficients for relative years

1, 2, and 3, and standard errors are clustered at the cohort-by-sector level.

**External shock.** The export shock  $Z_j^{\text{Ext}}$  is defined as the mean growth in world import demand for each firm  $j$ , so we fit Equation (8) in the sample of exporting firms for which the shock can be defined. The event year  $t^*$  is set to 2007. As our data for Portugal starts in 2004, we set  $\mathcal{T} = \{-3, \dots, 4\}$  excluding  $-1$ . Reduced-form effects are defined as the mean of coefficients for years 2009, 2010, and 2011, and standard errors are clustered at the firm level. As the sample of firms is significantly smaller, we expect the estimates to be less precise.

**Heterogeneity.** Our theoretical model implies that the observed responsiveness to demand shocks will differ between constrained and unconstrained firms. To explore this result, we split the estimation sample between “plausibly constrained” and “plausibly unconstrained” firms, using the classifications described in Section 3.4. Briefly, in Portugal and Colombia, we classify firms that pay close to the MW wage as constrained, whereas in Norway, we classify firms covered by a sectoral CBA as constrained. Within the group of CBA firms in Norway, we further split the sample into high- and low-union-density firms.

## 4.2 Specification Exploiting Variation in Constraints

Building on Section 2.3, we introduce a strategy to test the effect of changes in wage-setting constraints on the responsiveness to demand shocks. To do so, we leverage a national MW hike in Portugal in 2015–2017. We classify firms into several groups that reflect the changing “bite” of wage constraints they face and study their differential responsiveness to demand shocks. Leveraging the flexibility of the internal shock, we define a demand shock using the change in value added between 2015 and 2016. We frame our discussion using static models for simplicity, though we allow for dynamic effects in the implementation.

As a guide to our preferred specification, consider estimating the effect of the shock  $Z_j$  on firms *within* different groups. As discussed in Section 3.4.2, we classify firms into five groups based on their average wage in 2015: SC (constrained at any MW level), UCC (unconstrained at the 2015 MW but constrained at the 2016 MW), UUC (unconstrained at the 2016 MW but constrained at the 2017 MW), UUU (“just” unconstrained at the 2017 MW), and SU (remaining firms). One can then estimate the responsiveness to shocks with

$$\ln Y_{jt} = \beta_Y^G Z_j \text{Post}_t + \delta_{k(j)t}^{Y,G} + \omega_j^{Y,G} + v_{jt}^{Y,G} \quad (9)$$

where  $G \in \{SC, UCC, UUC, UUU, SU\}$  denotes the groups and  $Y \in \{W, L, VA\}$  denotes the outcomes, and the other objects are defined as before. The coefficients  $\{\beta_W^G, \beta_L^G, \beta_{VA}^G\}$

give the group-specific response of wages, employment, and value added to the shock.

We can then compare the shock responsiveness across groups. While such comparisons are straightforward given group-specific reduced-form estimates, we implement them formally in a triple-differences specification comparing, say, UCC and UUU firms:

$$\begin{aligned} \ln Y_{jt} = & \beta_Y^{UCC,UUU} Z_j \text{Post}_t \mathbf{1}\{G_j = \text{UCC}\} + \beta_Y^{UUU} Z_j \text{Post}_t \\ & + \gamma_Y \text{Post}_t \mathbf{1}\{G_j = \text{UCC}\} + \lambda_Y Z_j \mathbf{1}\{G_j = \text{UCC}\} \\ & + \delta_{k(j)t}^Y + \omega_j^Y + v_{jt}^Y. \end{aligned} \quad (10)$$

Let us unpack (10) step by step. Setting all indicators to zero, we note that the baseline category corresponds to the subset of UUU firms that have not been shocked.  $\beta_Y^{UUU}$  thus gives the effect of the shock  $Z_j$  among UUU firms. Setting  $G_j = \text{UCC}$  with  $Z_j = 0$  instead, we note that  $\gamma_Y$  gives the differential evolution of wages for UCC firms without a shock. This coefficient effectively captures the MW hike. Finally,  $G_j = \text{UCC}$  and  $Z_j = 1$  reveal that the effect of the shock among UCC firms is  $\beta_Y^{UUU} + \beta_Y^{UCC,UUU}$ . Thus, the triple-differences coefficient  $\beta_Y^{UCC,UUU}$  gives the differential response to the shock among UCC firms relative to UUU firms. We interpret  $\beta_Y^{UCC,UUU}$  as the effect of the constraint on shock responsiveness.<sup>36</sup>

Using the reduced-form effects, we construct elasticities to test the implications of the theoretical model. The rent-sharing elasticity for UCC firms, for example, can be constructed as  $(\beta_W^{UUU} + \beta_W^{UCC,UUU}) / (\beta_{VA}^{UUU} + \beta_{VA}^{UCC,UUU})$ , which we expect to be smaller than the elasticity for UUU firms,  $\beta_W^{UUU} / \beta_{VA}^{UUU}$ . As long as employment and value-added responses are similar across groups, we expect larger labor supply and smaller rent-sharing elasticities for UCC firms than for UUU firms. We also construct elasticities for firms in groups SC, UUC, and SU, which allows us to test whether a more intense “treatment” (i.e., being constrained by the MW for a longer period) leads to a stronger attenuation of the wage response.

The interpretation that this design reveals the causal effect of wage constraints on firms’ responsiveness relies on two assumptions on the two groups being compared. First, they should have a similar structural labor supply elasticity. Second, the relative evolution of other determinants of shock responsiveness should evolve in parallel. Under these assumptions, the group that did not experience a change in constraints is a valid counterfactual for the group that did. As discussed in Section 2.3, these assumptions are weaker than the ones underlying the heterogeneity analysis between constrained and unconstrained firms.

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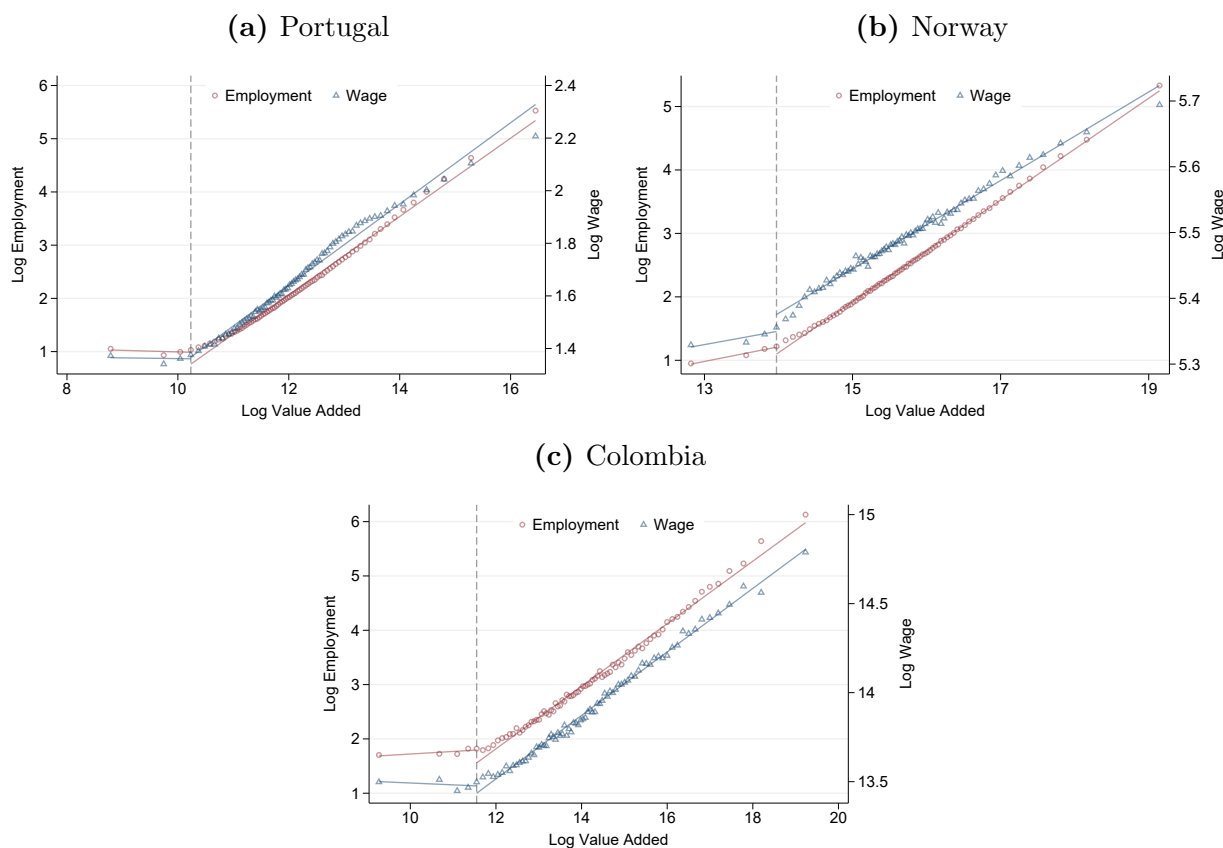
<sup>36</sup>We expect the following signs for the reduced-form triple-differences coefficients. First, we expect  $\beta_W^{UCC,UUU} < 0$  as the theoretical model predicts muted wage pass-through for constrained firms. Second, the model’s prediction for employment is ambiguous, thus we let data inform us about  $\beta_L^{UCC,UUU}$ . Finally, by construction of the internal shock, we expect  $\beta_{VA}^{UCC,UUU}$  to be close to zero.

## 5 Results

This section presents our empirical results and interprets them in the light of our theoretical model. First, we provide descriptive evidence on the wage-productivity relationship for each country. Next, we show responses to firm-level demand shocks and obtain implied elasticities across all firms, before we document the role of wage-setting constraints by examining the responses of firms by their constrained status. Finally, we exploit within-firm variation in the exposure to constraints to study their causal effect on the shock responsiveness.

### 5.1 Descriptive Evidence

**Figure 3:** Relationship Between Firm Value Added and Average Wages and Employment.



Notes: The figure shows the relationship between log wages and log employment against log value added, for Portugal (Panel (a)), Norway (Panel (b)), and Colombia (Panel (c)). In all countries, variables are residualized with respect to two-digit industry indicators and year fixed effects, and in Norway, we further control for firm-level union density and collective bargaining agreement coverage dummies, interacted with dummies for exposure to sectoral wage floor extensions. For Colombia and Portugal, we restrict the sample to the 2012 cohort, while for Norway, we use observations from 2015 onward. We construct 80 equal-sized bins of log value added and compute means of log employment and log wages within each bin. Solid lines correspond to linear fits on the binned data.

Our theoretical framework suggests a strong positive relationship between firm productivity and wages, and more importantly, predicts that this relationship should be kinked in the presence of a wage-setting constraint (Panel (e) of Figure 1). While it is possible to statistically test whether the empirical wage-productivity relationship is kinked, such evidence requires important caveats. First, while the model predictions concern the potential wage and employment outcomes for different values of productivity holding all other parameters fixed, the cross-sectional relationships may confound differences in firm productivity and unobserved correlates (e.g., amenities). Second, the theoretical wage-productivity relationship is less sharp in our framework with local bargaining (Panel (d) of Appendix Figure A.1), which effectively reduces the inaction region where firms do not adjust wages in response to changes in productivity. Subject to these caveats, we show the empirical relationships between wages and value added and employment and value added across firms in Figure 3. For each country, we find strong positive associations between value added and wages and employment. We also find clear evidence of “hockey sticks” for Portugal and Colombia, which have national MW policies, and a “kink” in Norway, which reflects the presence of both CBA wage floors and local bargaining, as predicted by our theoretical framework.<sup>37</sup>

## 5.2 Firm Responses to Demand Shocks

We now present causal evidence on firm-level responses to demand shocks as well as the implied estimates of labor supply and rent-sharing elasticities.

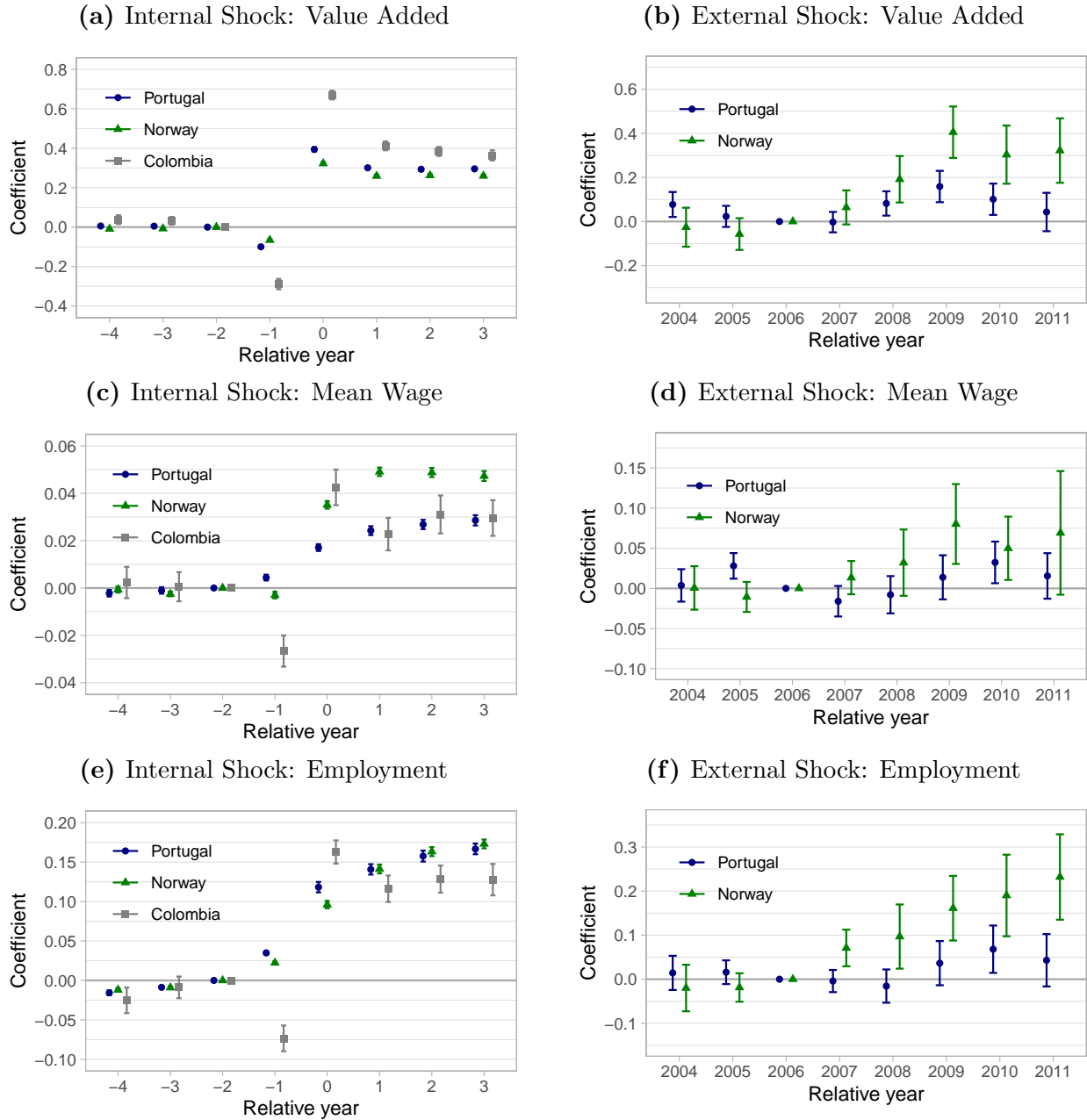
**Shock responses.** We now exploit firm-level demand shocks, following the empirical strategy from Section 4. We report the reduced-form estimates of the firm-level responses on value added, mean firm wage, and employment, using both the internal (left column) and external (right column) shocks in Figure 4. Panels (a) and (b) show that both shocks predict changes in value added, showing that the shocks are relevant. The wage and employment effects reveal significant differences in responsiveness across countries. Panels (c) and (d) show that wage responses appear stronger in Norway, regardless of the shock considered.<sup>38</sup> Panels (e)

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<sup>37</sup>Similar evidence on the “hockey-stick-like” relationship between firm productivity and wages is found in several countries (Kline 2024, p. 121). Recently, Bassier and Budlender (2025) propose an identification approach to infer the theoretically implied value-added thresholds from the cross-sectional relationship between wages and value added and use the inferred thresholds to classify wage-constrained firms. As our model features firm-specific production technologies and unobserved amenities, such thresholds are inherently firm-specific, so consistently implementing this approach in our theoretical framework requires strong assumptions on the cross-sectional distribution of the unobserved amenities and production technologies.

<sup>38</sup>To mitigate concerns of compositional effects in our wage variable, we measure mean wages only for stayers in Portugal and Norway, and for permanent workers in Colombia. For Portugal and Norway, we also provide evidence on mean wage impacts for incumbents in Appendix Figure F.7, where we allow incumbents to potentially work in a different firm after the shock. Wage impacts are similar for incumbents and stayers.

**Figure 4: Firm Responses to Demand Shocks: Internal and External Shocks.**



Notes: The figure shows estimates of the effects of a demand shock, using the internal and external shock designs, on firm outcomes. Panels (a)–(b) show log value added, Panels (c)–(d) shows the log mean wage, and Panels (e)–(f) shows log employment, for the internal and external shock, respectively. For the internal shock, we estimate the effect of a treatment indicator for having value-added growth between period  $-1$  and  $0$  above median within the firm’s 2-digit industry. For the external shock, we estimate the effect of a change in the mean world import demand using a shift-share strategy, where the shares correspond to 2005–2007 value, and the shifts represent the symmetric growth rate in world import demand for each country-product between 2006–2007 and 2009–2010. We restrict to firms exporting for at least three years up to 2007, with exports exceeding 1% of their revenue in 2005–2007, and, for Portugal, we further exclude firms that primarily export to Angola or Spain.

and (f) show that employment responses are similar in Norway and Portugal, and slightly smaller in Colombia with the internal shock, and with the external shock, they appear larger for Norway. For all outcomes, countries, and shocks, we find pre-treatment coefficients that are mostly insignificant or close to zero, supporting the parallel trends assumption.<sup>39</sup>

**Implied elasticities.** We report the implied rent-sharing and labor supply elasticities based on the conventional estimands defined in Section 2.2. To compute these elasticities we obtain  $\{\beta_L, \beta_W, \beta_{VA}\}$  by averaging the post-period coefficients and compute  $\eta^{CE} = \beta_L/\beta_W$  and  $\theta^{CE} = \beta_W/\beta_{VA}$ . Using the internal shock, we provide the implied estimates for  $\{\beta_L, \beta_W, \beta_{VA}\}$  and  $\{\eta^{CE}, \theta^{CE}\}$  in Columns (1), (4), and (7) of Table 2. The implied labor supply elasticities are 5.84 (SE = 0.245) in Portugal, 3.29 (SE = 0.085) in Norway, and 4.47 (SE = 0.631) in Colombia.<sup>40</sup> The implied rent-sharing elasticities are 0.090 (SE = 0.003) in Portugal, 0.186 (SE = 0.004) in Norway, and 0.072 (SE = 0.009) in Colombia, values that are consistent with reviews by Card et al. (2018) and Jäger et al. (2020).

Interestingly, the differences in elasticities that we estimate using the internal shock across the three countries are consistent with the theoretical implications of key differences in institutional settings. As discussed in Section 3, the national MW has a significant “bite” in Colombia and Portugal, and most workers in Portugal have CBA coverage, thus reflecting settings with relatively high wage floors. By comparison, around 50% of private sector workers are covered by CBAs in Norway, with a strong presence of labor unions. In line with our theoretical predictions, we should thus expect higher rent-sharing elasticities for Norway. Although other explanations can account for the cross-country differences, we view this as evidence motivating the within-country heterogeneity analysis in the next sections.<sup>41</sup>

**Comparison of internal and external shock designs.** Reduced-form estimates based on the external shock in Figure 4 yield elasticities that are less precise though still consistent with the internal shock design. We provide comparisons of the rent-sharing and labor supply elasticities from the two designs in Appendix Table F.3, while adjusting for differences in

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<sup>39</sup>For the internal shock design, we find evidence of mean reversion, which is expected given how the demand shock is defined (see discussions in Lamadon et al. 2022). However, trends remain stable at the start (for  $\tau < -1$ ) and end (for  $\tau > 1$ ) of the time window, consistent with the parallel trends assumption.

<sup>40</sup>Our estimate for Colombia is higher than the one reported by Amodio and De Roux (2024), who find a labor supply elasticity of 2.5 using similar data but identifying demand shocks through exchange rate-driven variation in export sales. Beyond the choice of shock, differences may also reflect their focus on exporting plants and on all types of workers (not only permanent) for measuring plant-level wages and employment.

<sup>41</sup>For a large sample of developing countries, we also examined the relationship between labor supply elasticities, as estimated by Amodio et al. (2024), and the share of workers covered by collective bargaining. Our framework predicts that countries with higher CBA coverage should have larger estimates of labor supply elasticities, due to a stronger bite of constraints. Appendix Figure F.14 confirms this positive relationship.

sample composition. For Norway, we find fairly similar estimates across the internal and external designs, with estimates of rent-sharing elasticities ranging between 0.13 and 0.2 and labor supply elasticities between 2.78 and 4.27. However, the confidence intervals for the external shock estimates are wider, which prevent us from drawing strong conclusions.

### 5.3 The Role of Wage-Setting Constraints

We proceed by examining differences in responsiveness to demand shocks depending on firms' exposure to wage-setting constraints. If some firms are constrained, the discussion in Section 2 implies that our estimated elasticities for all firms do not reflect the underlying structural parameters that firms face. We would further expect the observed labor supply elasticities to be larger for constrained firms, and the rent-sharing elasticities to be smaller.

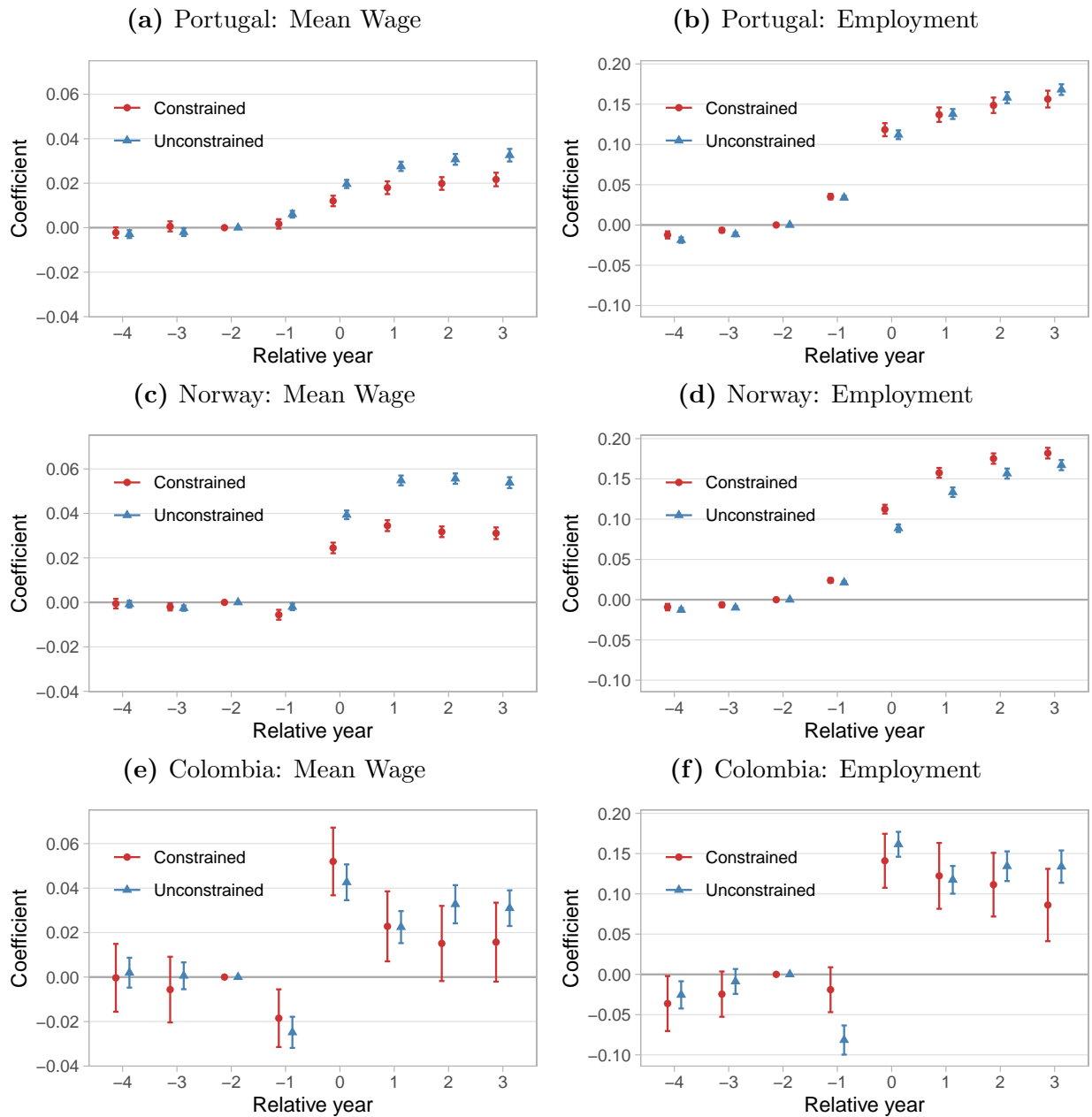
**Heterogeneity in shock responses.** We find significant differences in shock responses across plausibly constrained (C) and plausibly unconstrained (U) firms using the internal shock design. Figure 5 presents reduced-form estimates of wage and employment responses by firms' constrained status, separately for each country. In line with our theoretical predictions, we find that wage responses are smaller among constrained firms. Table 2 shows significant differences in wage responses for both Portugal (C= 0.020 vs. U= 0.030) and Norway (C= 0.032 vs. U= 0.055), while the estimates are consistent but noisier for Colombia (C= 0.018 vs. U= 0.029). For Norway, we also find a larger employment response for constrained than unconstrained firms. We note that the heterogeneity in shock responses does not reflect the intensity of demand shocks: Appendix Figure F.8 shows that value-added responses are very similar for unconstrained and constrained firms in each country.

