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Price Discovery in Labor Markets: Why Do Firms Say They Cannot Find Workers?

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Abstract

Why do firms report that they cannot find workers instead of preemptively raising wages? Using German administrative data, we show labor-constrained firms pay lower wages and quasi-exogenous wage increases alleviate constraints, consistent with monopsony. Yet constrained firms' delayed wage increases point beyond this mechanism. We develop a dynamic matching model combining wage-setting power with incomplete information and downward wage rigidity. Consistent with the model, firms raise wages when initial wage plans prove too low, especially for peripheral occupations, and face constraints after wage shocks to adjacent sectors, suggesting that firms' inaccurate beliefs and learning about market wages shape labor constraints.

Keywords: Hiring difficulties, wage adjustments, outside options, information frictions
JEL Codes: J23, J31, D83, E24, M51

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1 Introduction

Firms frequently report that they "cannot find workers," and that this is a major obstacle to their growth.[1] Basic economic theory suggests that these constraints can be mitigated by raising wages. Yet, firms may *not want to* raise wages, possibly because higher pay for new hires spills over to incumbents, is hard to reverse, or does not attract substantially more workers. Alternatively, firms may *fail to* raise wages even when doing so would be profitable, for example, because imperfect price discovery in labor markets prevents them from recognizing that they should pay more. Although labor constraints are widespread and important for firm growth and labor allocation, their sources are neither well characterized empirically nor well understood conceptually.

In this paper, we study both theoretically and empirically why firms report labor constraints instead of preempting them by raising wages. We document new facts about labor constraints using firm-level administrative data from Germany, combining information on the hiring process, wages, and firm outcomes, and exploiting the institutional setting with collective wage bargaining. A unique advantage of the data is that it contains direct measures of labor constraints. Motivated by the empirical patterns, we develop a dynamic model of hiring, which augments a standard labor search model with incomplete information about the state of the economy and downward wage rigidity. We then show in the data that, while monopsony power and low wages can explain some of the basic patterns of labor constraints, incomplete information and gradual learning are also important for understanding why labor constraints arise and why wages adjust after firms experience them.

Our first contribution is to demonstrate that while labor constraints are widespread and negatively affect firm growth, they can be mitigated by higher wages. We show that constrained firms initially pay lower wages and we use quasi-exogenous wage variation generated by collective bargaining to confirm that wages are effective in alleviating labor constraints. These results suggest that firms choose to keep wages below what is needed to attract new workers, implying that monopsony power partly explains labor constraints. However, we also find that firms pay more after longer searches, that they increase their wages after reporting labor constraints, and that the most profitable firms are most likely to be constrained, all of which are at odds with the predictions of canonical monopsony models.

Our second contribution is thus conceptual: we develop a dynamic model of hiring, which augments the standard search framework with additional mechanisms to explain the empirical regularities. Introducing downward wage rigidity and information frictions, such that firms face noisy signals about market conditions and learn over time, generates delayed wage adjustment and reproduces the joint dynamics of wages and labor constraints in the data.

[1] In the 2021 Small Business Credit Survey (SBCS) run by US Federal Reserve Banks, 59% of firms declared that it is "very difficult" to fill jobs, with an additional 32% declaring it is somewhat difficult. Similarly, the Vacancy Survey (VS) run by the German Employment Agency reveals that 9.5% of firms declare labor shortages to be a significant impediment to growth in the near future. For comparison, only 21% of firms in SBCS and 6.4% in VS declared that credit availability posed a substantial difficulty.

Our third contribution is to test the model's central mechanism empirically. We first provide direct evidence that firms revise wages upward during recruiting when their initial wage beliefs are too low. These revisions are more common among constrained firms, after longer searches, and after failed searches in which applicants' wage demands were too high. We then compare hiring of more and less informed firms and show that firms face lower constraints and are less likely to raise wages above plan when hiring for core rather than peripheral occupations. In addition, we trace the dynamics of learning using wage shocks in other industries and find that firms are slow to adjust, raising pay only after reported labor constraints increase. Together with evidence that constraints are more pronounced at firms with more downward-rigid wage structures, these findings support the importance of incomplete information and gradual learning about workers' outside options in the presence of wage rigidity.