Although our model predicts that firms subject to a pure wage floor should not adjust wages to demand shocks, our empirical results for Portugal and Colombia indicate that firms we classify as plausibly constrained do respond, but do so significantly less than unconstrained firms. We view our measurement as approximating actual firm-level constraints and thus argue that our empirical results support the key theoretical predictions. Consistent with this interpretation, Appendix Figure F.9 shows that when constrained firms are defined more narrowly closer to the MW or CBA wage floor in Portugal (Panels a and b) and Colombia (Panels c and d), their wage responses are smaller and hence the elasticities will differ even more from unconstrained. Moreover, Appendix Figure F.10 shows that defining constrained firms in Portugal based on CBA wage floors yields very similar results to the baseline definition, reinforcing the view that minimum wages capture most of wage floors.<sup>42</sup>

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<sup>42</sup>We find no evidence that the average wage floors  $\underline{W}_j$  that CBA firms face change in response to firm-level demand shocks (see Appendix Figure F.11), consistent with such floors being externally determined.

**Figure 5:** Firm Responses to Demand Shocks: Exposure to Wage-Setting Constraints.



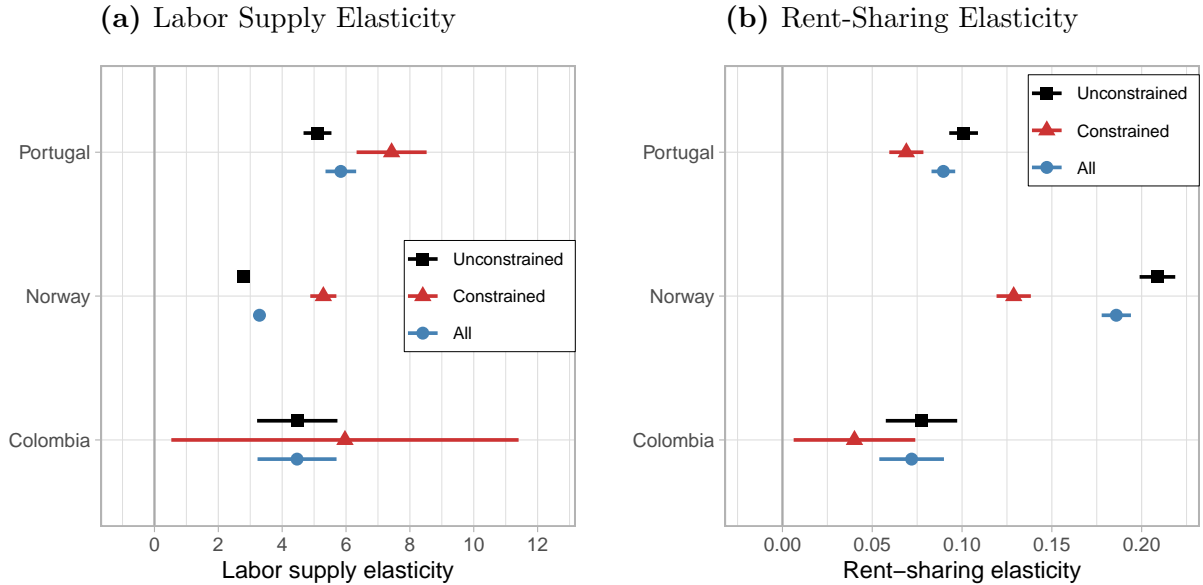
Notes: The figure shows estimates of the effects of a demand shock, using the internal shock design, on firm-level outcomes, by constrained status of firms. Constrained refers to firms with an average wage within 15% of the national MW for Portugal and Colombia, or firms being covered by a CBA for Norway, whereas Unconstrained refers to the remaining group of firms in each country. Panels (a) and (b) show estimates for Portugal, Panels (c) and (d) for Norway, and Panels (e) and (f) for Colombia. The left column uses the log of the firm wage as the outcome, whereas the right column uses the log of employment. In each case, we estimate the effect of a treatment indicator for having value-added growth between period  $-1$  and  $0$  above the median within the firm's 2-digit sector.

**Table 2:** Firm Responses to Demand Shocks and Estimated Elasticities: Internal Shock Design.

	Portugal			Norway			Colombia		
	A	C	U	A	C	U	A	C	U
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
<b>(a) Estimated effects</b>									
Employment	0.1550 (0.0034)	0.1473 (0.0049)	0.1547 (0.0033)	0.1592 (0.0028)	0.1716 (0.0031)	0.1524 (0.0031)	0.1241 (0.0083)	0.1066 (0.0196)	0.1285 (0.0086)
Wage	0.0266 (0.0009)	0.0198 (0.0013)	0.0303 (0.0012)	0.0484 (0.0009)	0.0324 (0.0011)	0.0548 (0.0011)	0.0278 (0.0035)	0.0179 (0.0076)	0.0287 (0.0037)
Value added	0.2965 (0.0034)	0.2875 (0.0053)	0.3003 (0.0040)	0.2604 (0.0032)	0.2520 (0.0037)	0.2623 (0.0037)	0.3865 (0.0109)	0.4458 (0.0243)	0.3711 (0.0117)
<b>(b) Elasticities</b>									
Labor Supply	5.84 (0.24)	7.43 (0.56)	5.11 (0.22)	3.29 (0.08)	5.29 (0.21)	2.78 (0.08)	4.47 (0.63)	5.97 (2.77)	4.47 (0.64)
Rent-Sharing	0.090 (0.003)	0.069 (0.005)	0.101 (0.004)	0.186 (0.004)	0.129 (0.005)	0.209 (0.005)	0.072 (0.009)	0.040 (0.017)	0.077 (0.010)

Notes: The table summarizes the estimates of the effects of a demand shock, using the internal shock design, on firm-level outcomes (log employment, log wages, and log value added), and the implied labor supply and rent-sharing elasticities, by constrained status of firms. “A” stands for All firms, “C” for Constrained firms, and “U” for Unconstrained firms. Constrained refers to firms with an average wage within 15% of the national MW for Portugal and Colombia, or covered by a CBA for Norway, whereas Unconstrained refers to the rest of the firms. Each reduced-form estimate is computed as the average among relative times 1 through 3. The labor supply elasticity is defined as the ratio of the employment and wage effects, and the rent-sharing elasticity is defined as the ratio of the wage and value-added effects. Standard errors clustered at the cohort-sector level are reported in parentheses. The standard errors of the elasticities are computed using the delta method.

**Figure 6:** Estimated Firm-Level Elasticities: Exposure to Wage-Setting Constraints.



Notes: The figure shows estimated elasticities implied by reduced-form effects of a demand shock on value added, wages, and employment, using the internal shock design, by constrained status of firms. Constrained refers to firms with an average wage within 15% of the national MW for Portugal and Colombia, or covered by a CBA for Norway, whereas Unconstrained refers to the rest of the firms. Panel (a) shows labor supply elasticities, defined as the ratio of the employment to wage responses to the shock. Panel (b) shows rent-sharing elasticities, defined as the ratio of the wage to value added responses to the shock. The elasticities are obtained from the reduced-form estimates from Figure 5.

**Heterogeneity in implied elasticities.** In line with the reduced-form estimates, our implied elasticities in the internal shock strategy differ significantly by constrained status within countries. Figure 6 presents the results visually, and Table 2 shows exact estimates. In Portugal, labor supply elasticities are higher for constrained ( $C = 7.43$ ) than for unconstrained firms ( $U = 5.11$ ). A similar pattern holds in Norway ( $C = 5.29$  vs.  $U = 2.78$ ) and, though less precisely estimated, in Colombia ( $C = 5.97$  vs.  $U = 4.47$ ). For the implied rent-sharing elasticities, constrained firms exhibit smaller values in Portugal ( $C = 0.069$  vs.  $U = 0.101$ ), Norway ( $C = 0.129$  vs.  $U = 0.209$ ), and Colombia ( $C = 0.040$  vs.  $U = 0.077$ ).

Taken together, the rent-sharing elasticities are between 32 and 48 percent smaller for constrained firms relative to unconstrained ones, while the implied labor supply elasticities are between 33 and 90 percent larger. This evidence is consistent with wage-setting constraints that limit wage responses, making constrained firms appear to face a more elastic labor supply, seemingly close to the competitive benchmark.<sup>43</sup>

<sup>43</sup>Using the external shock design, our findings broadly align with the internal shock, though results are much noisier. Appendix Figure F.12 presents the reduced-form effects, and Appendix Figure F.13 shows the estimated elasticities and compares with the internal shock. As expected, the elasticities are noisier given the smaller samples used in the external shock design. For Norway, the results by constrained status align

**Table 3:** Estimated Firm-Level Elasticities: Local Union Strength in Norway.

	Elasticity		Pre-Period Mean		
	Labor Supply	Rent-Sharing	Log Wage	Union Share	Share Sample
	(1)	(2)	(3)	(4)	(5)
All firms	3.29 (0.08)	0.186 (0.004)	5.37	0.196	
<i>Baseline definition</i>					
Constrained	5.29 (0.21)	0.129 (0.005)	5.38	0.346	0.29
Unconstrained	2.78 (0.08)	0.209 (0.005)	5.37	0.136	0.71
<i>Constrained firms</i>					
High local union share	6.59 (0.36)	0.098 (0.005)	5.41	0.546	0.50
Low local union share	4.46 (0.20)	0.161 (0.007)	5.36	0.145	0.50

Notes: The table summarizes the elasticities estimated for different samples of Norwegian firms using the internal shock design. Constrained firms as those covered by a CBA, whereas Unconstrained refers to the remaining group of firms. For constrained firms, we split the sample between above- and below-median union share. The labor supply elasticity is defined as the ratio of the employment to wage effects, and the rent-sharing elasticity as the ratio of wage to value added effects. The pre-period descriptives are computed as in Table 1. Standard errors clustered at the cohort-sector level are reported in parentheses. The standard errors for elasticities are calculated using the delta method.

**Local bargaining constraints in Norway.** We study heterogeneity in shock responses across high and low union density firms within the set of constrained—CBA-covered—firms in Norway. Table 3 shows that, although baseline wage levels are higher in high-union firms, the estimated rent-sharing elasticity is smaller, whereas the estimated labor supply elasticity is significantly larger. These findings are in line with our theoretical model predictions for local bargaining constraints derived in Section 2. The model suggests that high wage demands from strong unions may push some firms to the demand-constrained regime, and thus we expect to find higher labor supply and lower rent-sharing elasticities for such firms.<sup>44</sup>

closely with the internal shock design, with the ranking of estimated elasticities between constrained and unconstrained firms preserved across designs. For Portugal, the external shock estimates are more volatile and less informative. We adopt a higher constraint threshold in Portugal to ensure a meaningful sample size, so firms are now classified as constrained when their mean wage lies roughly 50 percent above the MW.

<sup>44</sup>Notably, this evidence concerns the heterogeneity in responsiveness to demand shocks across firms with different levels of pre-shock labor union density, but does not provide causal evidence on the role of local

**The “bias” in the conventional estimands.** Our heterogeneity analysis suggests that the implied estimates of rent-sharing and labor supply elasticities differ significantly across firms depending on their constrained status. Interpreting this evidence in the light of Proposition 2, we argue that the conventional estimands yield biased estimates of the underlying structural elasticities, where the extent of demand-constrained firms is informative about the degree of bias. To quantify the bias we compare the elasticities obtained using only unconstrained firms to those using all firms, where the latter corresponds to what the conventional estimands would pick up. Using all firms results in a labor supply elasticity of up to 15% higher, and a rent-sharing elasticity of up to 13% lower, relative to unconstrained firms only.

We further rely on Corollary 1 to quantify the share of demand-constrained firms. For this exercise, we take the estimated value for unconstrained firms as the “correct” estimate of  $\eta$ , and the value for all firms as  $\eta^{CE}$ . Noting that the relative magnitude of the shocks is similar across groups, we set  $\zeta_{dd}/\zeta_{uu} = \zeta_{ss}/\zeta_{uu} = 1$ , and we take the share of unconstrained firms from Table 2. Our estimates are consistent with a share of demand-constrained firms of 0.061 in Portugal and 0.059 in Norway, and with a negligible share for large manufacturing firms in Colombia. Notably, even with a small share of demand-constrained firms, we observe large differences in implied elasticities between the constrained and unconstrained groups.

We also quantify the share of demand-constrained firms in Norway due to local bargaining constraints. We rely on values from Table 3 and the formula from Appendix C. We use the “Constrained” estimate as  $\eta^{CE}$ , and the “Low Local Union Share” estimate as  $\eta$ . We try two values for the share  $\lambda = \kappa\Upsilon / (\kappa\Upsilon + (1 - \kappa)\underline{W})$ , 0.2 and 0.8, and obtain shares of demand-constrained firms of 0.070 and 0.093, respectively, which again suggests that a relatively small share of Norwegian firms is demand-constrained, despite the presence of strong unions.

## 5.4 Combining Demand Shocks with Variation in Constraints

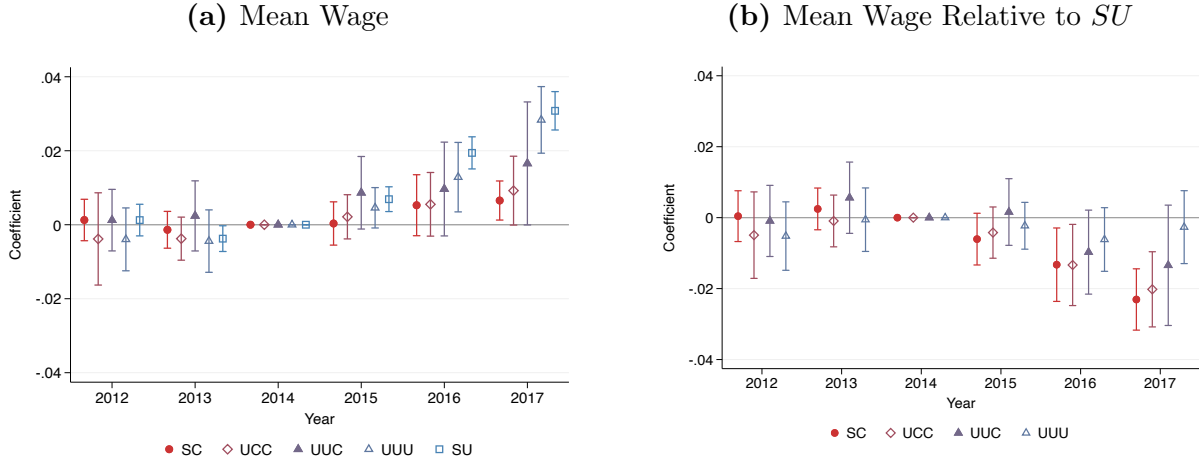
The differential responses between constrained and unconstrained firms may reflect unobserved firm-specific factors that correlate with firms’ exposure to wage-setting constraints. To address this concern, we study how imposing a wage constraint affects firms’ responsiveness to demand shocks. We leverage Portugal’s MW increase which, after staying constant at 485 euros per month since 2011, increased steadily between 2014 and 2017, reaching 557 euros, a cumulative nominal increase of 14.8%.

We implement the triple-differences design discussed in Section 4.2. To do so, we use the average pre-demand shock firm wages in 2015 to classify firms in five groups that capture the

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union density as such. In recent work, [Dodini et al. \(2023\)](#) provide causal evidence using changes in the tax deductibility of union dues as sources of exogenous variation in union density. Their findings show that, in the average private-sector firm, higher union density raises labor costs and lowers employment and profits.

**Figure 7:** Wage Responses to Demand Shocks: Variation in Constraints in Portugal.



Notes: The figure shows estimates of the wage effects of a demand shock for Portuguese firms, using the internal shock design, by constrained status of firms. We classify firms based on their 2015 mean base wages. SC denotes constrained by the 2015 MW, UCC denotes unconstrained by the 2015 MW but would be constrained by the 2016 MW, UUC denotes unconstrained by the 2015 and 2016 MW but would be constrained by the 2017 MW, UUU denotes they had a wage between the 2017 MW and up to 15% above it, and SU are all the remaining firms. Panel (a) shows event-study coefficients, estimated separately for each group, and Panel (b) shows the differences with respect to the SU group.

changing “bite” of the MW as defined in Section 3.4.2: SC, UCC, UUC, UUU, and SU. We also construct a demand shock using value-added growth between 2015 and 2016. Then, we estimate the effects of imposing wage constraints on the responses of firms to demand shocks, using a panel of firms between 2012 and 2017. We illustrate the triple-differences approach in Appendix E, where we first isolate the effect of minimum wage hikes, then introduce demand shocks, and finally estimate the joint specification.<sup>45</sup>

**Shock responses with variation in constraints.** The results show that the MW constraints mute the wage responses to shocks. Figure 7 presents the event-study estimates, separately for the five groups. Panel (a) shows that firms that are constrained at the 2015, 2016, and 2017 MW (the SC group) show almost no response in wages. On the other hand, firms that are “just” unconstrained at 2017 MW (the UUU group) and strongly unconstrained firms (the SU group) show large and significant increases in wages due to the MW shock. Panel (b) shows differences in the coefficients relative to the most unconstrained group, SU. The estimates show a clear pattern: the longer the exposure to the MW constraint, the smaller the wage response to the demand shock. At the same time, we find no

<sup>45</sup>We find that the MW hike increased hourly wages in strongly constrained firms by 1.7% without affecting employment. For firms that became constrained with the hike, both wage and employment effects were statistically insignificant. Full estimation details and robustness checks are provided in Appendix E.

evidence of differential employment or value added responses across groups. Panels (a) and (c) of Appendix Figure F.15 show the event study estimates, and Panels (b) and (d) present differences relative to the SU group, confirming no evidence of differential responses.

**Table 4:** Estimated Firm-Level Elasticities: Variation in Constraints in Portugal.

	Labor Supply Elasticity	Rent-Sharing Elasticity	Share of Firms (%)
Constrained at 2015 MW (SC)	19.77 [-3.54, 43.09]	0.02 [-0.00, 0.05]	9.92
Constrained at 2016 MW (UCC)	15.72 [-5.79, 37.23]	0.03 [-0.01, 0.08]	7.04
Constrained at 2017 MW (UUC)	8.56 [-1.49, 18.60]	0.06 [-0.01, 0.13]	6.03
Unconstrained at 2017 MW (UUU)	5.27 [2.71, 7.84]	0.10 [0.06, 0.14]	16.78
Strongly Unconstrained (SU)	4.66 [3.11, 6.22]	0.11 [0.08, 0.13]	60.23
All Firms	5.52 [3.71, 7.32]	0.09 [0.07, 0.11]	100.0

Notes: The table shows the estimated elasticities for different groups of Portuguese firms, using the internal shock design, by constrained status of firms. We classify firms based on their 2015 mean base wages. SC denotes constrained by the 2015 MW, UCC denotes unconstrained by the 2015 MW but would be constrained by the 2016 MW, UUC denotes unconstrained by the 2015 and 2016 MW but would be constrained by the 2017 MW, UUU denotes they had a wage between the 2017 MW and up to 15% above it, and SU are all remaining firms. We use the 2017 coefficients  $\beta_{2017}^Y$  to construct the implied elasticities, for  $Y \in \{L, W, VA\}$ . Labor supply elasticities are computed as the ratio  $\beta_{2017}^L/\beta_{2017}^W$ , and rent-sharing elasticities as  $\beta_{2017}^W/\beta_{2017}^{VA}$ . Share of firms refers to the distribution of firms in the pre-shock year, 2015. The 95 percent confidence intervals in brackets are constructed using the delta method.

**Implied elasticities with variation in constraints.** We find that, as firms become more constrained, the implied labor supply elasticities increase and the rent-sharing elasticities decrease. Table 4 shows the results. We compare the firms that receive the wage-floor hike with the UUU group, which represents “just” unconstrained firms that are plausibly more similar to them. The confidence interval for the implied labor supply elasticity among UUU firms excludes the point estimates for all less constrained groups, and the rent-sharing interval excludes the point estimates for the UCC and SC groups, highlighting that greater exposure to constraints results in larger distortions in elasticities. In fact, firms with the strongest bite (SC and UCC) behave as one would expect in a competitive benchmark, showing large

implied labor supply elasticities and almost no rent-sharing. These results suggest that constraints have a causal impact on the estimated firm responsiveness to demand shocks.

Finally, we note that strongly unconstrained firms (SU) exhibit elasticities similar to unconstrained firms (UUU). This indicates limited heterogeneity in responsiveness across the wage distribution, supporting the interpretation that wage-setting constraints partly drive the heterogeneity analyses in Section 5.3.

## 6 Discussion and Conclusion

This paper provides new perspectives on our understanding of the responses of firms to demand shocks in the presence of wage-setting constraints. First, we introduce an economic model of firm optimization that implies that responses to demand shocks depend on the firm’s exposure to wage-setting constraints. Second, using comparable demand shocks for Portugal, Norway and Colombia, we document a consistent pattern: within each country, wage responses to demand shocks are significantly lower among constrained firms. And, leveraging an increase in the national minimum wage in Portugal, we provide evidence suggesting that wage-setting constraints have a causal impact on the observed wage responsiveness to shocks. Overall, the results are consistent with our theoretical model and imply that wage-setting constraints are a key driver of the adjustment to demand shocks across firms.

Our economic model implies that wage-setting constraints distort conventional measures of labor supply and rent-sharing elasticities. The extent of such distortions depends on the “bite” of the constraints and the relative strength of the demand shocks between demand-constrained, supply-constrained, and unconstrained firms, as well as the value of the structural parameters. We provide an approach to quantify the share of demand-constrained firms based on the heterogeneity in shock responses, which can guide an assessment of what conventional measures of labor supply and rent-sharing elasticities capture across different institutional contexts. Descriptive statistics and heterogeneity in responses to the demand shock can thus inform a discussion of the potential magnitude of the bias, as we do in this paper. Importantly, we also provide a framework that incorporates general wage-setting constraints, building on insights from models of monopsony and bargaining. This complements recent work suggesting that labor adjustment frictions may also affect firm responsiveness to shocks (Dhyne et al. 2025, Chan et al. 2025, Agostinelli et al. 2025).

We discuss avenues to identify structural parameters taking into consideration the role of wage-setting constraints. First, if one is willing to assume that the labor supply elasticity is constant across firms, then using the conventional estimators on a sample of plausibly unconstrained firms may yield the structural parameters of interest. Two recent papers

follow this approach to estimate the within-market labor supply elasticity by fitting an inverse labor supply equation: [Ahlfeldt et al. \(2023\)](#) does so in Germany, focusing on the period before the introduction of the minimum wage, and [Jáñez and Delgado-Prieto \(2024\)](#) in Portugal, using a sample of firms that pay a certain threshold above the minimum wage. Both papers rely on instrumental strategies to instrument for employment. While the validity of the instruments should be evaluated on a case-by-case basis, our discussion implies that these estimates are less likely to be distorted by the differential behavior of constrained firms.

A second remedy to the identification problem is to develop a structural estimation strategy that explicitly accounts for the potential distortions generated by wage-setting constraints. An interesting example is [Wong \(2026\)](#), who proposes a model in which wages are determined by union-firm bargaining and the outside option of workers depends on the firm’s productivity, invalidating a rent-sharing approach to estimate the union bargaining power. As a result, [Wong \(2026\)](#) develops a model-consistent approach that is robust to this issue. Relatedly, [Azkarate-Askasua and Zerecero \(2025\)](#) studies rent-sharing in a monopsony model with firm-union bargaining, and [Hermo \(2025\)](#) studies responsiveness to shocks in a context of sectoral collective bargaining. Both of these papers estimate the union bargaining power parameters leveraging the model structure instead of fitting regressions that may be distorted by the wage-setting constraints.

While the optimal strategy may depend on the institutional context, our paper highlights the importance of accounting for institutional wage-setting constraints in estimating structural labor supply and rent-sharing parameters. Much of our analysis focused on cases where the underlying structural labor supply elasticity is either constant across firms, or where these elasticities are firm-specific yet invariant to demand shocks. In frameworks where the structural firm-level elasticities endogenously move in response to demand shocks—e.g., through changes in the local oligopsonistic equilibrium ([Berger et al. 2022](#)), worker composition ([Volpe 2024](#)), or simply non-iso-elastic preferences ([Kline 2025](#))—the empirical identification challenges are more severe as firms may be differentially exposed to institutional wage-setting constraints across the productivity distribution. A natural direction for future work is to devise empirical strategies for richer models of imperfect competition that allow firms to be differentially exposed to wage-setting constraints.

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# Appendix

## A Details on Theoretical Model

### A.1 Micro-Foundation of Labor Supply Equation

This appendix provides a micro-foundation for the labor supply function in Equation (1).

We consider a unit measure of homogeneous workers choosing among a finite set of firms  $\mathcal{J}$ . The indirect utility for worker  $i$  from working at firm  $j$  is  $V_{ij} = \psi_j W_j \xi_{ij}$ , where  $\psi_j W_j$  is the expected wage and  $\xi_{ij}$  is an idiosyncratic preference shock. We assume that  $\xi_{ij}$  is drawn independently for each worker-firm pair from a Fréchet distribution with common shape parameter  $\eta > 0$  and a firm-specific scale parameter (or amenity level)  $A_j > 0$ , so the cdf is  $F_j(\xi) = \exp(-A_j \xi^{-\eta})$ .

Each worker selects the firm that offers the highest utility. Let  $u_j = \psi_j W_j$  denote the systematic component of utility, standard results imply that the probability of a worker choosing firm  $j$  is  $P_j = (A_j u_j^\eta) / (\sum_{k \in \mathcal{J}} A_k u_k^\eta)$ . (See, for example, [McFadden 1978](#) and [Train 2009](#).) Substituting the expression for the systematic utility component yields

$$P_j = \frac{A_j (\psi_j W_j)^\eta}{\sum_{k \in \mathcal{J}} A_k (\psi_k W_k)^\eta}.$$

Given that the total measure of workers is assumed to be one, the labor supply to firm  $j$  equals this probability. We define an aggregate wage index  $\mathbb{W}$  that summarizes the attractiveness of all employment options in the market as  $\mathbb{W} = (\sum_{k \in \mathcal{J}} A_k (\psi_k W_k)^\eta)^{1/\eta}$ . Substituting this definition into the expression for  $P_j$  gives the desired result:

$$H(\psi_j W_j, \mathbb{W}; A_j) = \frac{A_j (\psi_j W_j)^\eta}{\mathbb{W}^\eta} = A_j \left( \frac{\psi_j W_j}{\mathbb{W}} \right)^\eta,$$

where the shape parameter  $\eta$  corresponds to the labor supply elasticity. The notation  $H(\cdot; A_j)$  reflects that heterogeneity in supply for a given wage arises only due to  $A_j$ .

### A.2 Wage Floor Constraint

This appendix discusses the solutions to the firm optimization problem under wage floor constraints. Recall that the firm maximizes profits  $\Pi = \Phi f(L) - WL$  subject to the labor supply constraint  $L \leq H(\psi W, \mathbb{W}; A)$ , with  $H(\psi W, \mathbb{W}; A) = A (\psi W)^\eta \mathbb{W}^{-\eta}$ , and the wage-setting constraint  $W \geq \underline{W}$ . For simplicity, we abstract from the sub-index  $j$ .

### A.2.1 Unconstrained Regime

To start with *unconstrained* firms we assume  $W_U^* > \underline{W}$ . First, note that the labor supply must bind as, for a given wage, profits are strictly increasing in  $L$ , so  $\psi^* = 1$ . We can then write profits as  $\Pi(W) = \Phi f(H(W, \mathbb{W}; A)) - WH(W, \mathbb{W}; A)$ . The first order condition (FOC) that determines the optimal wage  $W_U^*$  equates the marginal revenue product of labor (MRPL) to the marginal cost of labor (MCL):

$$\underbrace{\Phi f'(H(W_U^*, \mathbb{W}; A)) \frac{dH(W_U^*, \mathbb{W}; A)}{dW}}_{MRPL} = \underbrace{H(W_U^*, \mathbb{W}; A) + W_U^* \frac{dH(W_U^*, \mathbb{W}; A)}{dW}}_{MCL},$$

where we assumed the firm takes  $\mathbb{W}$  as exogenous, as in the main text. We can rearrange to obtain the usual wage equation in monopsonistic labor markets

$$W_U^* = \left( \frac{\eta}{\eta + 1} \right) \Phi f'(H(W_U^*, \mathbb{W}; A)) = \mu \Phi f'(L_U^*), \quad (\text{A.1})$$

where the second equality defines  $\mu \equiv \eta/(\eta + 1)$  and replaces  $L_U^* = H(W_U^*, \mathbb{W}; A)$ .

### A.2.2 Supply- and Demand-Constrained Regimes

In the case of *supply-constrained* firms both constraints are binding. The firm equilibrium is given by the constraints, resulting in  $W_S^* = \underline{W}$  and  $L_S^* = H(\underline{W}, \mathbb{W}; A)$ .

Finally, for *demand-constrained* firms the wage-setting constraint is binding, implying  $W_D^* = \underline{W}$ , but the labor supply constraint is not, thus  $L_D^* < H(\psi_D^* \underline{W}, \mathbb{W}; A)$ . The problem is then choosing  $L$  to maximize  $\Pi = \Phi f(L) - \underline{W}L$ , with FOC given by  $\Phi f'(L_D^*) = \underline{W}$ . We can recover the hiring probability with  $\psi_D^* = L_D^*/H(\psi_D^* \underline{W}, \mathbb{W}; A)$ , resulting in  $\psi_D^* = (L_D^*/A)^{1/(\eta+1)} (\mathbb{W}/\underline{W})^{\eta/(\eta+1)}$ .

### A.2.3 Observed Markdown Factor and Labor Share

We derive the observed markdown factor  $\tilde{\mu} \equiv W/(\Phi f'(L))$  and the labor share  $\tilde{s}_L \equiv WL/(\Phi f(L))$  for the three regimes. Note that these quantities are linked by the output elasticity of labor  $\alpha(L) \equiv Lf'(L)/f(L)$  such that  $\tilde{s}_L = \tilde{\mu}\alpha(L)$ .

For *unconstrained* firms, the optimality condition (A.1) applies. Thus,  $\tilde{\mu}_U = \mu = \eta/(\eta+1)$  and  $\tilde{s}_{L,U} = \mu\alpha(L_U^*)$ . Under a homogeneous production function (Assumption 1(a)), the labor share is constant at  $\mu\alpha$ . For *supply-constrained* firms, employment and wages are fixed at  $L_S^* = H(\underline{W}, \mathbb{W}; A)$  and  $\underline{W}$ . The observed markdown is then  $\tilde{\mu}_S = \underline{W}/(\Phi f'(L_S^*))$ , which is strictly decreasing in productivity  $\Phi$ . Similarly, the labor share  $\tilde{s}_{L,S}$  declines with pro-

ductivity as revenue increases while the wage bill remains constant. For *demand-constrained* firms, the first order condition requires the MRPL to equal the wage floor  $\underline{W}$ . Since the firm pays exactly  $\underline{W}$ , we have  $\tilde{\mu}_D = \underline{W}/\underline{W} = 1$ . Consequently, the labor share is  $\tilde{s}_{L,D} = \alpha(L_D^*)$ . Under Assumption 1(a),  $\tilde{s}_{L,D} = \alpha$ .

### A.3 Local Bargaining Constraint

This appendix discusses the solution when the constraint takes the form  $W \geq g(\Upsilon, \underline{W}) = \kappa\Upsilon + (1 - \kappa)\underline{W}$ . As discussed in the main text,  $\Upsilon = \Phi f(L)/L$  is the average revenue product of labor and  $\kappa \in (0, 1)$  is the bargaining power of the local union. We assume  $\Upsilon > \underline{W}$ , which guarantees that the “negotiated wage”  $\kappa\Upsilon + (1 - \kappa)\underline{W}$  is above the wage floor  $\underline{W}$ . This is a natural restriction since firms operating with an average product below  $\underline{W}$  would have negative profits. We omit sub-indexes  $j$  for simplicity.

The solution  $(W^*, L^*)$  is characterized by three regimes, as illustrated in Appendix Figure A.1. First, firms in the unconstrained regime set wages according to Equation (A.1), and employment is given by the labor supply curve. The solution for supply-constrained firms is again determined by the intersection of the two constraints, i.e.,  $W_S^* = \kappa\Phi f(L_S^*)/L_S^* + (1 - \kappa)\underline{W}$  and  $L_S^* = A(W_S^*)^\eta \mathbb{W}^{-\eta}$ . Demand-constrained firms choose employment on the labor demand curve. Replacing the wage constraint into the profit function we find  $\Pi = (1 - \kappa)(\Phi f(L) - \underline{W}L)$ , with FOC  $\underline{W} = \Phi f'(L_D^*)$  and final wage  $W_D^* = \kappa\Phi f(L_D^*)/L_D^* + (1 - \kappa)\underline{W}$ . Given our assumptions, employment is not distorted by the local union bargaining power  $\kappa$ .<sup>1</sup>

Appendix Figure A.1 displays the case of a homogeneous production function (Assumption 1(a)), so it is useful to prove that the demand-constrained wage is constant in this case. First, recall that Euler’s theorem for homogeneous functions states  $Lf'(L) = \alpha f(L)$ , or  $\Phi f(L)/L = \Phi f'(L)/\alpha$ , where  $\alpha \in (0, 1)$  is the degree of homogeneity. Additionally, in equilibrium the FOC  $\Phi f'(L_D^*) = \underline{W}$  holds, so  $\Phi f(L_D^*)/L_D^* = \underline{W}/\alpha$  and we obtain

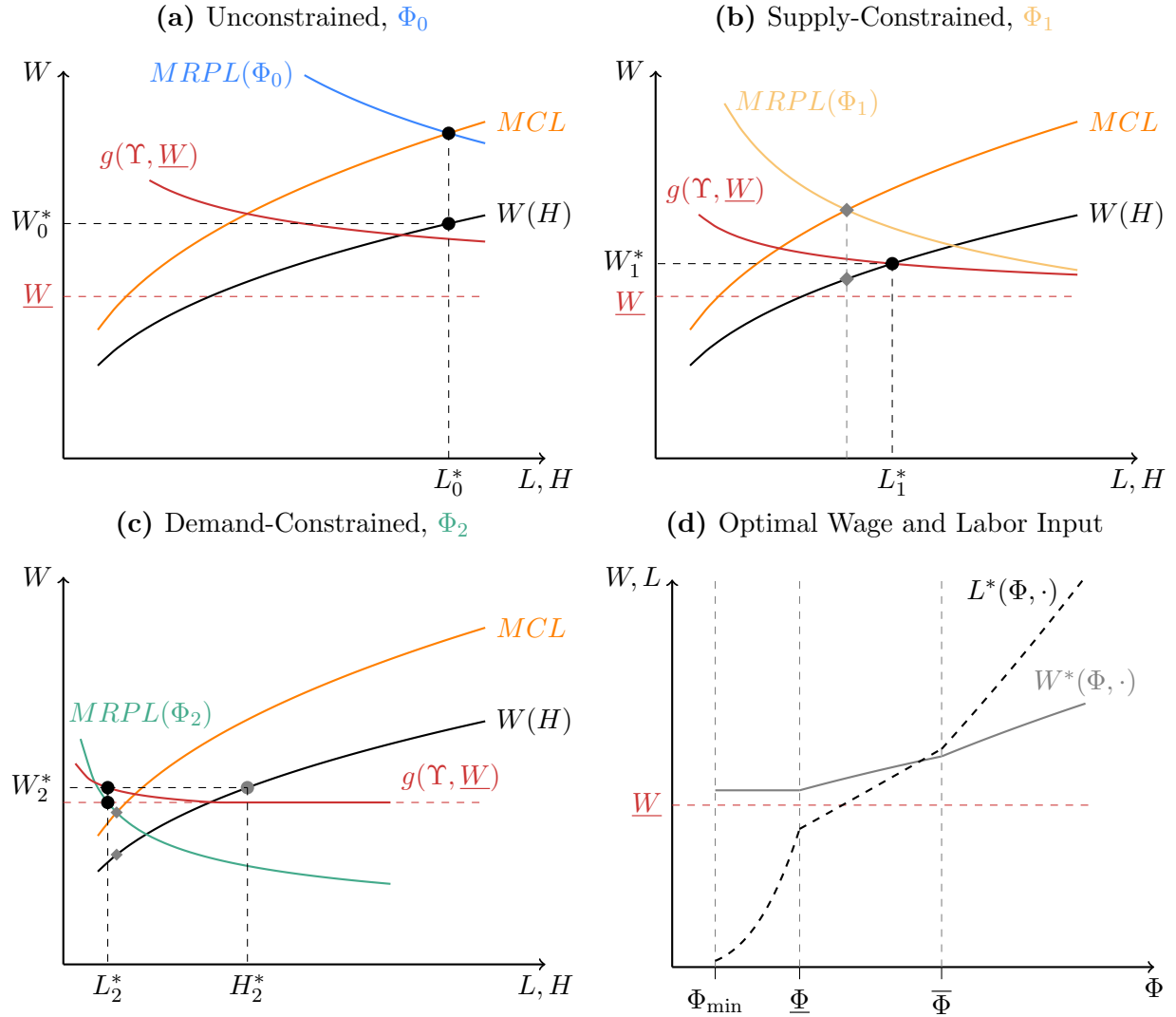
$$W_D^* = \underline{W} \left( 1 + \kappa \frac{1 - \alpha}{\alpha} \right). \quad (\text{A.2})$$

Let us briefly discuss an interpretation of the three regimes when  $\kappa > 0$ . The unconstrained case corresponds to a highly productive firm that faces a relatively weak union. The firm finds it optimal to pay a higher wage than the negotiated one to expand. The constrained case corresponds to stronger unions that set a binding negotiated wage. If the negotiated wage is not too high, then the firm will still find it optimal to locate on the labor

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<sup>1</sup>This result follows from the functional form we imposed on  $g(\cdot, \cdot)$ . Holden (1988), for example, discusses a “work-to-rule” setting where the average product of labor is multiplied by a constant lower than 1, which results in distortions of employment levels for demand-constrained firms.

**Appendix Figure A.1:** Solution to Firm Optimization Problem with Local Bargaining.



Notes: The figure illustrates the equilibrium of the model of firm maximization subject to a labor supply constraint and a local bargaining constraint introduced in Section 2.1, assuming a homogeneous production function with decreasing returns to scale in labor. Panel (a) shows an unconstrained firm for which the bargaining constraint is not binding. Panel (b) shows a supply-constrained firm for which both constraints are binding. Panel (c) shows a demand-constrained firm for which only the bargaining constraint is binding. In the three panels,  $W(H)$  stands for the inverse labor supply curve,  $MCL$  for the marginal cost of labor,  $MRPL$  for the marginal revenue product of labor, and  $g(\Upsilon, \underline{W})$  for the bargaining constraint. The black-filled circles indicate the equilibrium point, and the gray-filled diamonds in Panels (b) and (c) show the latent equilibrium point in the absence of the bargaining constraint. Panel (d) shows the optimal wage (solid line) and optimal labor choice (dashed line) as functions of productivity  $\Phi$ , where  $\Phi_{\min}$  indicates the minimum level of productivity.

supply curve. However, if the wage is high enough, it is better for the firm to choose an employment level lower than the one implied by the labor supply constraint.

**Threshold productivity levels.** We can determine what regime a firm operates in by comparing its productivity  $\Phi$  with threshold productivity levels  $\bar{\Phi}$  and  $\underline{\Phi}$ . The lower threshold is determined by the productivity level at which the demand-constrained employment coincides with the labor supply constraint at the demand-constrained wage  $W_D^*$  from (A.2). The FOC for demand-constrained firms is  $\underline{W} = \Phi f'(L)$ , then

$$\underline{\Phi} f'(H(W_D^*, \mathbb{W}; A)) = \underline{W} \implies \underline{\Phi} = \frac{\underline{W}}{f'(H(W_D^*, \mathbb{W}; A))}.$$

The upper threshold is given by the level of  $\Phi$  at which the unconstrained wage equation equals the wage constraint. Denoting the unconstrained employment by  $L_U^*(\Phi)$  we have

$$\mu \bar{\Phi} f'(L_U^*(\bar{\Phi})) = \kappa \bar{\Phi} \frac{f(L_U^*(\bar{\Phi}))}{L_U^*(\bar{\Phi})} + (1 - \kappa) \underline{W} \implies \bar{\Phi} = \frac{(1 - \kappa) \underline{W}}{\mu f'(L_U^*(\bar{\Phi})) - \kappa \frac{f(L_U^*(\bar{\Phi}))}{L_U^*(\bar{\Phi})}},$$

which is only defined when the denominator is positive. By concavity we know that  $f(L)/L > f'(L)$ , so we will only have unconstrained firms whenever  $\mu$  is sufficiently big relative to  $\kappa$ . In other words, with strong unions ( $\kappa \rightarrow 1$ ) there will not be any unconstrained firms.

We can more easily compare the thresholds in the case of a homogeneous production function. If so,  $f(L)/L = f'(L)/\alpha$  and we can write the upper threshold as

$$\bar{\Phi} = \frac{(1 - \kappa) \underline{W}}{\mu f'(L_U^*(\bar{\Phi})) - \kappa \frac{f'(L_U^*(\bar{\Phi}))}{\alpha}} = \left( \frac{1 - \kappa}{\mu - \frac{\kappa}{\alpha}} \right) \frac{\underline{W}}{f'(L_U^*(\bar{\Phi}))}.$$

We see that  $\bar{\Phi}$  is defined whenever  $\mu > \kappa/\alpha$ . To compare with  $\underline{\Phi}$  note that  $L_U^*(\bar{\Phi}) > H(W_D^*)$ , implying  $f'(L_U^*(\bar{\Phi})) < f'(H(W_D^*))$ . Then

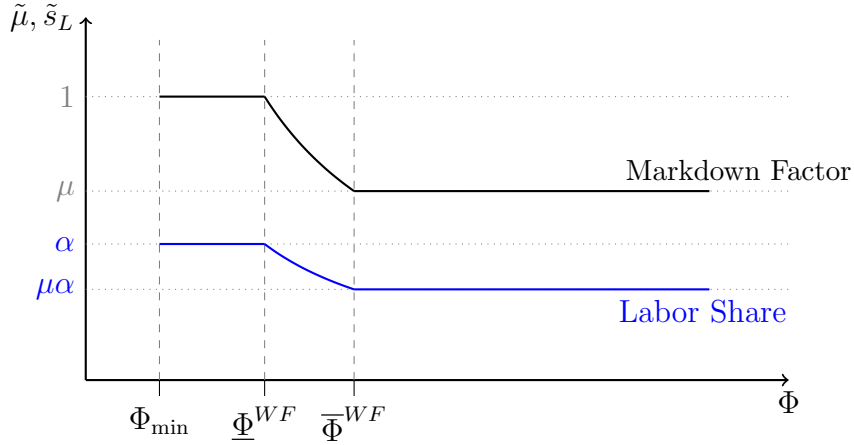
$$\bar{\Phi} > \left( \frac{1 - \kappa}{\mu - \frac{\kappa}{\alpha}} \right) \underline{\Phi}.$$

Since  $\mu < 1$  and  $\alpha \in (0, 1)$ , it follows that  $(1 - \kappa)/(\mu - \kappa/\alpha) > 1$ , implying  $\bar{\Phi} > \underline{\Phi}$  and the thresholds are ranked as expected.

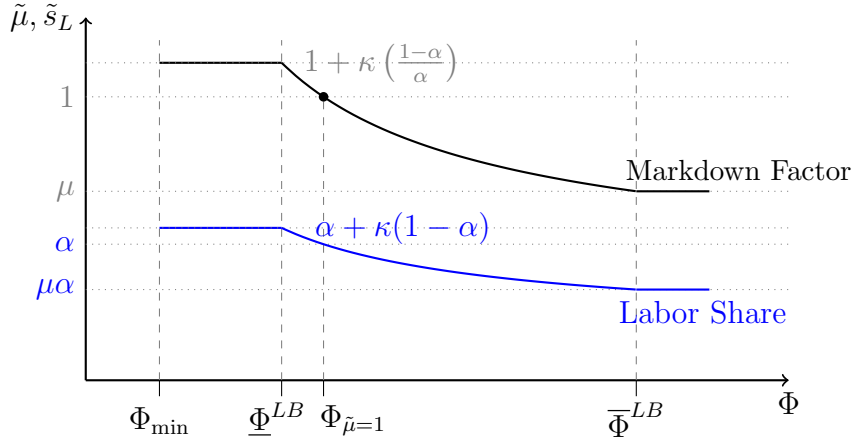
**Observed markdown factor and labor share.** Finally, we characterize the observed markdown  $\tilde{\mu}$  and labor share  $\tilde{s}_L$  under local bargaining. For *unconstrained* firms, the results are identical to the wage floor case ( $\tilde{\mu}_U = \mu$ ). However, *demand-constrained* firms exhibit a markdown strictly larger than 1. Using the wage equation (A.2) derived for homogeneous

**Appendix Figure A.2:** Markdowns and Labor Shares with Wage-Setting Constraints.

(a) Wage Floor Constraint



(b) Local Bargaining Constraint



Notes: This figure compares the theoretical predictions for the observed markdown factor  $\tilde{\mu} = W/\text{MRPL}$  (black lines) and the labor share  $\tilde{s}_L = WL/PY$  (blue lines) as a function of firm productivity  $\Phi$ . Panel (a) presents the case of a pure wage floor constraint. Here, demand-constrained firms ( $\Phi \leq \underline{\Phi}^{WF}$ ) exhibit a markdown of 1, which declines in the supply-constrained regime as productivity rises, eventually stabilizing at the structural markdown  $\mu$  in the unconstrained regime ( $\Phi > \bar{\Phi}^{WF}$ ). Panel (b) presents the case of local bargaining. In contrast to the wage floor case, demand-constrained firms ( $\Phi \leq \underline{\Phi}^{LB}$ ) exhibit a markdown strictly greater than 1, equal to  $1 + \kappa \frac{1-\alpha}{\alpha}$ , and a labor share strictly greater than the output elasticity  $\alpha$ , equal to  $\alpha + \kappa(1-\alpha)$ . The superscripts  $WF$  and  $LB$  denote the wage floor and local bargaining models, respectively. All else equal,  $\underline{\Phi}^{LB} > \underline{\Phi}^{WF}$  and  $\bar{\Phi}^{LB} > \bar{\Phi}^{WF}$ .

production functions, and noting that the MRPL equals  $\underline{W}$  in this regime, we obtain

$$\tilde{\mu}_D = \frac{W_D^*}{\text{MRPL}} = \frac{W(1 + \kappa \frac{1-\alpha}{\alpha})}{\underline{W}} = 1 + \kappa \left( \frac{1-\alpha}{\alpha} \right).$$

Since  $\alpha \in (0, 1)$  and  $\kappa > 0$ , it follows that  $\tilde{\mu}_D > 1$ . Correspondingly, the labor share is  $\tilde{s}_{L,D} = \tilde{\mu}_D \alpha = \alpha + \kappa(1 - \alpha) > \alpha$ . This implies that firms facing binding bargaining constraints pay wages above the marginal revenue product, rationalizing “markdowns” greater than one. In the *supply-constrained* regime, the markdown transitions from the high level  $\tilde{\mu}_D$  to the structural parameter  $\mu$  as productivity increases, as illustrated in Panel (b) of Figure A.2.

## A.4 General Wage-Setting Constraint

The firm maximizes profits subject to the labor supply constraint (2) and the wage-setting constraint  $W \geq g(\Upsilon, \underline{W})$ , where  $\Upsilon(\Phi, L) = \Phi f(L)/L$ . We assume that  $\Upsilon \geq g(\Upsilon, \underline{W}) \geq \underline{W}$ , so the negotiated wage must be below the average product, guaranteeing positive profits, and above the outside option  $\underline{W}$ . We also assume that  $\partial g(\Upsilon, \underline{W})/\partial \Upsilon > 0$  and  $\partial g(\Upsilon, \underline{W})/\partial \underline{W} > 0$ , so the negotiated wage is increasing in the average revenue product of labor and the outside option. We omit the sub-index  $j$  for simplicity.