Our analysis relies on administrative data from Germany, using two main data sources. First, vacancy survey data provide detailed information about firms' decisions and difficulties during their most recent hiring process. Second, establishment panel data from surveys on firms' operations include measures of staffing and recruiting difficulties and can be linked to worker-level data for the entire workforce. Both data sources contain questions about key obstacles to firm operations. We define a firm as labor constrained if it reports being unable to find enough suitable workers or facing a staff shortage.

A premise of our analysis is that reported labor constraints capture real recruiting difficulties with consequences for firms, rather than noise. Although skepticism about self-reported measures is common (Hyman, 1954), the data confirm this premise. Labor-constrained firms have longer job searches and a higher probability of failing to fill a vacancy. Fast-growing firms are more likely to report labor constraints, but market-level increases in labor constraints are associated with slower growth in firm employment, sales, and profitability.

Canonical monopsony models predict that firms keep wages low to avoid raising pay for incumbent workers, implying that higher wages should alleviate labor constraints. Our evidence supports this mechanism. First, we show that firms that initially pay lower wages are more likely to be labor constrained. Second, using quasi-exogenous variation in wages from central wage bargaining, which covers a large share of the German economy, we find that higher wages significantly reduce labor constraints. Third, the effects of bargaining-induced wages are particularly pronounced for firms likely to have higher monopsony power over their employees, due to employing workers in occupations concentrated in a few industries.

However, other data patterns suggest that monopsony power alone cannot explain the nature of observed constraints. First, after declaring labor constraints, firms raise both nominal wages for new hires and residualized wages after controlling for worker characteristics. Second, wages paid to new hires are higher if the search duration is longer. Third, labor constraints are more prevalent among more profitable, faster-growing firms. In classic monopsony models, we would not expect firms to adjust their wages after experiencing labor constraints, and in some key models we would expect that higher-productivity firms would be less likely to report constraints.[2]

To explain these patterns, we propose a dynamic model of hiring with incomplete information about aggregate conditions and downward wage rigidity. As in a standard search framework, firms face a trade-off between posting a high wage and filling a vacancy quickly, or posting a lower wage and accepting a longer expected search. Yet, because firms do not observe the aggregate state of the labor market directly, they post wages based on beliefs. Firms that underestimate competition make offers that are too low and therefore more likely to be rejected. Firms learn from these hiring failures, revise their beliefs upward, and increase wages gradually. Downward wage rigidity slows that adjustment further, because firms hesitate to commit to high wages if tight conditions may prove temporary. The model thus predicts that labor constraints can be explained not only by the degree of monopsony power, but also by firms' limited information about market wages and downward wage rigidity.

In the last part of the analysis, we test the model predictions in the data. We begin with direct evidence from the vacancy survey on wage revisions during the hiring process. Firms that report labor constraints are more likely to raise wages above their initial plans, especially after longer searches for workers with more experience and in markets where wages are more difficult to observe. In addition, when a search fails because applicants' wage demands are too high, firms are also more likely to plan higher wages for the next search. Inspired by the distinction between informed- and noise-traders in financial markets (Kyle, 1985; Glosten and Milgrom, 1985), we then compare hiring in core occupations (e.g., production workers at a manufacturing firm) to peripheral occupations (e.g., an accountant at a manufacturing firm). Firms hiring in occupations that are central to their existing workforce face fewer wage-related hiring difficulties and are less likely to raise wages above plan, consistent with better information about market wages. Additional appendix evidence based on market-level proxies for wage information quality—including wage dispersion in relevant occupations and the synchronicity of local wages with national wage trends—points in the same direction. Next, we use quasi-exogenous wage variation from collective bargaining in *other* industries to analyze the dynamics of wage adjustments. We show that wage increases by other firms in the same labor market raise labor constraints before firms adjust their own wages. Firms in the focal industry report more labor constraints the next year, followed by higher wages and fewer constraints one year later. Finally, we show that firms with higher downward wage rigidity, as measured by an index of missing wage cuts in the wage growth distribution following Ehrlich and Montes (2024), face more labor constraints.

Overall, our results highlight how imperfect and slow price discovery in labor markets and employer market power contribute to labor constraints, with implications for firms and policymakers. For firms, investing in wage intelligence and more flexible pay practices can mitigate labor

[2] In a basic static model (Robinson, 1933) the monopsonist rationally chooses the wage and has no incentive to change it later. In standard dynamic search frameworks (Burdett and Mortensen, 1998) more profitable firms will, on average, pay more and will be more likely to fill a vacancy; and firms that offer higher wages will be faster to fill a vacancy.

constraints—echoing the growing use of salary-benchmarking tools and integrated labor-market analytics. For policymakers, faster, more accurate wage reporting and stronger pay-transparency measures may meaningfully alleviate hiring difficulties. Information alone may not suffice, however, when firms have incentives to keep wages below the levels needed to attract workers. Policies that reduce barriers to job-to-job mobility and strengthen workers' outside options can complement transparency measures by limiting firms' monopsony power.

While an established literature in labor economics has studied why wages do not go down and unemployment arises (Stiglitz, 1984; Blinder and Choi, 1990; Bewley, 1999), our contribution is to explain why wages do not go up and why labor constraints arise instead. Recent work by D'Acunto, Weber and Yang (2020) and Le Barbanchon, Ronchi and Sauvagnat (2023) takes labor constraints as given and studies their impact on firm growth.[3] Like us, D'Acunto, Weber and Yang (2020) analyze the German labor market, and use an immigration shock to show that labor constraints increase firms' order backlogs and capital expenditures. Le Barbanchon, Ronchi and Sauvagnat (2023) use data from France and a shift-share strategy based on firms' occupational mix to document the adverse impact of constraints on firm employment, sales, and profits. We build on this recent work, confirm adverse impacts in our setting, and focus on why firms do not preempt labor constraints by raising wages. This focus on wage setting also relates to recent work in the vacancy literature on how wages affect job-filling rates and vacancy duration (Mueller et al., 2023; Bassier, Manning and Petrongolo, 2025).

Our findings confirm that labor shortages can be partially explained by firms' monopsonistic behavior (Sullivan, 1989; Jarosch, Nimczik and Sorkin, 2024; Kline, 2025) while also pointing to the importance of information frictions. Existing work has focused on firms' uncertainty about the quality of candidates (e.g., Schönberg, 2007; Kahn, 2013; Pallais, 2014; Burks et al., 2015; Stanton and Thomas, 2025; Friedrich, 2025) and applicants' uncertainty about firm fundamentals (Hacamo and Kleiner, 2022). We focus on firms' uncertainty about market wages, complementing recent work on the role of workers' beliefs about outside options (Jäger et al., 2024; Caldwell, Haegele and Heining, 2025).[4] Related work by Bertheau, Larsen and Zhao (2026) uses a firm survey to elicit firms' beliefs and demonstrates that they matter for hiring decisions, while Cullen, Hoffman and Koenig (2024) conduct a field experiment on an online staffing platform that shows the value of wage information when hiring short-term workers. Our analysis uses administrative data representative of the national economy and studies information frictions alongside other mechanisms in a long worker-firm panel spanning occupations, industries, and economic conditions.

While price rigidity is widely assumed to be important in macroeconomics, standard modeling approaches often lack clear microfoundations (e.g., "Calvo fairy." Calvo, 1983). Several mechanisms for sluggish price adjustments have been proposed, such as menu costs (Mankiw, 1985),

[3] A separate strand of literature analyzes systemic sources of skill shortages, including technical change (Haskel and Martin, 2001), educational regulation (Cappelli, 2015), occupational licensing (Friedrich and Hackmann, 2021), and migration policy (Kerr et al., 2016).

[4] This work is part of a growing literature on the impact of workers' outside options on search behavior and wages (Beaudry, Green and Sand, 2012; Jäger et al., 2020; Caldwell and Danieli, 2024).

coordination failures (Ball and Romer, 1991), or sticky information (Mankiw and Reis, 2002)—the last being most closely related to our work. However, in the typical scenario of positive inflation, existing models explain downward rather than upward wage rigidity (Ball and Mankiw, 1994; Golosov and Lucas Jr, 2007). In our model, the expected downward rigidity in the future may contribute to upward rigidity today (Elsby, 2009; Stüber and Beissinger, 2012).