The firm may operate in one of three regimes, depending on its productivity. *Unconstrained* firms voluntarily pay above the negotiated wage  $g(\cdot, \cdot)$ . The labor supply binds and we have  $L_U^* = H(W_U^*, \mathbb{W}; A)$ . The solution for  $W_U^*$  is given by Equation (A.1), where the wage is a markdown over the marginal revenue product of labor  $\Phi f'(L_U^*)$ .

The firm is *supply constrained* when both constraints bind. In this case its optimal choice is determined by the intersection of the constraints, so that  $W_S^* = g(\Upsilon(\Phi, L_S^*), \underline{W})$  and  $L_S^* = H(W_S^*, \mathbb{W}; A)$ . It is useful to compare the equilibrium wage with  $\Phi f'(L_S^*)$ . By assumption  $\Upsilon(\Phi, L_S^*) = \Phi f(L_S^*)/L_S^* \geq W_S^* \geq \underline{W}$ , and by concavity  $\Phi f(L_S^*)/L_S^* > \Phi f'(L_S^*)$ . Both the wage and the marginal revenue product are strictly bounded above by the average revenue product. However, the ranking between  $W_S^*$  and  $\Phi f'(L_S^*)$  is ambiguous.

The firm is *demand constrained* if the wage constraint binds but the labor supply constraint does not. In this case, the firm’s optimization problem is to choose  $L$  to maximize profits  $\Pi = \Phi f(L) - g(\Upsilon(\Phi, L), \underline{W})L$ . Define the elasticities

$$\lambda(L) = \frac{\partial g(\Upsilon(\Phi, L), \underline{W})}{\partial \Upsilon} \frac{\Upsilon(\Phi, L)}{g(\Upsilon(\Phi, L), \underline{W})} \quad \text{and} \quad \chi(L) = \frac{\partial \Upsilon(\Phi, L)}{\partial L} \frac{L}{\Upsilon(\Phi, L)}.$$

The first order condition that implicitly determines  $L_D^*$  is then

$$\Phi f'(L_D^*) = [1 + \lambda(L_D^*)\chi(L_D^*)] g(\Upsilon(\Phi, L_D^*), \underline{W}) = \omega(L_D^*) g(\Upsilon(\Phi, L_D^*), \underline{W}) \quad (\text{A.3})$$

where  $\omega(L_D^*) \equiv 1 + \lambda(L_D^*)\chi(L_D^*)$ .<sup>2</sup> By assumption we have  $\lambda(L) > 0$  and, under concave

<sup>2</sup>In the case of local bargaining discussed in Section A.3 we have  $\lambda = \kappa \Upsilon / (\kappa \Upsilon + (1 - \kappa) \underline{W})$  and  $\psi = (\Phi f'(L) - \Upsilon) / \Upsilon$ . Replacing  $g(\cdot, \cdot)$  and these expressions in Equation (A.3) yields  $\Phi f'(L^*) = \underline{W}$ ,

production functions,  $\chi(L^*) < 0$  implying  $\omega(L^*) < 1$ . Demand-constrained firms unambiguously set employment to equate the marginal revenue product of labor to a *fraction*  $\omega(L^*)$  of the wage  $g(\Upsilon, \underline{W})$ . To see this more clearly, we can write (A.3) as

$$\underbrace{g(\Upsilon(\Phi, L^*), \underline{W})}_{\text{wage}} = \frac{1}{\underbrace{\omega(L^*)}_{\text{wedge} > 1}} \underbrace{\Phi f'(L^*)}_{MRPL}.$$

The firm hires beyond the point where the marginal revenue product equals the wage. Because each additional worker dilutes the average product of labor, thereby relaxing the wage constraint, the effective marginal cost of labor is strictly lower than the final wage.

## A.5 Equilibrium

In this appendix, we close the theoretical model. First, we show that all firms have positive operative profits  $\Pi_j$  in equilibrium. Second, we show that a labor market equilibrium with wage-setting constraints exists and is unique.

### A.5.1 Partial Equilibrium Profits

We claim that equilibrium operative profits ( $\Pi^*$ ) are always positive. First, for unconstrained firms, we have

$$\begin{aligned} \Pi_U^* &= \Phi f(L_U^*) - \mu \Phi f'(L_U^*) L_U^* \\ &= \Phi [f(L_U^*) - \mu f'(L_U^*) L_U^*]. \end{aligned}$$

We know that  $\mu \in (0, 1)$ . Additionally, by the concavity of  $f(\cdot)$ , we have  $f(L) > f'(L)L$  for any  $L > 0$ . These facts imply that the expression in brackets is positive, and thus  $\Pi_U^* > 0$ .

Second, consider firms for which the wage-setting constraint binds, regardless of its specific form. For these firms, profits can be written as

$$\Pi^* = \Phi f(L^*) - W^* L^* = L^* \left[ \Phi \frac{f(L^*)}{L^*} - W^* \right],$$

where  $L^*$  denotes equilibrium employment and  $W^*$  the constrained wage. By assumption, the constraint satisfies  $g(\Upsilon(\Phi, L^*), \underline{W}) \leq \Upsilon(\Phi, L^*) = \Phi f(L^*)/L^*$ , so profits are weakly positive. To establish strict positivity, start with demand-constrained firms. Rearranging the FOC in Equation (A.3) implies  $g(\Upsilon(\Phi, L_D^*), \underline{W}) = \Phi f'(L_D^*)/\omega(L_D^*)$ . Since  $f(\cdot)$  is strictly

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the optimality condition discussed before.

concave,  $\Phi \frac{f(L_D^*)}{L_D^*} > \Phi f'(L_D^*)$ , so  $\omega(L_D^*) < 1$  implies  $\Phi f(L_D^*)/L_D^* > g(\Upsilon(\Phi, L_D^*), \underline{W})$  and profits are positive. An analogous argument applies to supply-constrained firms, for which both constraints bind and  $g(\Upsilon(\Phi, L_S^*), \underline{W}) < \Upsilon(\Phi, L_S^*)$  by construction.

### A.5.2 General Equilibrium

We assume that the economy is populated by a unit measure of workers and a discrete set of firms  $\mathcal{J}$ . We define the equilibrium below.

**Definition 2** (Equilibrium). *Given productivities  $\{\Phi_j\}_{j \in \mathcal{J}}$  and strictly concave production technologies  $\{f_j(\cdot)\}_{j \in \mathcal{J}}$ , amenities  $\{A_j\}_{j \in \mathcal{J}}$ , continuous wage-setting constraints  $\{g_j(\cdot, \cdot)\}_{j \in \mathcal{J}}$ , and a wage floor  $\underline{W}$ , an equilibrium is an aggregate wage index  $\mathbb{W}^*$  and firm-level allocations  $\{(W_j^*, L_j^*, \psi_j^*)\}_{j \in \mathcal{J}}$  such that:*

1. (Firm optimality) For each firm  $j$ , taking  $\mathbb{W}^*$  as given,  $(W_j^*, L_j^*, \psi_j^*)$  solves the firm optimization problem.
2. (Consistent rationing) If  $L_j^* < H(\psi_j^* W_j^*, \mathbb{W}^*; A_j)$ , then the equilibrium hiring probability satisfies  $\psi_j^* = L_j^*/H(\psi_j^* W_j^*, \mathbb{W}^*; A_j)$ .
3. (Labor market clearing)  $\sum_{j \in \mathcal{J}} H(\psi_j^* W_j^*, \mathbb{W}^*; A_j) = 1$ .

The following proposition establishes the existence and uniqueness of the equilibrium. As in the main text, we maintain the assumption that  $g_j(\Upsilon_j, \underline{W}) \in [\Upsilon_j, \underline{W}]$  is continuous and weakly increasing in both of its arguments.

**Proposition 3** (Existence and Uniqueness of Equilibrium). *Given primitives  $\{\Phi_j\}_{j \in \mathcal{J}}$ ,  $\{f_j(\cdot)\}_{j \in \mathcal{J}}$ ,  $\{A_j\}_{j \in \mathcal{J}}$ ,  $\{g_j(\cdot, \cdot)\}_{j \in \mathcal{J}}$ , and  $\{\underline{W}_j\}_{j \in \mathcal{J}}$ , an equilibrium*

$$(\mathbb{W}^*, \{W_j^*, L_j^*, \psi_j^*\}_{j \in \mathcal{J}})$$

*exists and is unique.*

*Proof of Proposition 3.* Fix any  $\mathbb{W} > 0$ . For any firm  $j$ , taking  $\mathbb{W}$  as given, the labor supply function  $H(\psi_j W_j, \mathbb{W}; A_j)$  is continuous and strictly increasing in the expected wage. The wage-setting constraint  $W_j \geq g_j(\Upsilon_j(L_j), \underline{W})$  is continuous and bounded. As the production function  $f_j(L_j)$  is strictly concave ( $f_j''(L_j) < 0$ ), the firm's MRPL intersects the relevant marginal cost of labor (determined by either the labor supply curve or the wage-setting constraint) at a single crossing point. Therefore, the firm optimization problem admits a unique solution  $(W_j^*(\mathbb{W}), L_j^*(\mathbb{W}), \psi_j^*(\mathbb{W}))$  for each  $\mathbb{W} > 0$ .

Next, we show that  $H(\psi_j^* W_j^*, \mathbb{W}; A_j)$  is strictly decreasing in  $\mathbb{W}$  for all  $j$ . Unconstrained and supply-constrained firms have a hiring probability of  $\psi_j^* = 1$ . An increase in  $\mathbb{W}$  shifts the

labor supply curve inward. For unconstrained firms, this raises the marginal cost of labor, reducing optimal employment. For supply-constrained firms, employment is determined by the intersection of the inward-shifting labor supply curve and the non-increasing wage-setting constraint  $g_j(\cdot)$ , which strictly lowers employment  $L_j^*$ . Thus, as  $H(\psi_j^* W_j^*, \mathbb{W}; A_j) = L_j^*(\mathbb{W})$  is strictly decreasing in  $\mathbb{W}$ . For demand-constrained firms, the wage constraint binds such that the firm rations employment ( $\psi_j^* < 1$ ). In this regime, the optimal employment level is independent of  $\mathbb{W}$ . However, the number of workers attached to the firm,  $H_j$ , adjusts to satisfy the consistent rationing condition. Substituting  $\psi_j^* = L_j^*/H_j$  into the supply equation yields  $H_j = A_j(L_j^* W_j^*/(H_j \mathbb{W}))^\eta$ . Solving for  $H_j$  implies  $H_j(\mathbb{W}) \propto \mathbb{W}^{-\frac{\eta}{1+\eta}}$ . Since  $L_j^*$  and  $W_j^*$  are constant with respect to  $\mathbb{W}$ ,  $H_j(\mathbb{W})$  is strictly decreasing.

Define the aggregate labor supply as

$$S(\mathbb{W}) = \sum_{j \in \mathcal{J}} H(\psi_j^*(\mathbb{W}) W_j^*(\mathbb{W}), \mathbb{W}; A_j).$$

Since  $H(\psi_j^* W_j^*, \mathbb{W}; A_j)$  is continuous and strictly decreasing in  $\mathbb{W}$ ,  $S(\mathbb{W})$  is also continuous and strictly decreasing. As  $\mathbb{W} \rightarrow 0$ , labor supply to each firm diverges and  $S(\mathbb{W}) \rightarrow \infty$ . As  $\mathbb{W} \rightarrow \infty$ , labor supply to every firm vanishes and  $S(\mathbb{W}) \rightarrow 0$ . By the intermediate value theorem, there exists a unique  $\mathbb{W}^* > 0$  such that  $S(\mathbb{W}^*) = 1$ . Given  $\mathbb{W}^*$ , the firm-level allocations  $\{(W_j^*, L_j^*, \psi_j^*)\}_{j \in \mathcal{J}}$  are uniquely determined, which establishes existence and uniqueness of the equilibrium.  $\square$

The wage index  $\mathbb{W}^*$  depends on the parameters of the model, in particular the wage floor  $\underline{W}$ , the productivities  $\{\Phi_j\}_{j \in \mathcal{J}}$ , and the amenities  $\{A_j\}_{j \in \mathcal{J}}$ . The value of  $\mathbb{W}^*$  relative to the wage floor  $\underline{W}$  determines the threshold productivity levels and the constrained status of every firm. Since we assumed away fixed costs and profits are positive for firms in every regime, then all  $|\mathcal{J}|$  firms will be active in equilibrium. If, in equilibrium, there are demand-constrained firms, there will be labor rationing, i.e.,  $\sum_{j \in \mathcal{J}} L_j^* < 1$ .

## A.6 Comparative Statics

### A.6.1 Wage Floor Constraint

We start with the pure wage floor constraint discussed in Section A.2. The goal is to derive the elasticities of  $L^*(\Phi, \underline{W}, \mathbb{W}, A)$  and  $W^*(\Phi, \underline{W}, \mathbb{W}, A)$  with respect to  $\Phi$ , where we abstract from the sub-index  $j$ . As in the main text, we define  $\alpha(L) \equiv \frac{L f'(L)}{f(L)} > 0$  as the employment elasticity of revenue and  $\gamma(L) \equiv -\frac{L f''(L)}{f'(L)} > 0$  as the negative of the employment elasticity of the marginal revenue product of labor.

Totally differentiating the equations that determine the unconstrained equilibrium with respect to  $\ln \Phi$  yields

$$\frac{d \ln W_U^*}{d \ln \Phi} = \frac{1}{1 + \eta\gamma(L_U^*)} > 0 \quad \text{and} \quad \frac{d \ln L_U^*}{d \ln \Phi} = \frac{\eta}{1 + \eta\gamma(L_U^*)} > 0.$$

Next, let us analyze wage-constrained firms. For *supply-constrained* firms, wages and employment are independent of the firm's productivity, so

$$\frac{d \ln W_S^*}{d \ln \Phi} = 0 \quad \text{and} \quad \frac{d \ln L_S^*}{d \ln \Phi} = 0.$$

Finally, for *demand-constrained* firms, we differentiate the first order condition to obtain

$$\frac{d \ln W_D^*}{d \ln \Phi} = 0 \quad \text{and} \quad \frac{d \ln L_D^*}{d \ln \Phi} = \frac{1}{\gamma(L^*)} > 0.$$

In this case, an increase in productivity allows the firm to hire more workers at  $\underline{W}$ , with the magnitude of the increase depending on the curvature of the production function.

We compute the ratio between the employment and wage derivatives, following the usual approach to derive labor supply elasticities. For unconstrained firms we have  $\eta_U = \eta$ . For supply-constrained firms both employment and wage responses are zero, and thus the ratio is undefined. For demand-constrained firms we have  $\eta_D \rightarrow \infty$ .

We continue by deriving the response of value added to the shock, which in this model is simply equal to equilibrium revenue  $VA^*(\Phi, \underline{W}) = \Phi f(L^*(\Phi, \underline{W}))$ , to find

$$\frac{d \ln VA^*}{d \ln \Phi} = 1 + \frac{d \ln f(L^*)}{d \ln \Phi} = 1 + \alpha(L^*) \frac{d \ln L^*}{d \ln \Phi}. \quad (\text{A.4})$$

For an *unconstrained* firm, we substitute the elasticity of labor to obtain

$$\frac{d \ln VA_U^*}{d \ln \Phi} = 1 + \frac{\eta\alpha(L_U^*)}{1 + \eta\gamma(L_U^*)} = \frac{1 + \eta(\gamma(L_U^*) + \alpha(L_U^*))}{1 + \eta\gamma(L_U^*)}.$$

Value added increases due to the direct productivity effect and because the firm increases its employment in response to the shock. For a *supply-constrained* firm, employment does not react to the firm's own productivity shock, resulting in

$$\frac{d \ln VA_S^*}{d \ln \Phi} = 1.$$

Finally, for a *demand-constrained* firm, the response of value added is

$$\frac{d \ln VA_D^*}{d \ln \Phi} = 1 + \frac{\alpha(L_D^*)}{\gamma(L_D^*)}.$$

We conclude by computing the rent-sharing induced by a productivity shock as the ratio of the wage response to the value-added response. Using previous expressions we have

$$\theta_U = \frac{1}{1 + \eta(\gamma(L^*) + \alpha(L^*))} \quad \text{and} \quad \theta_S = \theta_D = 0.$$

**Homogeneous production function.** Assume that  $f(L) = L^\alpha$  for some constant  $\alpha \in (0, 1)$ . Then, we have  $\alpha(L) = \alpha$  and  $\gamma(L) = 1 - \alpha$ . It is straightforward to solve for the derivatives above, all of which turn out to be constant. Interestingly, the rent-sharing elasticity for unconstrained firms becomes  $\theta_U = 1/(1 + \eta)$ .

### A.6.2 Local Bargaining Constraint

We now discuss the local bargaining case introduced in Section A.3. We start by deriving the elasticities of  $L^*(\Phi, \underline{W}, \mathbb{W}, A)$  and  $W^*(\Phi, \underline{W}, \mathbb{W}, A)$  with respect to  $\Phi$  under the local bargaining constraint  $g(\Upsilon, \underline{W}) = \kappa\Upsilon + (1 - \kappa)\underline{W}$ . As before, define  $\alpha(L) \equiv \frac{Lf'(L)}{f(L)} > 0$  and  $\gamma(L) \equiv -\frac{Lf''(L)}{f'(L)} > 0$ . Also note that the elasticity  $\lambda(L) = (\partial g(\Upsilon, \underline{W})/\partial \Upsilon)(\Upsilon/g(\Upsilon, \underline{W})) = \kappa\Upsilon/(\kappa\Upsilon + (1 - \kappa)\underline{W})$  is simply the share of the average product in the wage.

For *unconstrained* firms, the solution is identical to the pure wage floor case.

For *supply-constrained* firms, both constraints bind. The equilibrium is determined by  $W_S^* = \kappa\Phi f(L_S^*)/L_S^* + (1 - \kappa)\underline{W}$  and  $L_S^* = A(W_S^*)^\eta \mathbb{W}^{-\eta}$ . Substituting the first equation into the second yields  $L_S^* = A\left(\kappa\Phi \frac{f(L_S^*)}{L_S^*} + (1 - \kappa)\underline{W}\right)^\eta \mathbb{W}^{-\eta}$ . Taking logs and totally differentiating with respect to  $\ln L_S^*$  and  $\ln \Phi$ :

$$d \ln L_S^* = \eta \lambda(L_S^*) [d \ln \Phi + (\alpha(L_S^*) - 1) d \ln L_S^*].$$

Thus, the employment response is

$$\frac{d \ln L_S^*}{d \ln \Phi} = \frac{\eta \lambda(L_S^*)}{1 + \eta \lambda(L_S^*) (1 - \alpha(L_S^*))} > 0. \quad (\text{A.5})$$

For the wage response, differentiate  $W_S^* = \kappa\Phi f(L_S^*)/L_S^* + (1 - \kappa)\underline{W}$  to obtain

$$d \ln W_S^* = \lambda(L_S^*) [d \ln \Phi + (\alpha(L_S^*) - 1) d \ln L_S^*]. \quad (\text{A.6})$$

Solving for  $d \ln L_S^*$  in terms of  $d \ln \Phi$  in (A.5) and replacing yields

$$\frac{d \ln W_S^*}{d \ln \Phi} = \frac{\lambda(L_S^*)}{1 + \eta \lambda(L_S^*) (1 - \alpha(L_S^*))} > 0. \quad (\text{A.7})$$

Unlike the pure wage floor case, supply-constrained firms' wages and employment both respond positively to productivity shocks. This occurs because higher productivity increases the average revenue product, which directly enters the wage constraint.

For *demand-constrained* firms, recall that the optimal employment satisfies  $\Phi f'(L_D^*) = \underline{W}$  and the wage is  $W_D^* = \kappa \Phi f(L_D^*)/L_D^* + (1 - \kappa)\underline{W}$ . Differentiating the FOC yields

$$\frac{d \ln L_D^*}{d \ln \Phi} = \frac{1}{\gamma(L_D^*)} > 0.$$

We can differentiate  $W_D^* = \kappa \Phi f(L_D^*)/L_D^* + (1 - \kappa)\underline{W}$  to obtain an expression analogous to (A.6) for demand-constrained firms. Replacing in  $d \ln L_D^* = -d \ln \Phi / \gamma(L_D^*)$  we get

$$\frac{d \ln W_D^*}{d \ln \Phi} = \lambda(L_D^*) \left( \frac{\alpha(L_D^*) + \gamma(L_D^*) - 1}{\gamma(L_D^*)} \right).$$

As  $\gamma(L_D^*) > 0$ , the sign of  $\frac{d \ln W_D^*}{d \ln \Phi}$  depends on the sign of  $\alpha(L_D^*) + \gamma(L_D^*) - 1$ , which boils down to whether the output elasticity of labor  $\alpha(L)$  is increasing or decreasing in  $L$ . As discussed before, under Assumption 1(a)  $\alpha(L_D^*) + \gamma(L_D^*) = 1$  and the wage response is zero, and under Assumption 1(b)  $\alpha(L_D^*) + \gamma(L_D^*) > 1$  and the wage response is positive.

Next, we compute the ratio of employment to wage responses. For unconstrained firms we have  $\eta_U = \eta$ , as in the pure wage floor case. For supply-constrained we also obtain  $\eta_S = \eta$ . Finally, for demand-constrained firms we have

$$\eta_D = \frac{d \ln L_D^*/d \ln \Phi}{d \ln W_D^*/d \ln \Phi} = \frac{1}{\lambda(L_D^*) (\alpha(L_D^*) + \gamma(L_D^*) - 1)}.$$

In the case of a homogeneous production function (Assumption 1(a)), the denominator is zero and so  $\eta_D \rightarrow \infty$ . More generally, provided that the production function exhibits "weak curvature" (i.e.,  $\alpha(L) + \gamma(L) \approx 1$ ), the denominator remains small. Consequently, we expect  $\eta_D$  to be large and, in particular,  $\eta_D > \eta$ .

We now derive the response of value added to productivity shocks. As before, value added is  $VA^*(\Phi, \underline{W}) = \Phi f(L^*(\Phi, \underline{W}))$  and so a general expression for the derivative is (A.4). Then, for *unconstrained* firms the result is identical to the wage floor case. For *supply-constrained*

firms we substitute the employment elasticity to get

$$\frac{d \ln VA_S^*}{d \ln \Phi} = \frac{1 + \eta \lambda(L_S^*)}{1 + \eta \lambda(L_S^*) (1 - \alpha(L_S^*))}.$$

For *demand-constrained* firms we get

$$\frac{d \ln VA_D^*}{d \ln \Phi} = 1 + \frac{\alpha(L_D^*)}{\gamma(L_D^*)},$$

which is identical to the wage floor case since the FOC is the same.

Finally, we compute the ratio of wage and value-added responses for firms in each regime. For *unconstrained* firms we have  $\theta_U = 1 / (1 + \eta [\gamma(L_U^*) + \alpha(L_U^*)])$ , which is identical to the wage floor case.

For *supply-constrained* firms:

$$\theta_S = \frac{\lambda(L_S^*)}{1 + \eta \lambda(L_S^*)} > 0.$$

Unlike the wage floor case, supply-constrained firms show a positive rent-sharing elasticity under local bargaining.<sup>3</sup> We can now discuss the ranking  $\theta_S < \theta_U$  discussed in the main text. The condition  $\theta_U > \theta_S$  holds whenever

$$\frac{1 - \lambda(L_S^*)}{\lambda(L_S^*)} > \eta (\alpha(L_U^*) + \gamma(L_U^*) - 1).$$

In the Cobb-Douglas case (Assumption 1(a)), the right-hand-side is zero, and since  $\lambda(L_S^*) \in (0, 1)$  the inequality holds trivially. In the general case, the inequality holds provided that the curvature of the production function is not sufficiently large to offset the dampening effect of the bargaining share. Thus, rent-sharing for supply-constrained firms is typically dampened relative to the unconstrained benchmark.

And, for *demand-constrained* firms,

$$\theta_D = \lambda(L_D^*) \frac{\gamma(L_D^*) + \alpha(L_D^*) - 1}{\gamma(L_D^*) + \alpha(L_D^*)}.$$

As discussed before, in the Cobb-Douglas case we obtain  $\theta_D = 0$ . In the general case,  $\theta_D$  remains small relative to  $\theta_U$  unless the production function exhibits strong curvature.

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<sup>3</sup>Panel (d) of Appendix Figure A.1 illustrates this result, showing a positive slope for the optimal wage with respect to  $\Phi$  in the range  $\Phi \in (\underline{\Phi}, \bar{\Phi})$ .

### A.6.3 General Wage-Setting Constraint

We briefly discuss the predictions for this case. Recall that  $\lambda(L) = \frac{\partial g(\Upsilon, W)}{\partial \Upsilon} \frac{\Upsilon}{g(\Upsilon, W)}$  is the elasticity of the wage constraint with respect to the average product, and note that  $\frac{\partial \ln \Upsilon}{\partial \ln L} = \alpha(L) - 1$ . For *unconstrained* firms, the predictions are identical to the previous cases. For *supply-constrained* firms, the expressions are identical to (A.5) and (A.7), subject to the general definition of  $\lambda(L)$ . Thus, provided  $\lambda(L) > 0$ , we again find positive but dampened rent-sharing ( $\theta_S < \theta_U$ ). For *demand-constrained* firms the exact expressions can be obtained by differentiating the system that determines  $(W_D^*, L_D^*)$  given by Equation (A.3) and the wage  $W_D^* = g(\Upsilon(\Phi, L_D^*), \underline{W})$ . Crucially, the qualitative predictions remain unchanged: under homogeneous production functions ( $\alpha(L)$  constant), the wage response is zero and the apparent labor supply elasticity diverges ( $\eta_D \rightarrow \infty$ ), just as in the local bargaining case.

## B Proofs

*Proof of Lemma 1.* We first derive the result for  $\bar{\eta}$ . In the counterfactual scenario without constraints, all firms are on the labor supply curve, thus Equation (1) implies  $\Delta \ln L_j^* = \eta \Delta \ln W_j^* - \eta \Delta \ln \mathbb{W}$ . Taking conditional expectations and differencing between  $Z_j = 1$  and  $Z_j = 0$ , the term involving the aggregate wage index  $\mathbb{W}$  cancels out and we obtain

$$\mathbb{E} [\Delta \ln L_j^* | Z_j = 1] - \mathbb{E} [\Delta \ln L_j^* | Z_j = 0] = \eta (\mathbb{E} [\Delta \ln W_j^* | Z_j = 1] - \mathbb{E} [\Delta \ln W_j^* | Z_j = 0]).$$

Dividing the employment response by the wage response yields  $\bar{\eta} = \eta$ .

Next, we derive the result for  $\bar{\theta}$ . From Result 1, the structural rent-sharing elasticity for firm  $j$  is  $\theta_j \equiv (1 + \eta[\alpha_j(L_j^*) + \gamma_j(L_j^*)])^{-1}$ . For this proof only, define the auxiliary coefficients  $\beta_j \equiv (1 + \eta\gamma_j(L_j^*))^{-1}$  and  $\delta_j \equiv (1 + \eta[\alpha_j(L_j^*) + \gamma_j(L_j^*)]) / (1 + \eta\gamma_j(L_j^*))$ , so that  $\theta_j = \beta_j / \delta_j$ . Using the comparative statics derived in Appendix A.6, the unconstrained responses can be written as  $\Delta \ln W_j^* \approx \beta_j \Delta \ln \Phi_j$  and  $\Delta \ln VA_j^* \approx \delta_j \Delta \ln \Phi_j$ . Substituting into Definition 1:

$$\bar{\theta} = \frac{\mathbb{E} [\beta_j \Delta \ln \Phi_j | Z_j = 1] - \mathbb{E} [\beta_j \Delta \ln \Phi_j | Z_j = 0]}{\mathbb{E} [\delta_j \Delta \ln \Phi_j | Z_j = 1] - \mathbb{E} [\delta_j \Delta \ln \Phi_j | Z_j = 0]}. \quad (\text{B.1})$$

To express this as a weighted average, let  $\tau_j = (\alpha_j, \gamma_j)$  denote firm  $j$ 's production technology, which determines  $\beta_j$ ,  $\delta_j$ , and  $\theta_j$ . Then, for notational convenience write the coefficients as  $\beta(\tau_j)$ ,  $\delta(\tau_j)$ ,  $\theta(\tau_j)$ . Define the  $\tau$ -specific first stage:

$$\pi(\tau) \equiv \mathbb{E} [\Delta \ln \Phi_j | Z_j = 1, \tau_j = \tau] - \mathbb{E} [\Delta \ln \Phi_j | Z_j = 0, \tau_j = \tau].$$

Applying the law of iterated expectations to the first term in the numerator of (B.1):

$$\mathbb{E}[\beta_j \Delta \ln \Phi_j | Z_j = 1] = \mathbb{E}[\beta(\tau_j) \mathbb{E}[\Delta \ln \Phi_j | Z_j = 1, \tau_j]].$$

Applying the same reasoning to all four terms in (B.1), the numerator becomes:

$$\begin{aligned} & \mathbb{E}[\beta(\tau_j) \mathbb{E}[\Delta \ln \Phi_j | Z_j = 1, \tau_j]] - \mathbb{E}[\beta(\tau_j) \mathbb{E}[\Delta \ln \Phi_j | Z_j = 0, \tau_j]] \\ &= \mathbb{E}[\beta(\tau_j) (\mathbb{E}[\Delta \ln \Phi_j | Z_j = 1, \tau_j] - \mathbb{E}[\Delta \ln \Phi_j | Z_j = 0, \tau_j])] \\ &= \mathbb{E}[\beta(\tau_j) \pi(\tau_j)]. \end{aligned}$$

By analogous reasoning, the denominator equals  $\mathbb{E}[\delta(\tau_j) \pi(\tau_j)]$  and so

$$\bar{\theta} = \frac{\mathbb{E}[\beta(\tau_j) \pi(\tau_j)]}{\mathbb{E}[\delta(\tau_j) \pi(\tau_j)]} = \frac{\mathbb{E}[\theta(\tau_j) \delta(\tau_j) \pi(\tau_j)]}{\mathbb{E}[\delta(\tau_j) \pi(\tau_j)]},$$

where we replaced  $\beta(\tau) = \theta(\tau) \delta(\tau)$ . Define the weight for a firm with technology  $\tau_j$  as:

$$\omega(\tau_j) \equiv \frac{\delta(\tau_j) \pi(\tau_j)}{\mathbb{E}[\delta(\tau_j) \pi(\tau_j)]}.$$

By construction,  $\mathbb{E}[\omega(\tau_j)] = 1$ . The target parameter can therefore be written as:

$$\bar{\theta} = \mathbb{E}[\omega(\tau_j) \theta(\tau_j)],$$

confirming that  $\bar{\theta}$  identifies a weighted average of the firm-specific rent-sharing elasticities  $\theta(\tau_j)$ . The weight  $\omega(\tau_j)$  is proportional to  $\delta(\tau_j) \pi(\tau_j)$ : firms receive more weight when their value-added responds more elastically to productivity shocks (higher  $\delta$ ) and when the instrument induces larger productivity shifts for their technology type (higher  $\pi$ ).

Finally, we consider the case under Assumption 1(a). Here, the output elasticity of labor is constant, implying  $\alpha_j(L) + \gamma_j(L) = 1$  for all firms. Consequently,  $\theta(\tau_j) = (1 + \eta)^{-1}$  is identical across all firms and  $\bar{\theta} = (1 + \eta)^{-1}$  follows directly.  $\square$

*Proof of Proposition 1. Part (i).* Under Assumption 2(b), firms are either unconstrained or supply-constrained. For these firms, the labor supply equation (1) holds:

$$\Delta \ln L_j = \Delta \ln A_j + \eta \Delta \ln W_j - \eta \Delta \ln \mathbb{W}.$$

Taking expectations conditional on  $Z_j = z$  for  $z \in \{0, 1\}$  and differencing:

$$\begin{aligned} \mathbb{E}[\Delta \ln L_j | Z_j = 1] - \mathbb{E}[\Delta \ln L_j | Z_j = 0] &= \mathbb{E}[\Delta \ln A_j | Z_j = 1] - \mathbb{E}[\Delta \ln A_j | Z_j = 0] \\ &\quad + \eta(\mathbb{E}[\Delta \ln W_j | Z_j = 1] - \mathbb{E}[\Delta \ln W_j | Z_j = 0]). \end{aligned} \tag{B.2}$$

By Assumption 4 (exogeneity), the term involving labor supply shifters  $\Delta \ln A_j$  equals zero. Supply-constrained firms may show no increase in the expected wage, if the shock is small, or some increase in the wage, if the shock is large and some firms move to the unconstrained region. In any case, they are on the labor supply curve and (B.2) holds. Solving for  $\eta$ :

$$\eta = \frac{\mathbb{E}[\Delta \ln L_j | Z_j = 1] - \mathbb{E}[\Delta \ln L_j | Z_j = 0]}{\mathbb{E}[\Delta \ln W_j | Z_j = 1] - \mathbb{E}[\Delta \ln W_j | Z_j = 0]} = \eta^{\text{CE}}$$

as desired. Note that Assumption 3 (relevance) ensures the denominator is non-zero, as the unconstrained firms are on the labor supply curve, so productivity shock affects wages.

*Part (ii).* Under Assumption 2(a) all firms are unconstrained. For simplicity, we focus on the case of homogeneous production functions (Assumption 1(a)). If so, the unconstrained equilibrium is determined by  $W_j = \mu \Phi_j \alpha L_j^{\alpha-1}$  and  $L_j = A_j (W_j / \mathbb{W})^\eta$ .

Let us start with the labor supply elasticity. As Assumption 2(a) implies Assumption 2(b), Part (i) applies directly and  $\eta^{\text{CE}}$  identifies  $\eta$ .

Next, we focus on the rent-sharing elasticity. Solving for the unconstrained equilibrium  $(W_j^*, L_j^*)$  and replacing  $VA_j^* = \Phi_j (L_j^*)^\alpha$  yields

$$\begin{aligned} \ln W_j^* &= \frac{1}{1 + \eta(1 - \alpha)} \ln \Phi_j - \frac{1 - \alpha}{1 + \eta(1 - \alpha)} \ln A_j + C^W, \\ \ln VA_j^* &= \frac{1 + \eta}{1 + \eta(1 - \alpha)} \ln \Phi_j + \frac{\alpha}{1 + \eta(1 - \alpha)} \ln A_j + C^{VA}, \end{aligned}$$

where  $C^W$  and  $C^{VA}$  are constants that are common to all firms. The expected changes in wages and value added due to the shocks are then:

$$\begin{aligned} \mathbb{E}[\ln W_j^* | Z_j = 1] - \mathbb{E}[\ln W_j^* | Z_j = 0] &= \frac{1}{1 + \eta(1 - \alpha)} (\mathbb{E}[\ln \Phi_j | Z_j = 1] - \mathbb{E}[\ln \Phi_j | Z_j = 0]) \\ &\quad - \frac{1 - \alpha}{1 + \eta(1 - \alpha)} (\mathbb{E}[\ln A_j | Z_j = 1] - \mathbb{E}[\ln A_j | Z_j = 0]), \\ \mathbb{E}[\ln VA_j^* | Z_j = 1] - \mathbb{E}[\ln VA_j^* | Z_j = 0] &= \frac{1 + \eta}{1 + \eta(1 - \alpha)} (\mathbb{E}[\ln \Phi_j | Z_j = 1] - \mathbb{E}[\ln \Phi_j | Z_j = 0]) \\ &\quad + \frac{\alpha}{1 + \eta(1 - \alpha)} (\mathbb{E}[\ln A_j | Z_j = 1] - \mathbb{E}[\ln A_j | Z_j = 0]). \end{aligned}$$

By Assumption 4 (exogeneity), the terms involving  $\Delta \ln A_j$  vanish, so  $\theta^{\text{CE}}$  yields

$$\begin{aligned}\theta^{\text{CE}} &= \frac{\frac{1}{1+\eta(1-\alpha)} (\mathbb{E}[\ln \Phi_j | Z_j = 1] - \mathbb{E}[\ln \Phi_j | Z_j = 0])}{\frac{1+\eta}{1+\eta(1-\alpha)} (\mathbb{E}[\ln \Phi_j | Z_j = 1] - \mathbb{E}[\ln \Phi_j | Z_j = 0])} \\ &= \left( \frac{1}{1 + \eta(1 - \alpha)} \right) \left( \frac{1 + \eta(1 - \alpha)}{1 + \eta} \right) = \frac{1}{1 + \eta} = \theta,\end{aligned}\tag{B.3}$$

as desired. Note that the second equality uses Assumption 3 (relevance) to ensure that  $\mathbb{E}[\Delta \ln \Phi_j | Z_j = 1] - \mathbb{E}[\Delta \ln \Phi_j | Z_j = 0] \neq 0$ .  $\square$

*Proof of Proposition 2.* We provide counterexamples where identification fails, starting with the labor supply elasticity. Under Assumption -2(b) (Strong Constraints), there are demand-constrained firms in the economy. For simplicity, we consider the case where  $\rho_{uu} + \rho_{ss} + \rho_{dd} = 1$ , with strictly positive shares of unconstrained ( $u$ ), supply-constrained ( $s$ ), and demand-constrained ( $d$ ) firms. For this proof only, we introduce the indicators  $UU_j, SS_j, DD_j \in \{0, 1\}$ , which are equal to 1 if firm  $j$  belongs to the respective group.

The average employment response is an average of the group-specific responses:

$$\begin{aligned}\mathbb{E}[\Delta \ln L_j | Z_j = 1] - \mathbb{E}[\Delta \ln L_j | Z_j = 0] \\ &= \rho_{dd} (\mathbb{E}[\Delta \ln L_j | Z_j = 1, DD_j = 1] - \mathbb{E}[\Delta \ln L_j | Z_j = 0, DD_j = 1]) + \rho_{ss} \cdot 0 \\ &\quad + \rho_{uu} (\mathbb{E}[\Delta \ln L_j^* | Z_j = 1, UU_j = 1] - \mathbb{E}[\Delta \ln L_j^* | Z_j = 0, UU_j = 1]).\end{aligned}$$

Similarly, the average wage response is:

$$\begin{aligned}\mathbb{E}[\Delta \ln W_j | Z_j = 1] - \mathbb{E}[\Delta \ln W_j | Z_j = 0] &= \rho_{dd} \cdot 0 + \rho_{ss} \cdot 0 \\ &\quad + \rho_{uu} (\mathbb{E}[\Delta \ln W_j^* | Z_j = 1, UU_j = 1] - \mathbb{E}[\Delta \ln W_j^* | Z_j = 0, UU_j = 1]).\end{aligned}$$

The conventional estimator  $\eta^{\text{CE}}$  is the ratio of these expressions. Using the fact that, for unconstrained firms, the ratio of responses identifies  $\eta$ , we obtain

$$\eta^{\text{CE}} = \eta + \frac{\rho_{dd}}{\rho_{uu}} \frac{\mathbb{E}[\Delta \ln L_j | Z_j = 1, DD_j = 1] - \mathbb{E}[\Delta \ln L_j | Z_j = 0, DD_j = 1]}{\mathbb{E}[\Delta \ln W_j^* | Z_j = 1, UU_j = 1] - \mathbb{E}[\Delta \ln W_j^* | Z_j = 0, UU_j = 1]}.\tag{B.4}$$

Note that  $\eta^{\text{CE}}$  does not identify the structural parameter  $\eta$ . The additional term, which arises from the employment response of demand-constrained firms, introduces bias.

We now turn to the rent-sharing elasticity, relying on -2(a) (Weak Constraints). To show that demand-constrained firms are not needed for this result, we consider  $\rho_{ss} + \rho_{uu} = 1$  with

$\rho_{ss} > 0$ . Since supply-constrained firms do not adjust wages, the average wage effect is:

$$\begin{aligned} & \mathbb{E} [\Delta \ln W_j | Z_j = 1] - \mathbb{E} [\Delta \ln W_j | Z_j = 0] \\ &= \rho_{uu} \left( \mathbb{E} [\Delta \ln W_j^* | Z_j = 1, UU_j = 1] - \mathbb{E} [\Delta \ln W_j^* | Z_j = 0, UU_j = 1] \right). \end{aligned}$$

However, value added responds for both groups. For supply-constrained firms, value added moves with productivity even if employment is fixed. The average value-added effect is:

$$\begin{aligned} & \mathbb{E} [\Delta \ln VA_j | Z_j = 1] - \mathbb{E} [\Delta \ln VA_j | Z_j = 0] \\ &= \rho_{ss} \left( \mathbb{E} [\Delta \ln VA_j | Z_j = 1, SS_j = 1] - \mathbb{E} [\Delta \ln VA_j | Z_j = 0, SS_j = 1] \right) \\ &+ \rho_{uu} \left( \mathbb{E} [\Delta \ln VA_j^* | Z_j = 1, UU_j = 1] - \mathbb{E} [\Delta \ln VA_j^* | Z_j = 0, UU_j = 1] \right). \end{aligned}$$

Constructing the estimator  $\theta^{\text{CE}}$ , we find:

$$\begin{aligned} \theta^{\text{CE}} &= \iota \times \frac{\mathbb{E} [\Delta \ln W_j^* | Z_j = 1, UU_j = 1] - \mathbb{E} [\Delta \ln W_j^* | Z_j = 0, UU_j = 1]}{\mathbb{E} [\Delta \ln VA_j^* | Z_j = 1, UU_j = 1] - \mathbb{E} [\Delta \ln VA_j^* | Z_j = 0, UU_j = 1]} \\ &= \iota \theta \end{aligned} \tag{B.5}$$

where the attenuation factor is

$$\iota = \frac{\rho_{uu} \left( \mathbb{E} [\Delta \ln VA_j^* | Z_j = 1, UU_j = 1] - \mathbb{E} [\Delta \ln VA_j^* | Z_j = 0, UU_j = 1] \right)}{\left\{ \begin{aligned} & \rho_{uu} \left( \mathbb{E} [\Delta \ln VA_j^* | Z_j = 1, UU_j = 1] - \mathbb{E} [\Delta \ln VA_j^* | Z_j = 0, UU_j = 1] \right) \\ & + \rho_{ss} \left( \mathbb{E} [\Delta \ln VA_j | Z_j = 1, SS_j = 1] - \mathbb{E} [\Delta \ln VA_j | Z_j = 0, SS_j = 1] \right) \end{aligned} \right\}} \in (0, 1).$$

The attenuation is driven by the responsiveness of value added among supply-constrained firms in the denominator. Thus, we conclude  $\theta^{\text{CE}} < \theta$ , as desired.  $\square$

*Proof of Corollary 1.* We proceed by obtaining expressions for the different components in Equation (B.4). Using the model's comparative statics with  $f(L) = L^\alpha$ , we have

$$\begin{aligned} & \mathbb{E} [\Delta \ln L_j | Z_j = 1, DD_j = 1] - \mathbb{E} [\Delta \ln L_j | Z_j = 0, DD_j = 1] \\ & \approx \frac{1}{1 - \alpha} \left( \mathbb{E} [\Delta \ln \Phi_j | Z_j = 1, DD_j = 1] - \mathbb{E} [\Delta \ln \Phi_j | Z_j = 0, DD_j = 1] \right) \\ & = \frac{1}{1 - \alpha} \zeta_{dd} \end{aligned} \tag{B.6}$$

and

$$\begin{aligned}
& \mathbb{E} [\Delta \ln W_j^* | Z_j = 1, UU_j = 1] - \mathbb{E} [\Delta \ln W_j^* | Z_j = 0, UU_j = 1] \\
& \approx \frac{1}{1 + \eta(1 - \alpha)} (\mathbb{E} [\Delta \ln \Phi_j | Z_j = 1, UU_j = 1] - \mathbb{E} [\Delta \ln \Phi_j | Z_j = 0, UU_j = 1]) \quad (\text{B.7}) \\
& = \frac{1}{1 + \eta(1 - \alpha)} \zeta_{uu},
\end{aligned}$$

where we introduced the notation

$$\zeta_{uu} = \mathbb{E} [\Delta \ln \Phi_j | Z_j = 1, UU_j = 1] - \mathbb{E} [\Delta \ln \Phi_j | Z_j = 0, UU_j = 1],$$

and  $\zeta_{dd}$  is defined analogously but conditioning on  $DD_j = 1$ . Plugging (B.6) and (B.7) into (B.4) we get Equation (4) in the corollary. Noting that  $\alpha > 0$ ,  $\eta > 0$ ,  $\rho_{dd} \geq 0$ , and noting that  $\text{sgn}(\zeta_{dd}) = \text{sgn}(\zeta_{uu})$ , we confirm that  $\eta^{\text{CE}} - \eta \geq 0$ .  $\square$

## C Additional Results for Local Bargaining Constraint

This appendix presents additional results for the setting with local bargaining constraints.

**Corollary 2** (Bias Decomposition Formula: Local Bargaining). *Consider the local bargaining constraint  $g_j(\Upsilon_j, W) = \kappa_j \Upsilon_j + (1 - \kappa_j)W$  under Assumption 1(a) (homogeneous production functions). Let Assumptions 3 and 4 hold. Furthermore, assume  $\rho_{uu} + \rho_{ss} + \rho_{dd} = 1$  and  $\text{sgn}(\zeta_{uu}) = \text{sgn}(\zeta_{ss}) = \text{sgn}(\zeta_{dd})$ . Then, the bias of the conventional labor supply elasticity estimator is*

$$\eta^{\text{CE}} - \eta = \frac{\rho_{dd} \frac{1}{1-\alpha} \frac{\zeta_{dd}}{\zeta_{uu}}}{\rho_{uu} \frac{1}{1+\eta(1-\alpha)} + \rho_{ss} \frac{\lambda}{1+\eta\lambda(1-\alpha)} \frac{\zeta_{ss}}{\zeta_{uu}}} > 0, \quad (\text{C.8})$$

where we assumed  $\lambda = \kappa_j \Upsilon_j / (\kappa_j \Upsilon_j + (1 - \kappa_j)W)$  is a constant, for simplicity, and  $\zeta_{dd}$ ,  $\zeta_{ss}$ , and  $\zeta_{uu}$  are the expected changes in productivity due to the shock in each group.

*Proof of Corollary 2.* We decompose the estimator  $\eta^{\text{CE}}$  into the contributions from the different groups of firms. Under the local bargaining constraint, firms in the unconstrained ( $u$ ) and supply-constrained ( $s$ ) regimes satisfy the labor supply condition so, for them,  $\Delta \ln L_j = \eta \Delta \ln W_j$ . In contrast, demand-constrained ( $d$ ) firms are rationed. Under Assumption 1(a) (homogeneous production functions), the wage for demand-constrained firms does not respond to productivity shocks; see Appendix A.6.2, where we discuss the comparative statics for the local bargaining constraint. Following the proof of Proof of Corollary 1,

we can then write

$$\eta^{\text{CE}} = \eta + \rho_{dd} \frac{\mathbb{E}[\Delta \ln L_j | Z_j = 1, DD_j = 1] - \mathbb{E}[\Delta \ln L_j | Z_j = 0, DD_j = 1]}{\mathbb{E}[\Delta \ln W_j^* | Z_j = 1, ON_j = 1] - \mathbb{E}[\Delta \ln W_j^* | Z_j = 0, ON_j = 1]}, \quad (\text{C.9})$$

where  $ON_j$  is a vector that equals one if the firm is “on” the labor supply curve (either unconstrained  $UU_j = 1$  or supply-constrained  $SS_j = 1$ ).

We construct the elements of  $\eta^{\text{CE}}$ . The average employment response among demand-constrained firms is given by (B.6). The average wage response for  $ON_j = 1$  firms is

$$\begin{aligned} & \mathbb{E}[\Delta \ln W_j^* | Z_j = 1, ON_j = 1] - \mathbb{E}[\Delta \ln W_j^* | Z_j = 0, ON_j = 1] \\ & \approx \rho_{uu} \frac{1}{1 + \eta(1 - \alpha)} (\mathbb{E}[\Delta \ln \Phi_j | Z_j = 1, UU_j = 1] - \mathbb{E}[\Delta \ln \Phi_j | Z_j = 0, UU_j = 1]) \\ & \quad + \rho_{ss} \frac{\lambda}{1 + \eta\lambda(1 - \alpha)} (\mathbb{E}[\Delta \ln \Phi_j | Z_j = 1, SS_j = 1] - \mathbb{E}[\Delta \ln \Phi_j | Z_j = 0, SS_j = 1]) \\ & = \rho_{uu} \frac{1}{1 + \eta(1 - \alpha)} \zeta_{uu} + \rho_{ss} \frac{\lambda}{1 + \eta\lambda(1 - \alpha)} \zeta_{ss}, \end{aligned}$$

where  $\lambda = (\kappa\Upsilon) / (\kappa\Upsilon + (1 - \kappa)\underline{W})$  is the share of the negotiated wage corresponding to the average productivity, assumed constant for simplicity. Then, we can write (C.9) as

$$\eta^{\text{CE}} - \eta = \frac{\rho_{dd} \frac{1}{1 - \alpha} \frac{\zeta_{dd}}{\zeta_{uu}}}{\rho_{uu} \frac{1}{1 + \eta(1 - \alpha)} + \rho_{ss} \frac{\lambda}{1 + \eta\lambda(1 - \alpha)} \frac{\zeta_{ss}}{\zeta_{uu}}},$$

as desired. Given that  $\alpha > 0$  and  $\eta > 0$ , and the assumptions on the signs of  $\zeta_{dd}$ ,  $\zeta_{ss}$ , and  $\zeta_{uu}$ , it is easy to see that  $\eta^{\text{CE}} - \eta > 0$ .  $\square$

Unlike in the wage floor case, supply-constrained firms mitigate the bias somewhat as they response in both employment and wages.