Finally, our theoretical framework connects to several strands of literature on dynamic markets with frictions. First, we build on dynamic urn-ball matching models where agents meet via Poisson processes that generate multilateral encounters (Wolinsky, 1988; Satterthwaite and Shneyerov, 2007; Lauermann, 2013) and we extend this framework to a labor market with belief updating. In contrast, the majority of the labor search and matching literature relies on random one-to-one matching or directed search with complete information about market conditions and partner types, as surveyed in Rogerson, Shimer and Wright (2005).[5] Despite these differences in matching processes, our model preserves static price dispersion insights from Burdett and Judd (1983) and the wage-posting tradition of Burdett and Mortensen (1998). Second, our model relates to the literature on price discovery in auctions, where bidders learn about the common value or the degree of competition through repeated interactions (Milgrom and Weber, 1982; Kremer, 2002). Third, our emphasis on aggregate uncertainty aligns with work on price discovery under incomplete information about market fundamentals (Lauermann, Merzyn and Virág, 2018; Shneyerov and Wong, 2020), where agents infer aggregate states from trading outcomes. However, we do not assume a common equilibrium price for labor in each state because applicants have heterogeneous outside options.

2 Data and Measurement of Labor Constraints

2.1 Data Sources

Our data come from the Institute for Employment Research (IAB) of the German Federal Employment Agency. We use two main datasets: the Vacancy Survey and the Establishment Panel. In addition, we create supplementary variables based on worker-level employment records (LIAB) and other auxiliary datasets. We briefly introduce the main data sources below and provide more information in Appendix A.

Vacancy Survey (SE, *IAB-Stellenerhebung*) The SE is a repeated cross-section covering 2000–2018, with over 230,000 establishment-year observations and 7,500–15,000 unique establishments per year. It reports the number and structure of vacancies, characteristics of the last hire, search duration, and indicators for difficulties and decisions associated with the recruiting process

[5] One important exception is Shimer (2005), who studies the assignment of heterogeneous workers to jobs under coordination frictions in a directed search framework, where multiple applicants may target the same vacancy, leading to rationing and inefficiencies in assortative matching, highlighting frictions beyond simple bilateral pairings that resonate with our multilateral Poisson meetings.

(Gurtzgen et al., 2023). The SE is merged with establishment-year administrative records (Establishment History Panel (BHP)), which contain information on employment and wages for the establishment by worker age, sex, education, and occupational group.

Establishment Panel (EP, *IAB-Betriebspanel*) The EP is a longitudinal survey of establishments covering 1993–2019. While fewer than 10,000 establishments were included in the initial waves, the sample contains around 15,000 establishments annually after 2000. The data combine the establishment survey and administrative employment and wage records. The survey is composed of a core module with the key characteristics of the establishment and additional rotating modules about various areas of business operations. We rely heavily on the "Staffing Challenges" module, which is included in the survey approximately every two years. We focus on establishments with at least 10 employees, and our main sample includes 77,330 establishment-year observations across 25,190 unique establishments.[6]

Linked employer-employee data (LIAB) To construct some variables, we use the cross-sectional LIAB-QM (Graf et al., 2023), which includes all workers employed at EP establishments as of June 30 each year. We use occupational information on workers (classified according to the German standard KldB 2010, which is similar to the International Standard Classification of Occupations, ISCO) to measure the occupational structure of establishments as well as occupation-level average wages and labor constraints as further discussed below.

2.2 Key Variables

The key variables in our analysis are measures of labor constraints and wages. We describe them in detail here, while also briefly discussing a larger set of variables that serve as controls or dimensions of heterogeneity in our tests. Summary statistics are presented in Table 1.

Labor Constraints. Both of our main datasets, SE and EP, contain measures of labor constraints. In the SE data, the labor constraints measure is a binary indicator of whether the establishment declares that "not having enough suitable workers" posed "economic difficulty during the past 12 months".[7] On average, 9.5% of establishments report labor constraints, compared to 6.4% reporting financial constraints.

In the EP data, the labor constraints measure is a binary indicator of whether the establishment expects a "staff shortage" to be among its problems over the next two years. Despite the different wording and time horizon of the question, the prevalence of labor constraints is similar, affecting 12.2% of establishments in the EP data on average.

[6] We use establishment and firm interchangeably throughout the paper. Typically, 75% of survey respondents are single-establishment firms.

[7] The exact wording of the question varies slightly over the years, see Appendix A.

In addition to the main measure of labor constraints, the SE also contains other related measures of difficulties during the last recruitment process. 24.8% of establishments declare that they have "experienced difficulties in filling the vacancy", which we denote as *search difficulties*. Unlike the rate of labor constraints among all surveyed firms, these search difficulties are conditional on hiring over the last 12 months. Establishments reporting difficulties are then asked what kind of difficulties they experienced, such as qualifications of applicants being too low or wage expectations being too high. The survey also contains search duration in days. Finally, firms also report whether they had to pay a higher wage than they initially expected and whether the wage was negotiated with the worker. In both SE and EP, we also compute the vacancy rate as the number of reported job vacancies divided by the number of current employees. In SE, vacancies include positions to be filled immediately and within the next 12 months. In EP, only immediate vacancies are included. As a result, the average vacancy rate is 2.9% in SE and 1.5% in EP.