We solve for  $\rho_{dd}$  in equation (C.8), assuming that the shock is equally strong across groups so  $\zeta_{dd}/\zeta_{uu} = \zeta_{ss}/\zeta_{uu} = 1$ . Recalling that  $\rho_{dd} + \rho_{ss} + \rho_{uu} = 1$ , we get

$$\rho_{dd} = \frac{\Lambda}{1 + \Lambda \frac{\lambda}{1 + \eta\lambda(1 - \alpha)}} \left[ \rho_{uu} \left( \frac{1}{1 + \eta(1 - \alpha)} \right) + (1 - \rho_{uu}) \frac{\lambda}{1 + \eta\lambda(1 - \alpha)} \right] \quad (\text{C.10})$$

where  $\Lambda = (\eta^{\text{CE}} - \eta)(1 - \alpha)$ . Analogous to the wage floor case discussed in Section 2.2.2, we use equation (C.10) to quantify the share of demand-constrained firms, after plugging in values for  $\eta^{\text{CE}}$ ,  $\eta$ , and  $\alpha$ , and given the share of unconstrained firms  $\rho_{uu}$  in the economy.

## D Details on Data Sources

### D.1 Portugal

We focus on individuals aged 18 to 65, excluding apprentices, workers in agriculture or mining, those in skilled agricultural occupations, individuals in fishing, and those with unidentified occupations. We also exclude workers based in the islands (*Azores* and *Madeira*) or abroad, those without reported earnings or remuneration, and individuals with invalid worker identifiers. When workers have multiple firms in a given year, we use the wage from the highest-wage firm in the year.

For the internal instrument, we define the mean firm wage for “stayer” workers who remain with the same firm for at least seven years within each eight-year cohort window, though we consider an alternative definition using “incumbent” workers. Additionally, we keep only firms with at least two stayers per year that appear continuously in QP over the eight years of each cohort. The resulting number of firms in each cohort used in the estimation with the internal instrument is reported in Appendix Table F.1.

For the external instrument we similarly define the firm wage using stayers. Additionally, we apply several sample restrictions. We restrict to firms that exported in 2005–2007 and whose cumulative real exports over the period accounted for at least 1% of the cumulative real revenue. Following [Garin and Silvério \(2024\)](#), we restrict to firms whose share of exports to Angola or Spain between 2005 and 2007 is less than 90% of total exports.<sup>4</sup>

**Estimating CBA wage floors.** While our baseline definition of constrained firms relies on the MW, we consider an alternative approach based on the CBA wage floor. To this end, we estimate the wage floors following [Cardoso and Portugal \(2005\)](#). We define the CBA wage floor as the modal base wage within each CBA–job title–year cell, retaining the mode only if it is concentrated in the lower part of the wage distribution. Overall, we identify 1,026 CBA wage floors. We follow the standard sample restrictions in [Card and Cardoso \(2022\)](#) for the period 2004–2017. We further exclude agreements with fewer than 50 workers in a given year and drop CBA-1-digit job title-year cells with fewer than 10 workers. The last two restrictions are that the mode can’t exceed 1.25 times the wage of the 10th percentile of the CBA–job title–year wage distribution, and that the modal wage floor lies within a 25% range of the median wage floor in each CBA–job title. We winsorize the remaining values at the 1st and 99th percentiles of the wage-floor distribution.

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<sup>4</sup>We exclude product–country shifts that are missing, either because they are the only suppliers to that market and are therefore mechanically dropped, or because the import demand shift is undefined in the post period. This results in dropping a small number of firms from the sample.

**Minimum wage hikes.** For the analysis in Section 5.4 in Portugal, we focus on the period from 2012 to 2017. After implementing the restrictions outlined above for this period, we transform it into a firm-level dataset. For this analysis, we focus on full-time workers who have worked at least 150 hours in total per month. To construct firm mean wages, we focus on stayers who remain with the same firm for at least five years during the 2012-2017 period. We retain only firms with at least two stayers. We then keep firms that have appeared continuously in QP for the six-year window. For firms with non-negative VA, we define the internal demand shock as the difference between 2016 and 2015, comparing firms in the top half of the VA growth distribution in each two-digit industry.

## D.2 Norway

We focus on the worker’s main job, defined as the job in which a worker obtains the highest cash earnings during a year. We drop spells that are shorter than 31 days or less than 4 hours per week. We measure average hourly labor earnings as annual wage-cash earnings within the job spell relative to the total hours of work in the spell during the year. We restrict our sample to the private sector.

For the internal instrument we proceed analogously to Portugal. We focus on the mean wages of stayers (workers staying in the same firm for at least seven years within each eight-year cohort window). We restrict the sample to firms with at least two stayers per year, observed continuously over the time window, and with non-negative value added. Appendix Table F.1 reports the number of firms per cohort used for the internal instrument.

When using the external instrument, we restrict the sample to firms that exported in at least three years prior to or including 2007 and whose cumulative real exports over 2005–2007 accounted for at least 1% of their cumulative real revenue during the same period.<sup>5</sup>

## D.3 Colombia

Because the EAM reports only firm-level outcomes from survey responses, we cannot impose worker-level restrictions. Instead, we restrict the sample to firms with non-negative value added that are present in at least seven years of each cohort window. To approximate the definition of stayers used in Portugal and Norway, our main analysis focuses on the employment and wages of permanent workers. The number of firms in each cohort for the internal instrument estimation in Colombia is reported in Appendix Table F.1.

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<sup>5</sup>Similarly to Portugal, we exclude country-products with missing shifts, resulting in dropping a small number of firms.

## E Impacts of Demand Shocks and Minimum Wage Hikes

We present an empirical design that disentangles the effects of demand shocks from those of minimum wage shocks developed in Section 5.4. To this end, we first use a specification to estimate the impact of minimum wage changes, then use another specification to estimate the effects of demand shocks, and finally assess the joint impact of both using a third specification akin to the one in the main text. This staged approach helps disentangle the effect of imposing wage-setting constraints more clearly and facilitates a more pedagogical presentation of our empirical strategy with variation in firm constraints.

We begin by estimating the effect of the minimum wage hike on constrained (SC) and partly constrained firms (UCC), using the same firm classification described in Section 3.4.2. Specifically, we define an indicator  $\text{Post}_t = 1$  for 2016, exclude 2015 in the estimation, and use 2014 as the base year to maintain consistency with the event-study estimates in Figure 7. The specification for identifying the effect of minimum wage shocks is:

$$\ln Y_{jt} = \lambda_Y^{SC} \text{Post}_t \mathbf{1}\{G_j = \text{SC}\} + \lambda_Y^{UCC} \text{Post}_t \mathbf{1}\{G_j = \text{UCC}\} + \delta_{k(j)t}^Y + \omega_j^Y + v_{jt}^Y, \quad (1C)$$

where  $\omega_j^Y$  are firm fixed effects,  $\delta_{k(j)t}^Y$  are industry-by-year fixed effects, and  $v_{jt}^Y$  is the error term. Firms unconstrained by the 2015 and 2016 MW but constrained by the 2017 MW (UUC) serve as the control group. The underlying identifying assumption is that firms that become constrained under the new MW would follow similar trends to those already constrained, absent the MW hike. Panels (a) and (b) of Appendix Figure E.3 present supporting evidence of this, showing pre-treatment estimates of wages and employment close to zero in 2013, although in 2012, the coefficients for employment are larger.<sup>6</sup>  $\lambda_Y^{SC}$  then captures the effect of the MW hike for constrained firms on  $Y \in \{W, L, VA\}$ , and  $\lambda_Y^{UCC}$  for firms transitioning from unconstrained to constrained.

The MW hike raises hourly wages by 1.7% without affecting employment for constrained firms (SC), implying an own-wage employment elasticity close to zero. For firms that become constrained (UCC), both wage and employment effects are insignificant, yielding an insignificant positive elasticity (see Column 1 of Appendix Table E.1). The insignificant employment effect is consistent with Card and Cardoso (2022), which examines the employment effects of CBA wage-floor increases in Portugal.<sup>7</sup>

Next, we define demand shocks as an indicator  $Z_j = 1$  for firms experiencing value-added

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<sup>6</sup>To avoid confounding the effects of multiple treatments, we restrict the sample to firms that are not simultaneously exposed to a demand shock in the same period.

<sup>7</sup>According to Dube and Lindner (2024), elasticities above zero are in the right tail of the distribution of estimates in the MW literature, consistent with firms being *supply-constrained* under our theoretical framework.

growth above the median within their 2-digit industry in 2016, and zero otherwise. We estimate the effect of  $Z_j$  only for firms that would be unconstrained under the MW. We group UUU and SU firms into a U group for simplicity in the exposition and estimate the following specification:

$$\ln Y_{jt} = \beta_Y^U \text{Post}_t Z_j + \delta_{k(j)t}^Y + \omega_j^Y + v_{jt}^Y. \quad (2C)$$

Here,  $\beta_Y^U$  captures the impact of  $Z_j$  on unconstrained firms for the different outcomes. Column 2 of Appendix Table E.2 shows that the demand shock increases VA by about 36%, and Column 2 of Appendix Table E.1 shows that average hourly wages of stayers increase by 2% and employment by 12.1%, in line with our baseline results with the internal shock. We then combine this model with the MW specification, adding two interaction terms to capture the differential effects of demand shocks across firms with varying wage constraints, and estimate the following:

$$\begin{aligned} \ln Y_{jt} = & \beta_Y^{SC,U} Z_j \text{Post}_t \mathbf{1}\{G_j = \text{SC}\} + \beta_Y^{UCC,U} Z_j \text{Post}_t \mathbf{1}\{G_j = \text{UCC}\} \\ & + \gamma_Y^{SC} \text{Post}_t \mathbf{1}\{G_j = \text{SC}\} + \gamma_Y^{UCC} \text{Post}_t \mathbf{1}\{G_j = \text{UCC}\} \\ & + \beta_Y^U Z_j \text{Post}_t + \delta_{k(j)t}^Y + \omega_j^Y + v_{jt}^Y. \end{aligned} \quad (3C)$$

The key parameters are  $\beta_Y^{SC,U}$  and  $\beta_Y^{UCC,U}$ , which measure how demand shock effects differ relative to firms that are always unconstrained. This equation is then the extended version of Equation (10) in the main text, including SC firms. Specifically,  $\beta_Y^U + \beta_Y^{SC,U}$  yields the impact for SC firms, and  $\beta_Y^U + \beta_Y^{UCC,U}$  for UCC firms that became constrained in 2016.<sup>8</sup>

We find that  $\beta_W^{SC,U} = -0.012$ , indicating that constrained firms respond less to demand shocks than unconstrained ones. While this aligns with the idea that wage-setting constraints reduce wage responsiveness, differences in underlying heterogeneous labor supply elasticities can rationalize this. Thus, comparing newly constrained firms to those that are always unconstrained provides cleaner evidence. In Column 3 of Appendix Table E.1, we find that  $\beta_W^{UCC,U} = -0.010$ , consistent with the idea that MW constraints limit wage adjustments.

Finally, we conduct a sensitivity analysis of the wage threshold definitions. Panel (a) of Appendix Figure E.4 shows effects fade as the definition of constrained goes farther away from the wage floor, consistent with the MW impacting firms paying closer to the old MW more profoundly. Panel (b) of Appendix Figure E.4 shows that newly constrained firms' wage responses converge toward those of unconstrained firms as thresholds are farther away from the MW, reinforcing the interpretation that constraints limit wage responsiveness to

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<sup>8</sup>The omitted category is always unconstrained firms not exposed to the demand shock. Industry-by-year fixed effects absorb the indicator for  $U_j = 1$ .

demand shocks.

**Appendix Table E.1:** Firm Wage and Employment Responses to Minimum Wage and Demand Shocks: Variation in Constraints in Portugal.

	(1)	(2)	(3)
	MW Shock	Demand Shock	Joint
<i>(a) Mean Wage</i>			
<i>SC</i>	0.017*** (0.006)		0.008 (0.007)
<i>UCC</i>	0.004 (0.005)		-0.002 (0.004)
<i>U, Z<sub>j</sub> = 1</i>		0.020*** (0.003)	0.017*** (0.003)
<i>SC, Z<sub>j</sub> = 1</i>			-0.012* (0.006)
<i>UCC, Z<sub>j</sub> = 1</i>			-0.011 (0.008)
<i>N</i>	10,984	60,998	101,370
<i>(b) Employment</i>			
<i>SC</i>	0.004 (0.009)		-0.007 (0.008)
<i>UCC</i>	-0.000 (0.007)		-0.010 (0.008)
<i>U, Z<sub>j</sub> = 1</i>		0.120*** (0.011)	0.122*** (0.011)
<i>SC, Z<sub>j</sub> = 1</i>			-0.005 (0.014)
<i>UCC, Z<sub>j</sub> = 1</i>			0.018 (0.019)
<i>N</i>	10,984	60,998	101,370

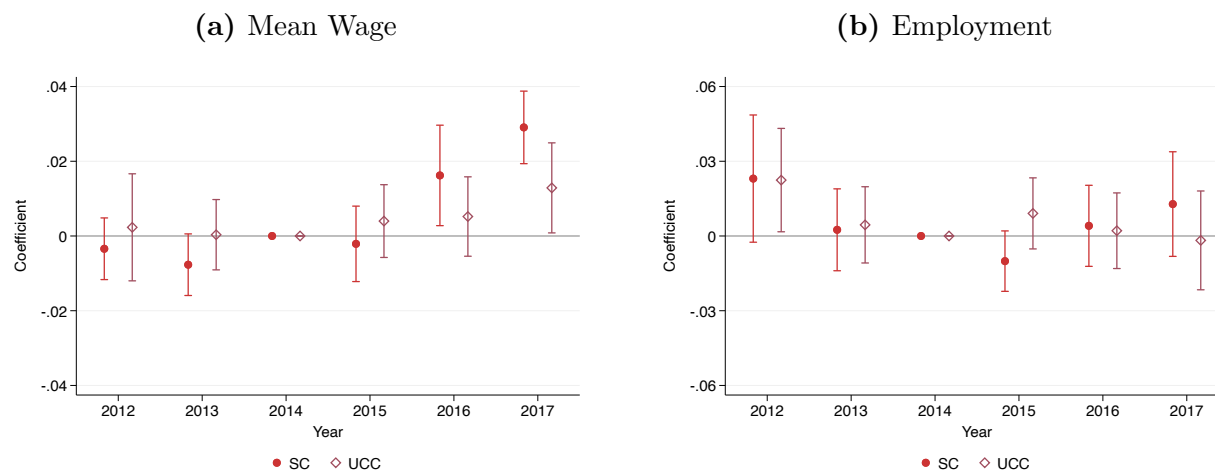
Notes: Each panel reports the coefficients for a different dependent variable. For Panel A, we focus on column (1) on the average wages of all workers, while in column (2), we focus on the average wages of stayers. We classify firms based on their 2015 mean base wages. SC denotes constrained by the 2015 MW, UCC denotes unconstrained by the 2015 MW but would be constrained by the 2016 MW, UUC denotes unconstrained by the 2015 and 2016 MW but would be constrained by the 2017 MW, and U denotes they had a wage higher than the 2017 MW. Standard errors in parentheses are clustered at the industry level. \*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

**Appendix Table E.2:** Firm Value Added Responses to Minimum Wage and Demand Shocks: Variation in Constraints in Portugal.

	(1)	(2)	(3)
	MW Shock	Demand Shock	Joint
<i>SC</i>	-0.014 (0.012)		0.030 (0.024)
<i>UCC</i>	-0.010 (0.016)		0.030 (0.020)
<i>U, Z<sub>j</sub> = 1</i>		0.360*** (0.021)	0.358*** (0.016)
<i>SC, Z<sub>j</sub> = 1</i>			0.021 (0.050)
<i>UCC, Z<sub>j</sub> = 1</i>			-0.019 (0.031)
<i>N</i>	10,984	60,998	101,370

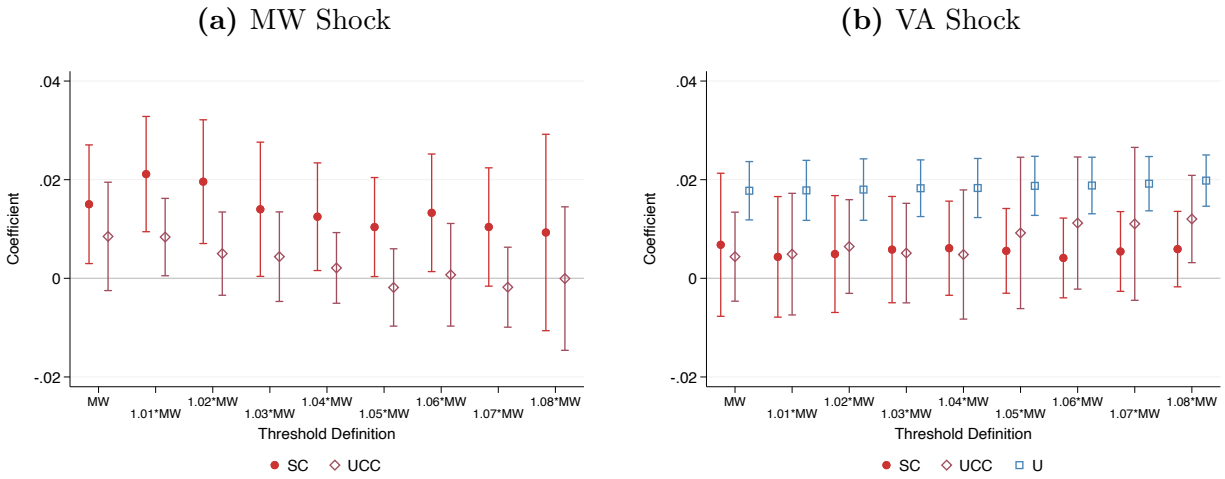
Notes: We report the coefficients for the log value added. We classify firms based on their 2015 mean base wages. SC denotes constrained by the 2015 MW, UCC denotes unconstrained by the 2015 MW but would be constrained by the 2016 MW, UUC denotes unconstrained by the 2015 and 2016 MW but would be constrained by the 2017 MW, and U denotes they had a wage higher than the 2017 MW. Standard errors in parentheses are clustered at the industry level. \*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

**Appendix Figure E.3:** Dynamic Responses to Minimum Wage Shocks: Variation in Constraints in Portugal.



Notes: We focus on the wages and employment of all workers in the firm. For quantifying the dynamic coefficients, we estimate a dynamic version of Equation (1C) separately for employment and mean wages. We classify firms based on their 2015 mean base wages. SC denotes constrained by the 2015 MW, and UCC denotes unconstrained by the 2015 MW but would be constrained by the 2016 MW.

**Appendix Figure E.4:** Wage Responses Varying the Definition of Constrained Firms in Portugal in 2016.



Notes: Panel (a) shows the effects of the minimum wage shock on total wages using Equation (1C). Panel (b) shows the effects of the demand shock on total wages using Equation (2C). We classify firms based on their 2015 mean base wages. SC denotes constrained by the 2015 MW, UCC denotes unconstrained by the 2015 MW but would be constrained by the 2016 MW, and U denotes they had a wage above the 2017 MW. The sample of firms in each panel varies as the control group differs. Panel (a) includes firms that would be constrained by the MW in 2017 (UUC) as control. Panel (b) includes only firms in the same group, but that did not receive a demand shock in 2016 as a control.

## F Supplementary Figures and Tables

**Appendix Table F.1:** Number of Firms Used in Estimations By Country and Cohort.

	Portugal	Norway	Colombia
2001		26,724	
2002		27,855	
2003		28,801	
2004		29,724	5,012
2005		30,187	5,022
2006		30,510	5,247
2007		30,592	5,253
2008	47,646	31,066	5,382
2009	50,459	31,956	5,377
2010	49,473	32,794	5,358
2011	50,588	33,939	5,799
2012	51,610	34,631	6,764
2013	54,329	34,868	7,420
2014	52,684	36,708	7,026
2015		38,103	
2016		39,266	

Notes: The table shows the number of firms per cohort that used for the internal shock design.

**Appendix Table F.2:** Descriptive Statistics for Exporting and Non-Exporting Firms in 2007.

	Portugal			Norway		
	A	E	N	A	E	N
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Number of firms</i>	48,921	4,859	44,062	31,540	2,966	28,574
(Share exporting)		(0.10)	(0.90)		(0.09)	(0.91)
<i>Log value added</i>						
Mean	12.27	13.40	12.14	15.64	16.76	15.52
SD	1.33	1.40	1.26	1.26	1.43	1.18
<i>Log Employment</i>						
Mean	2.25	3.31	2.14	2.37	3.21	2.28
SD	1.09	1.21	1.01	1.13	1.28	1.08
<i>Log Wages</i>						
Mean	1.64	1.75	1.63	5.47	5.60	5.46
SD	0.45	0.45	0.44	0.34	0.31	0.34

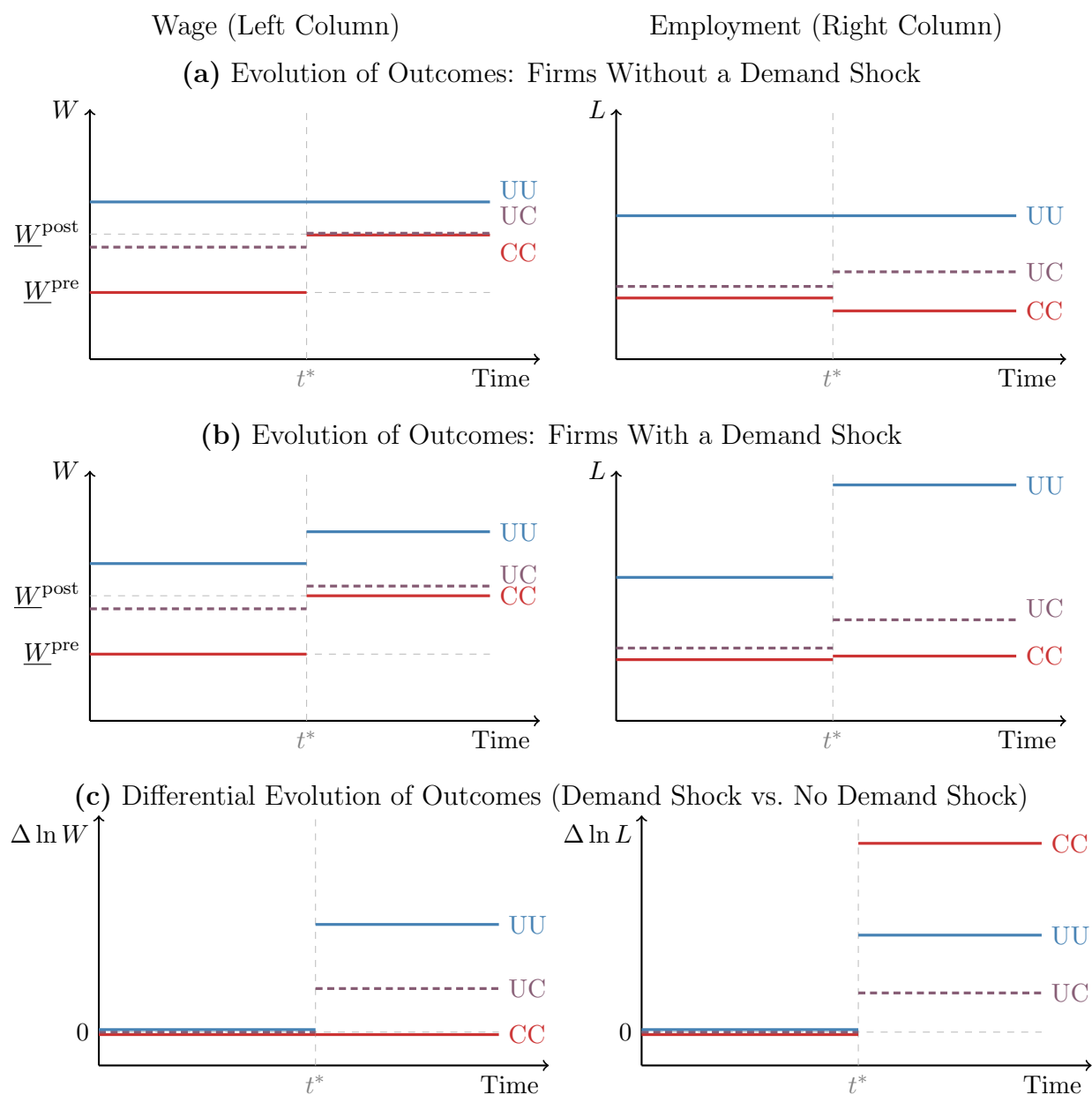
Notes: The table shows descriptive statistics for key firm-level variables in 2007. The sample is restricted to firms with non-negative value added, at least two stayers per year, and observed continuously over the cohort window, where stayers are workers who remain with the same firm for at least seven years within each eight-year cohort window. “A” (All) refers to all firms satisfying these restrictions. “E” (Exporters) refers to firms that exported in at least three years prior to or including 2007 and whose cumulative real exports over 2005–2007 accounted for at least 1% of their cumulative real revenue over the same period. For Portugal, we additionally exclude firms whose share of exports to Angola or Spain over 2005–2007 exceeded 90% of total exports, following [Garin and Silvério \(2024\)](#). “N” (Non-exporters) refers to all remaining firms in the sample. The wage variable corresponds to the mean hourly wage of stayers, measured in Euros (Portugal) and Norwegian Kroner (Norway).

**Appendix Table F.3:** Validating Labor Supply and Rent-Sharing Elasticities: Internal and External Instruments.

	Portugal				Norway			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Labor supply	5.84 (0.24)	4.67 (0.28)	3.98 (0.73)	2.39 (1.75)	3.29 (0.08)	2.78 (0.15)	4.27 (0.88)	2.93 (1.09)
Rent-sharing	0.09 (0.00)	0.12 (0.01)	0.12 (0.02)	0.20 (0.12)	0.19 (0.00)	0.20 (0.01)	0.13 (0.03)	0.19 (0.07)
Instrument	Int.	Int.	Int.	Ext.	Int.	Int.	Int.	Ext.
Cohort	All	GR	GR	GR	All	GR	GR	GR
Sample	All	All	Export	Export	All	All	Export	Export

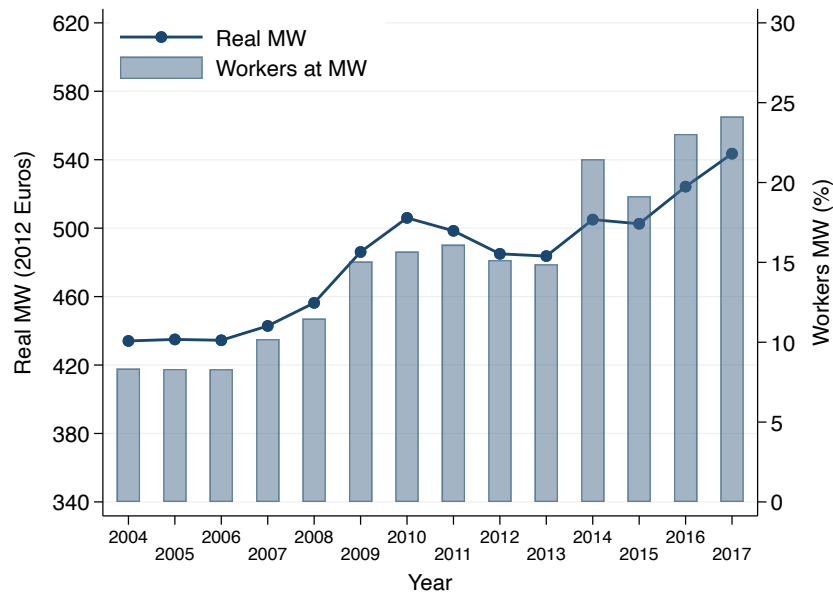
Notes: The table summarizes the estimates of the labor supply elasticity, defined as the ratio of the average employment and wage reduced form effects, and the rent-sharing elasticity, defined as the ratio of the average wage and value added reduced form effects, for Portugal and Norway, and for different specifications. “Int.” and “Ext.” refer to whether the reduced form effects are identified using the internal or the external instrument, respectively. “GR” refers to the Great Recession cohort used for the external instrument. Columns (1) and (4) report estimates from Table 2 for Portugal and Columns (5) and (8) report estimates from Table 2 for Norway. The sample includes firms with positive employment over 7 years and at least one stayer. In Columns (2)-(4) and (6)-(8), only one cohort around the Great Recession is included, and in Columns (3)-(4) and (7)-(8), the sample is further restricted to only cover exporting firms. The estimated elasticities are obtained by averaging reduced form estimates on value added, wages and employment over the post-periods 2009, 2010, 2011, and next taking ratios of these averaged estimates. Across all specifications, year and firm fixed effects are included. Standard errors, which are included in parenthesis, are calculated based on the delta method and in Columns (1) and (5) clustered at the 2-digit industry level, and in Columns (2)-(4) and (6)-(8) clustered at the firm level.

**Appendix Figure F.1:** Triple-Difference Design with Wage Floor Hike and Demand Shock.



Notes: The figure illustrates the predictions of the model with a wage floor constraint introduced in Section 2.1 for wage and employment outcomes when a wage floor hike from  $\underline{W}^{\text{pre}}$  to  $\underline{W}^{\text{post}}$  at time  $t^*$  concurs with a positive demand shock. Panels (a) and (b) show the evolution of outcomes for firms that do not receive a demand shock and firms that do receive it, respectively. Panel (c) plots the difference in outcomes between firms with and without the shock. As in Figure 2, we distinguish between three types of firms based on their pre-shock wage: those constrained at both wage floors (CC), those unconstrained before but unconstrained after the increase (UC), and those unconstrained at either wage floor (UU). For each outcome, the y-scale of Panels (a) and (b) is constant, whereas the y-scale of Panel (c) is not.

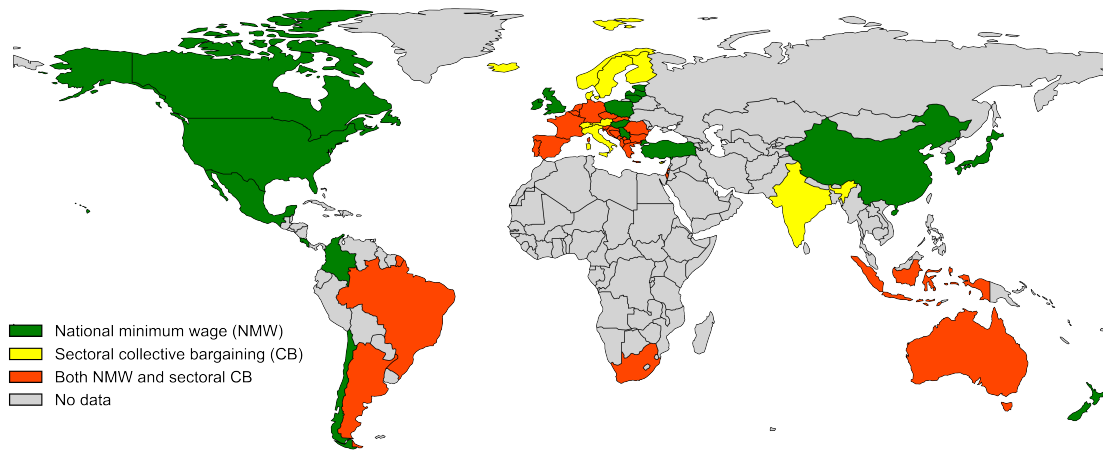
**Appendix Figure F.2:** Evolution of the National Minimum Wage and the Share of Minimum-Wage Workers in Portugal.



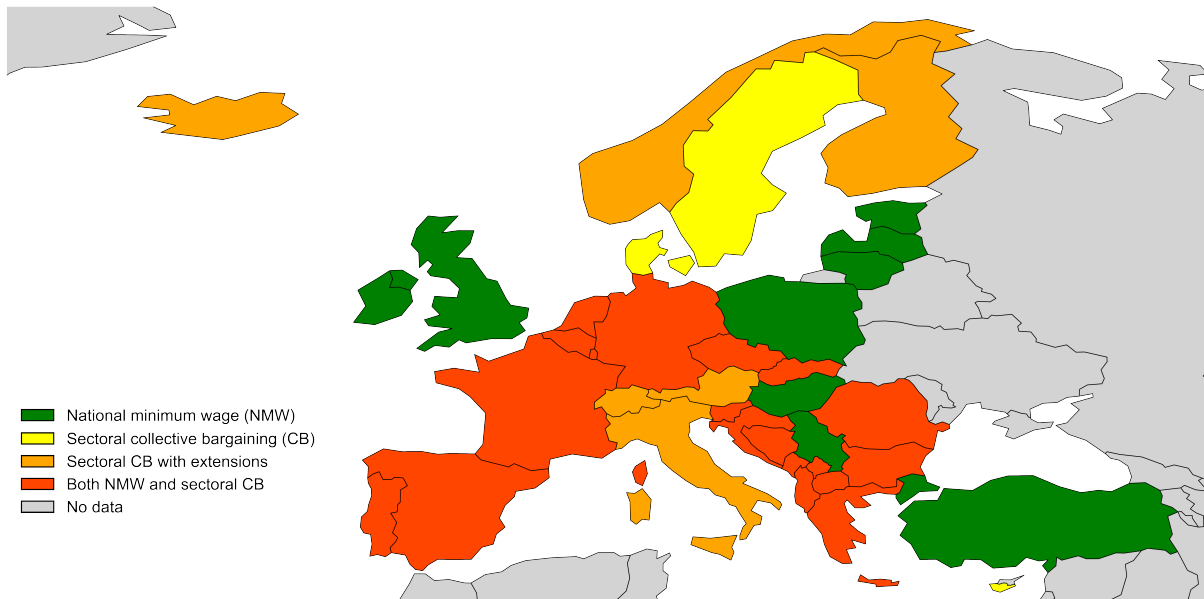
Notes: This figure shows the evolution of the national minimum wage and the share of minimum-wage workers in Portugal. The left axis shows the real monthly MW, while the right axis shows the share of workers bound by the MW. We focus on full-time workers (with over 150 monthly working hours), aged between 18 and 65, and exclude supplementary and bonus payments from their wages to define whether they are bound by the monthly MW. We include all firms in each year, without restricting to stayers or firms observed in multiple years.

Appendix Figure F.3: Wage-Setting Regimes around the World.

(a) World



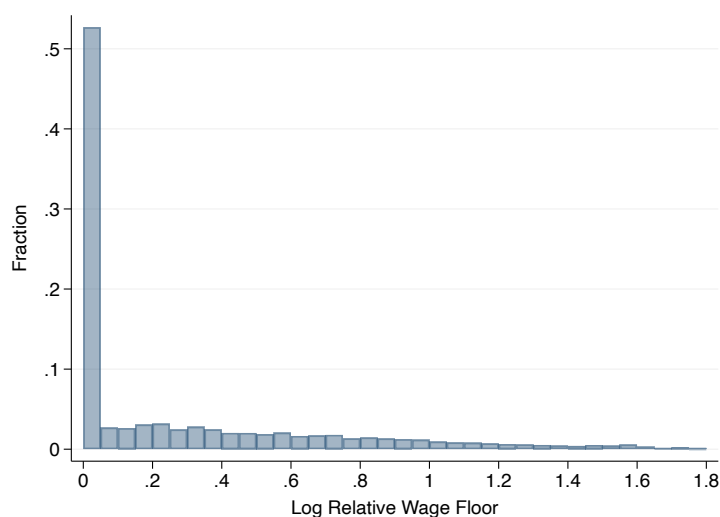
(b) Europe



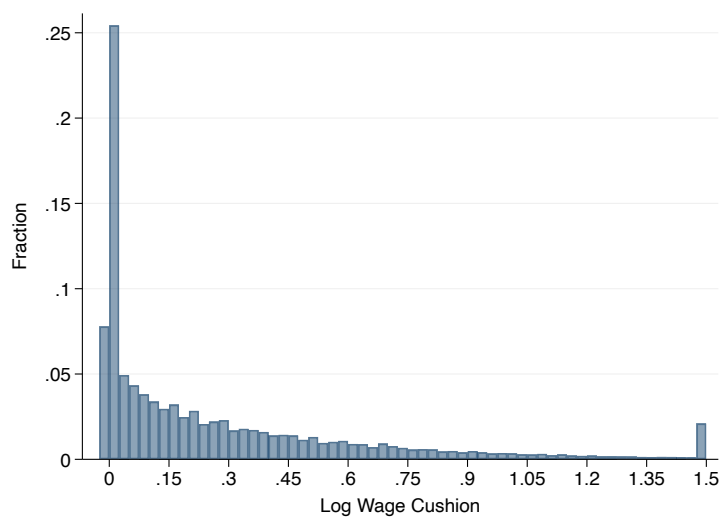
Notes: Data are from ICTWSS (OECD and AIAS 2023). The map displays the predominant wage-setting institutions in different countries in 2019. Panel (a) shows all countries, and Panel (b) shows a zoomed-in view of Europe. In Panel (a) we split the countries into three groups: countries with a national minimum wage (NMW) in green, countries with any sectoral collective bargaining (CB) in orange, and countries with both institutions in red. In Panel (b) we also indicate whether countries with sectoral CB also experienced extensions of the negotiated wage floors to non-covered workers.

**Appendix Figure F.4:** Wage Floors in Collective Bargaining Agreements in Portugal.

**(a)** Relative CBA Wage Floor

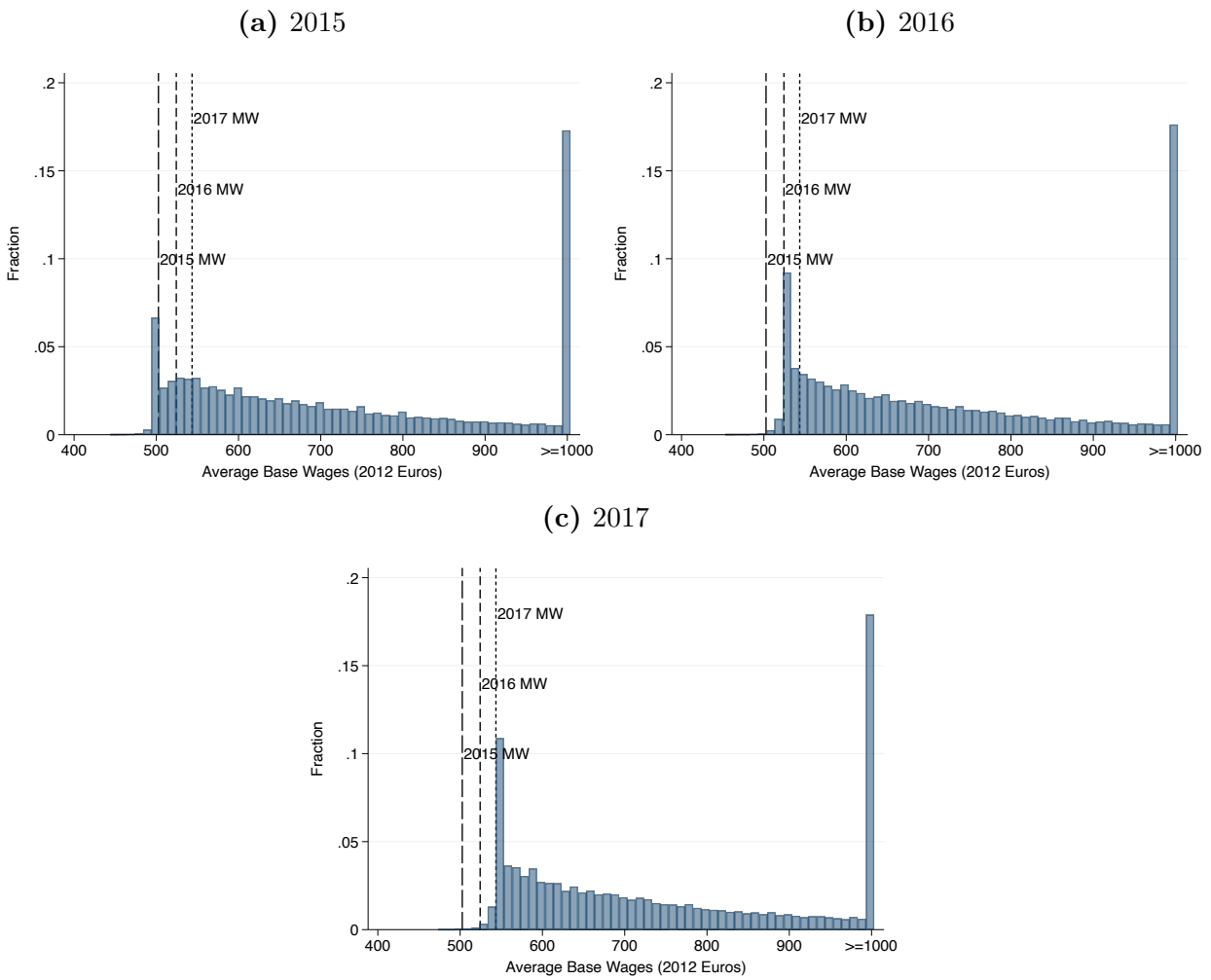


**(b)** Wage Cushions



Notes: This figure shows the distribution of relative CBA wage floors (panel a) and wage cushions (panel b) in Portugal. After applying our sample restrictions, we further drop CBAs with fewer than 50 workers per year, and CBA–job title–year cells with fewer than 10 workers. The relative wage floor is defined as  $\log(\text{CBA wage floor}/\text{MW})$ . The wage cushion is defined as  $\log(\text{base wage}_i/\text{CBA wage floor})$ , and it is censored at 1.5. CBA wage floors are identified from the modal base wage within each CBA–job title–year cell, retaining the mode only when it lies near the bottom of the wage distribution.

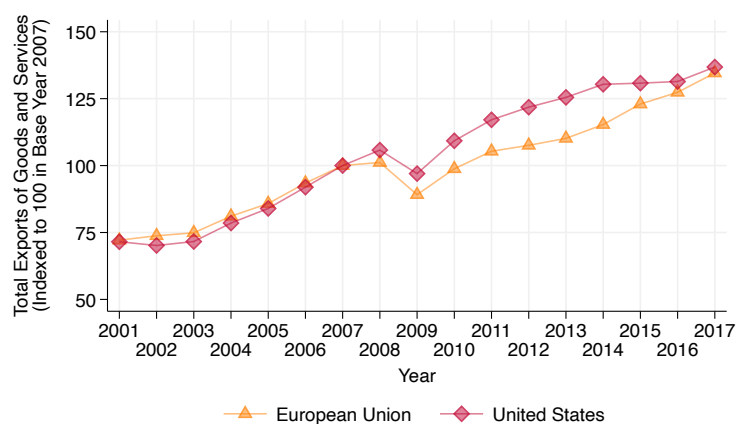
**Appendix Figure F.5:** Distribution of Firm Mean Wages in Portugal, 2015–2017.



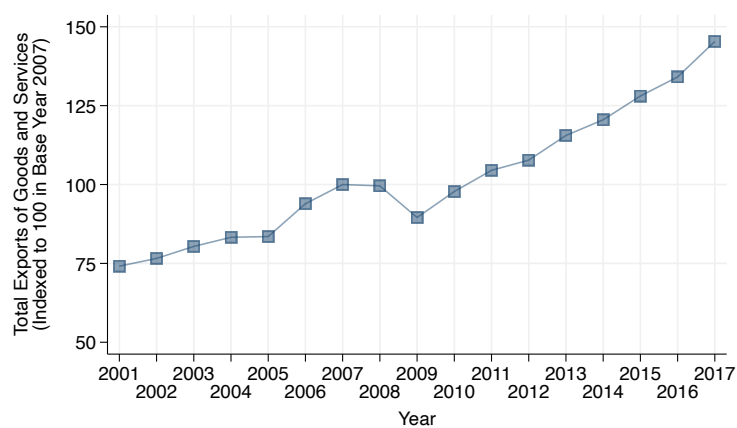
Notes: The figure shows the distribution of firm mean wages in 2015, 2016, and 2017, comparing them with the corresponding real minimum wage levels. The mean firm wage is computed using the base wage for full-time workers (with over 150 monthly working hours) aged between 18 and 65. We focus on firms with non-negative value added. Panel (a) presents the distribution for 2015, panel (b) for 2016, and panel (c) for 2017.

Appendix Figure F.6: Trends in Total Exports for Different Countries.

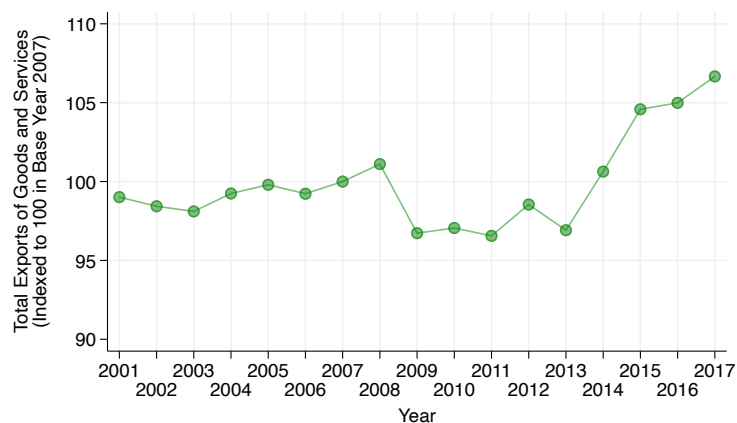
(a) EU and the US



(b) Portugal



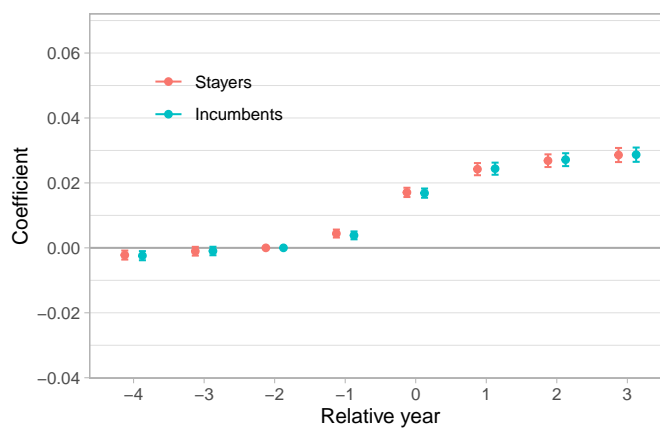
(c) Norway



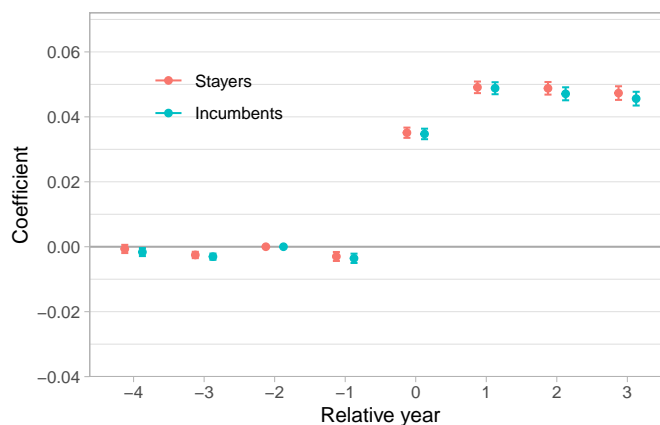
Notes: Total values of exports on goods and services are reported in constant 2015 USD and obtained from the World Bank. We index all values to 100 in 2007. Panel (a) compares the European Union (EU) and the United States (US), while panels (b) and (c) consider Portugal and Norway, respectively.

Appendix Figure F.7: Wage Responses to Demand Shocks: Stayers versus Incumbents.

(a) Portugal

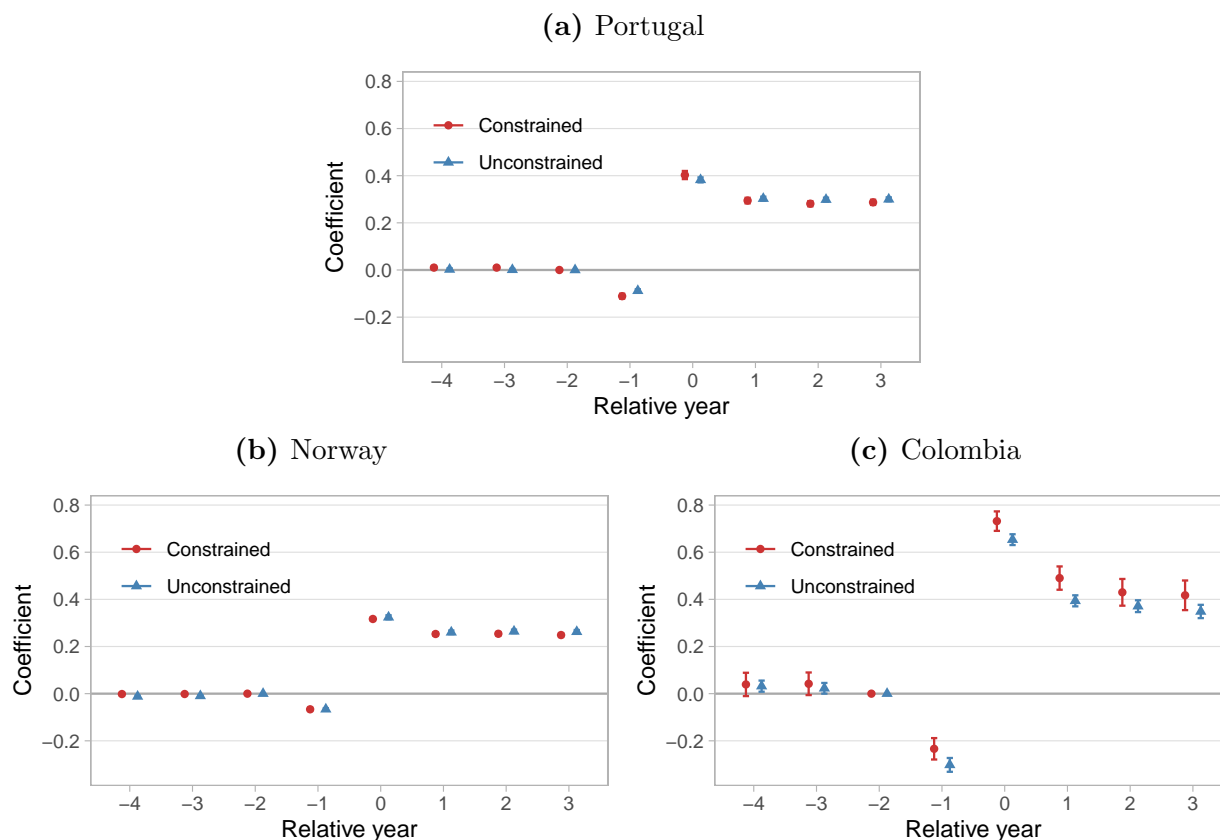


(b) Norway



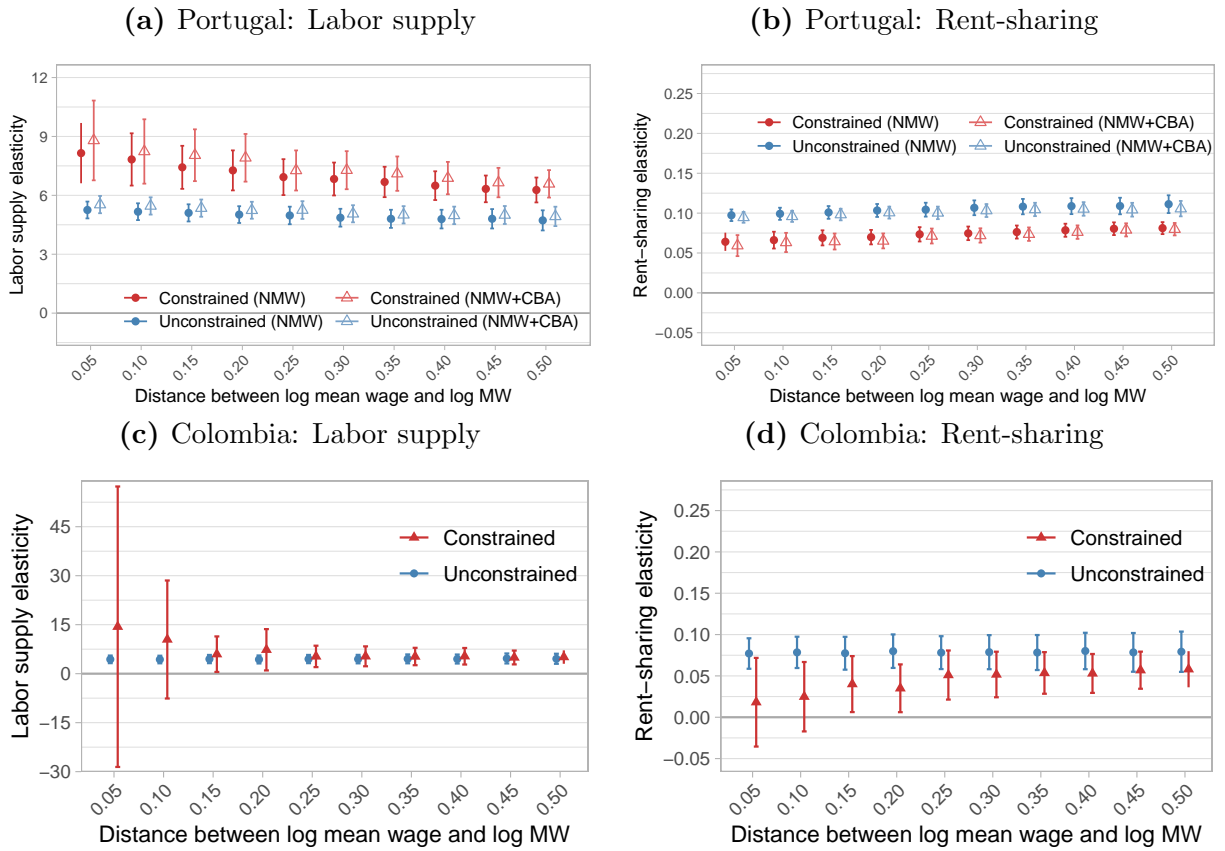
Notes: The figure shows estimates of the effects of exposure to a demand shock using the internal instrument ( $Z_j^{Int}$ ) on the wage of stayers and incumbents. Stayers are workers who remain in the firm for 7 years within the sample window, and incumbents are the subset of workers who remain in the firm in the first three periods. The sample includes firms that have positive employment for 7 years in the window and at least 2 stayers.

**Appendix Figure F.8:** Firm Value Added Responses to Demand Shocks: Exposure to Wage-Setting Constraints.



Notes: The figure shows estimates of the effects of exposure to a demand shock using the internal shock ( $Z_j^{\text{Int}}$ ) on the firm value added by constrained status. Constrained refers to firms with an average wage within 15% of the national MW for Portugal and Colombia, or covered by a CBA for Norway, whereas Unconstrained refers to the rest of the firms. The internal shock ( $Z_j^{\text{Int}}$ ) is defined as having value added growth between -1 and 0 above the median within the firm's 2-digit sector. For Portugal and Norway, the average firm wage is computed for stayers (workers who remain in the firm for 7 years within the sample window). The sample includes firms that have positive employment for 7 years in the window and, for Portugal and Norway, at least 2 stayers.

**Appendix Figure F.9:** Implied Labor Supply and Rent-Sharing Elasticities: Varying the Definition of Constrained firms.



Notes: The figure shows estimates of implied labor supply and rent-sharing elasticities using the internal shock design, varying the threshold used to define constrained firms. For Portugal (Panels a and b), we report estimates using two definitions: (i) proximity to the national MW (NMW), with filled circles, and (ii) proximity to CBA wage floors (NMW+CBA), with hollow triangles. For Colombia (Panels c and d), we split firms based on proximity to the NMW.

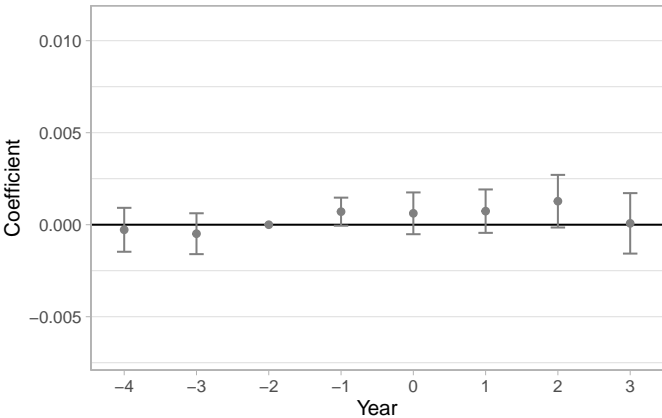
**Appendix Figure F.10:** Firm Response by the Type of Wage-Setting Constraints in Portugal.



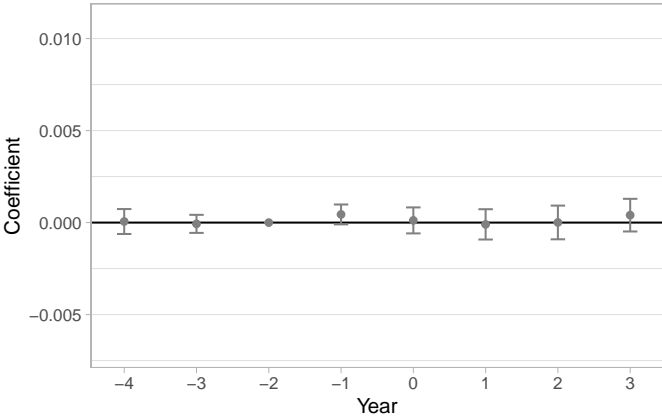
Notes: The figure shows estimates of the effects of a demand shock, using the internal shock design, on firm-level outcomes by constrained status, where constraints combine MW and CBA wage floors. Constrained refers to firms with an average wage within 15% of the national MW (NMW) or of the CBA wage floor (NMW+CBA), whereas Unconstrained refers to the rest of the firms. We estimate the effect of a treatment indicator for having value-added growth between period  $-1$  and  $0$  above the median within the firm's 2-digit sector.

**Appendix Figure F.11: Mean Wage Floor Responses to Internal Shocks.**

**(a) Portugal**

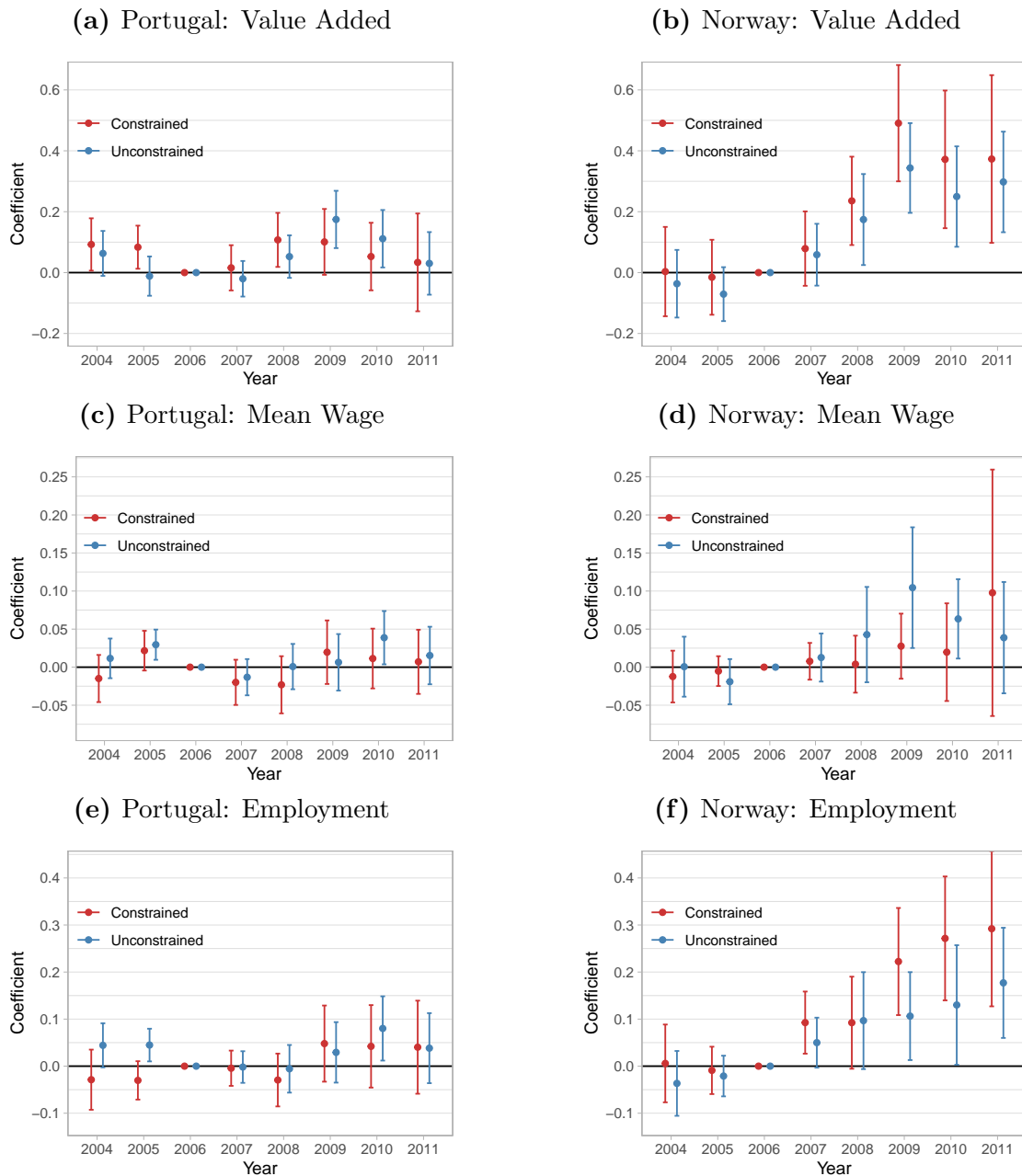


**(b) Norway**



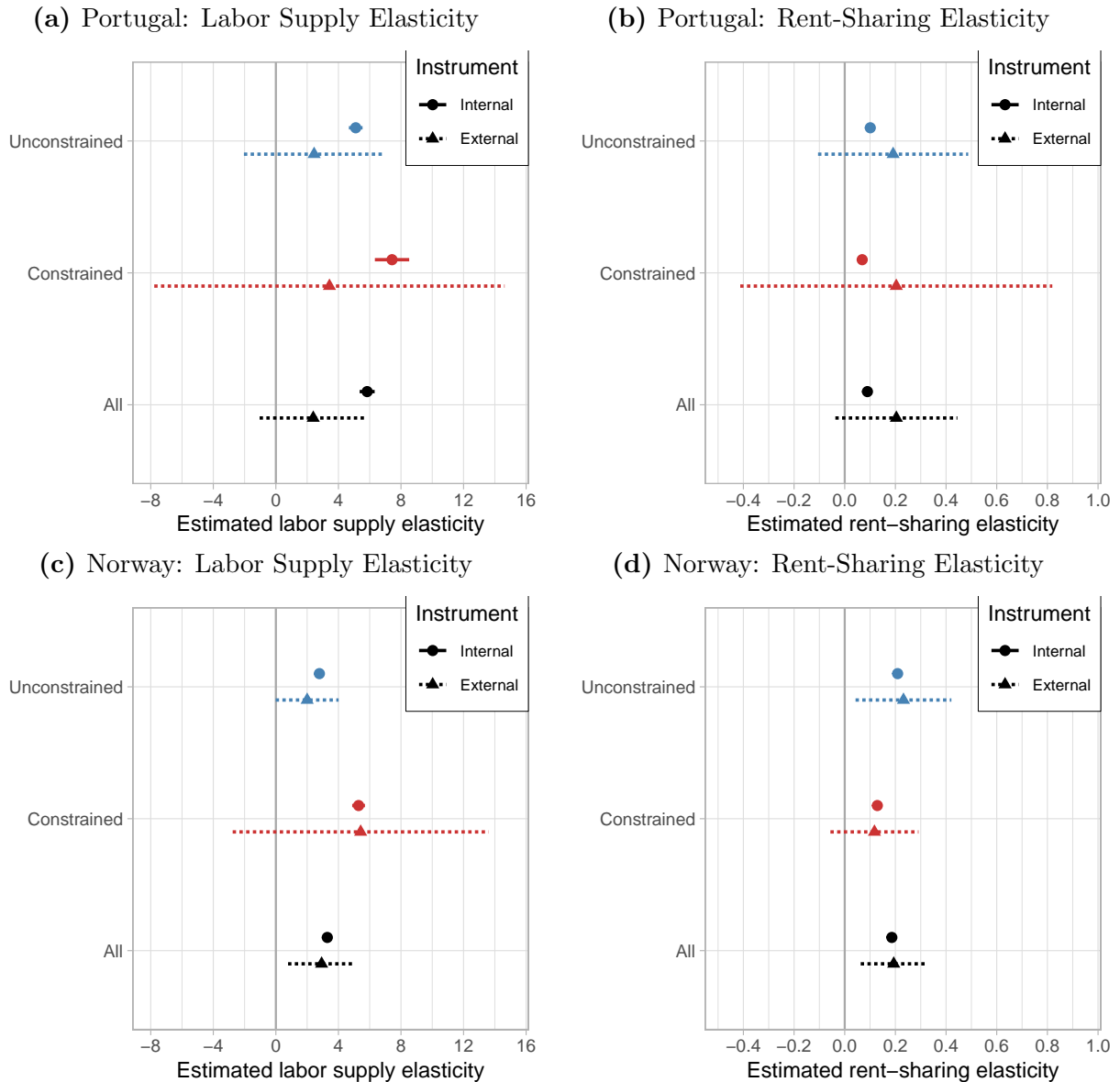
Notes: The figure shows estimates of the effects of exposure to a demand shock, using an internal shock design, on the mean wage floor among stayers in the firm. For Portugal, we focus on firms with non-missing CBA wage floors, as described in Appendix D, and for Norway, we focus on firms covered by CBAs. Stayers are workers who remain in the firm for at least 7 years within the sample window. The sample includes firms with at least 2 stayers within the sample window.

**Appendix Figure F.12:** Firm Responses to Demand Shocks: External Shock and Exposure to Wage-Setting Constraints.



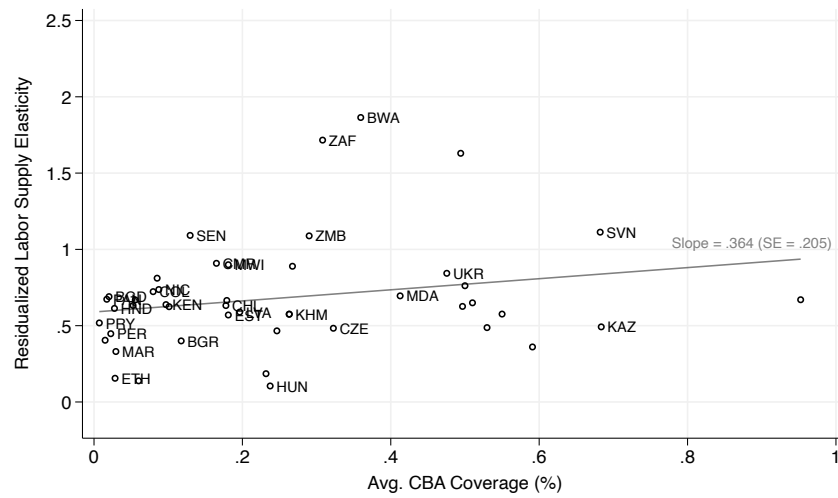
Notes: The figure shows estimates of the effects of a demand shock, using an external shock design, on firm-level outcomes, by constrained status of firms. The left column shows the effects for Portugal, whereas the right column shows the effects for Norway. We restrict to firms exporting for at least three years up to 2007, with exports exceeding 1% of their revenue in 2005–2007, and, for Portugal, we further exclude firms that primarily export to Angola or Spain. Panels (a)-(b) show effects on value added, panels (c)-(d) show wage effects, whereas panels (e)-(f) show employment effects. In each case, we estimate the effect of an increase in the demand for a firm’s export, defined as the mean change in world import demand for the firm’s 2005–2007 exports. The sample includes exporting firms that have positive employment for 7 years in the window and at least 2 stayers. In Portugal, firms are classified as constrained using a looser threshold of approximately 50 percent above the MW to ensure a sufficient sample.

**Appendix Figure F.13:** Estimated Firm-Level Elasticities: External Shock and Exposure to Wage-Setting Constraints.



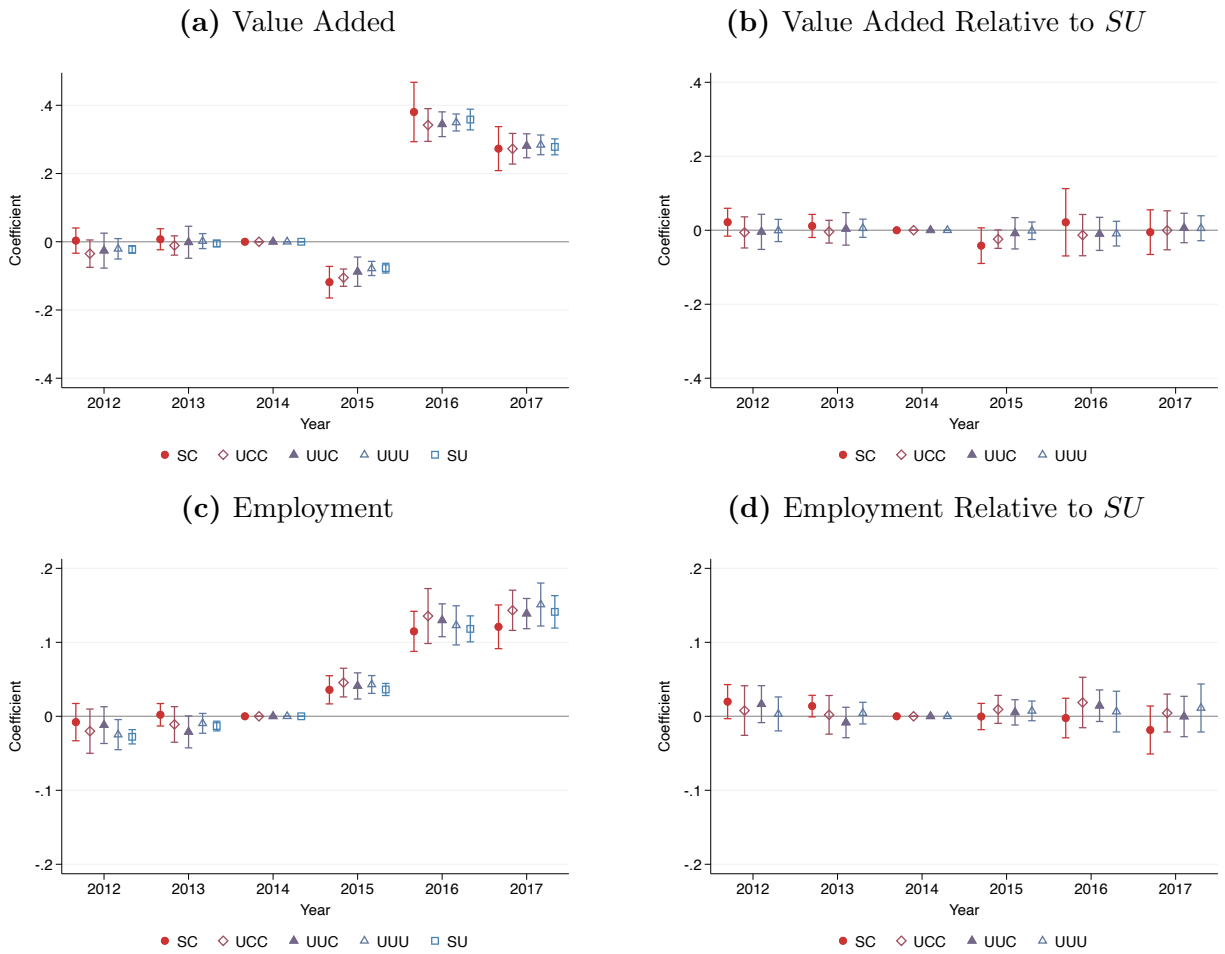
Notes: The figure shows estimated elasticities implied by reduced-form effects of a demand shock on value added, wages, and employment, using the external shock design, by constrained status. Constrained refers to firms with an average wage within 50% of the national MW for Portugal, or covered by a CBA for Norway, whereas Unconstrained refers to the rest of the firms. We restrict to firms exporting for at least three years up to 2007, with exports exceeding 1% of their revenue in 2005–2007, and, for Portugal, we further exclude firms that primarily export to Angola or Spain. Panels (a) and (b) show estimates for Portugal, and Panels (c) and (d) show estimates for Norway. The left column shows labor supply elasticities, and the right column shows rent-sharing elasticities. To construct the mean reduced-form responses, we use the dynamic estimates in Appendix Figure F.12 and average the post-periods 1 through 3.

**Appendix Figure F.14:** Estimates of Firm-Level Labor Supply Elasticities and Collective Bargaining Coverage in Developing Countries.



Notes: Estimates of the median firm markdown are taken from [Amodio et al. \(2024\)](#) and transformed into labor supply elasticities. These elasticities are residualized with respect to two polynomials in GDP per capita and self-employment rates, as well as a dummy for unemployment protection, and winsorized at the 5th and 95th percentiles. Data on the average CBA coverage are from the International Labour Organization's (IRData), which combines survey- and administrative-based sources. The average coverage rate is computed using available information for 2010–2019 and is defined as the share of employees covered by collective bargaining relative to those with the right to be covered. The resulting sample covers 46 of the 81 countries with markdown measures.

**Appendix Figure F.15:** Employment and Value Added Responses to Demand Shocks: Variation in Constraints in Portugal.



Notes: The figure shows estimates of the effects of a demand shock, using the internal shock design, by constrained group. The estimation approach follows Figure 7, but using employment and value added as outcomes. Panels (a) and (c) show event-study coefficients estimated separately for each group, and Panels (b) and (d) show differences with respect to the least constrained group, *SU*.