Wages and Collective Bargaining. Our wage analyses primarily use the EP data. We compute the average daily wage of all workers, and separately for new hires (defined as those who joined the firm over the last 12 months) and incumbent workers (employed for more than 12 months). The daily wages come from administrative employment records and represent contractual wages valid as of June 30 of a given year. In the SE data, average wages at the firm level or by worker group are also based on administrative daily wages, while the wage paid to the last hire is the monthly amount reported by firms in the survey. To isolate quasi-exogenous variation in wages, we rely on central-bargaining-induced wage levels, which we measure based on workers whose employer is bound by an agreement and pays at the collectively agreed rates (as further discussed in Section 4).

Other Variables. Other variables used in our analysis are more standard. We measure employment with the total number of workers employed as of June 30 according to administrative records. Sales are based on total volume of sales in EUR in the previous year, as reported by the firm in the EP survey. Capital expenditures in EUR also come from EP reports and reflect the total amount spent on investment in the previous year. The EP survey does not contain quantitative measures of profitability; instead, only a qualitative measure of the profit situation is collected. Based on that measure, we define a binary indicator for a "sufficient" or higher profit situation as reported by the firm.

2.3 Sample Descriptives

Table 1 presents summary statistics for key variables from both datasets. The average establishment in our EP sample has 261 employees and sales of EUR 164M per year, which grow at an annual rate of 1.8%. The average daily wage for incumbent workers is EUR 76.4, which corresponds to a monthly wage of EUR 2,324. In the SE data, the average search lasts for 93 days,

Table 1: Summary Statistics

The sample for the Vacancy Survey (SE) contains 232,696 firm-year observations. Firms that declare that they have been searching for a worker in the last 12 months are asked questions about search duration, search difficulties, and the wage offered related to their last completed worker search. Sample size varies because not all questions were asked in all waves of the survey. Labor constraints and financial constraints are declarations that firms' sales in the near future would be higher if not for these constraints. There are 77,330 firm-year observations in the IAB Establishment Panel sample filtered on the availability of employment and labor constraints data. Sales are missing for part of the sample; sales growth is winsorized at the 1st and 99th percentiles. For confidentiality reasons, bargaining-induced wage growth is not available for some observations belonging to industry/occupation-year cells with fewer than 20 observations.

	Mean	Std Dev	P5	P95	N
Vacancy Survey (SE)					
Search Difficulties	0.248	0.432	0	1	139,478
Search Duration [days]	92.6	78.5	12	245	112,570
Monthly Wage Offered [EUR]	2,367	1,040	1,030	4,400	39,641
Stopped Search	0.166	0.372	0	1	147,430
Vacancy Rate (SE)	0.029	0.194	0	0.143	232,696
Labor Constraints (SE)	0.095	0.294	0	1	232,696
Financial Constraints	0.064	0.244	0	1	232,696
% Workforce in Hired Occupation	0.179	0.257	0	0.771	46,578
Had to Pay More than Planned	0.117	0.321	0	1	57,930
IAB Establishment Panel (IAB-EP)					
Employment	261	1,060	12	1,026	77,330
Employment Growth	-0.010	0.291	-0.251	0.234	77,330
Sales [EUR Million]	164	2,964	0.6	281	56,360
Sales Growth (winsorized)	0.018	0.370	-0.383	0.386	52,723
Profitable	0.913	0.281	0	1	52,030
Incumbents Daily Wage [EUR]	76.4	29.0	34.0	128.7	77,273
New Hires Daily Wage [EUR]	51.8	28.3	12.1	103.5	71,083
Labor Constraints (IAB-EP)	0.122	0.327	0	1	77,330
Labor Constraints (shift-share)	0.132	0.113	0.018	0.378	77,310
Vacancy Rate (IAB-EP)	0.015	0.041	0	0.091	77,330

with 17% of firms giving up before finding a suitable worker. The monthly wage offered is very close to the average wage in the EP data and equals EUR 2,367.

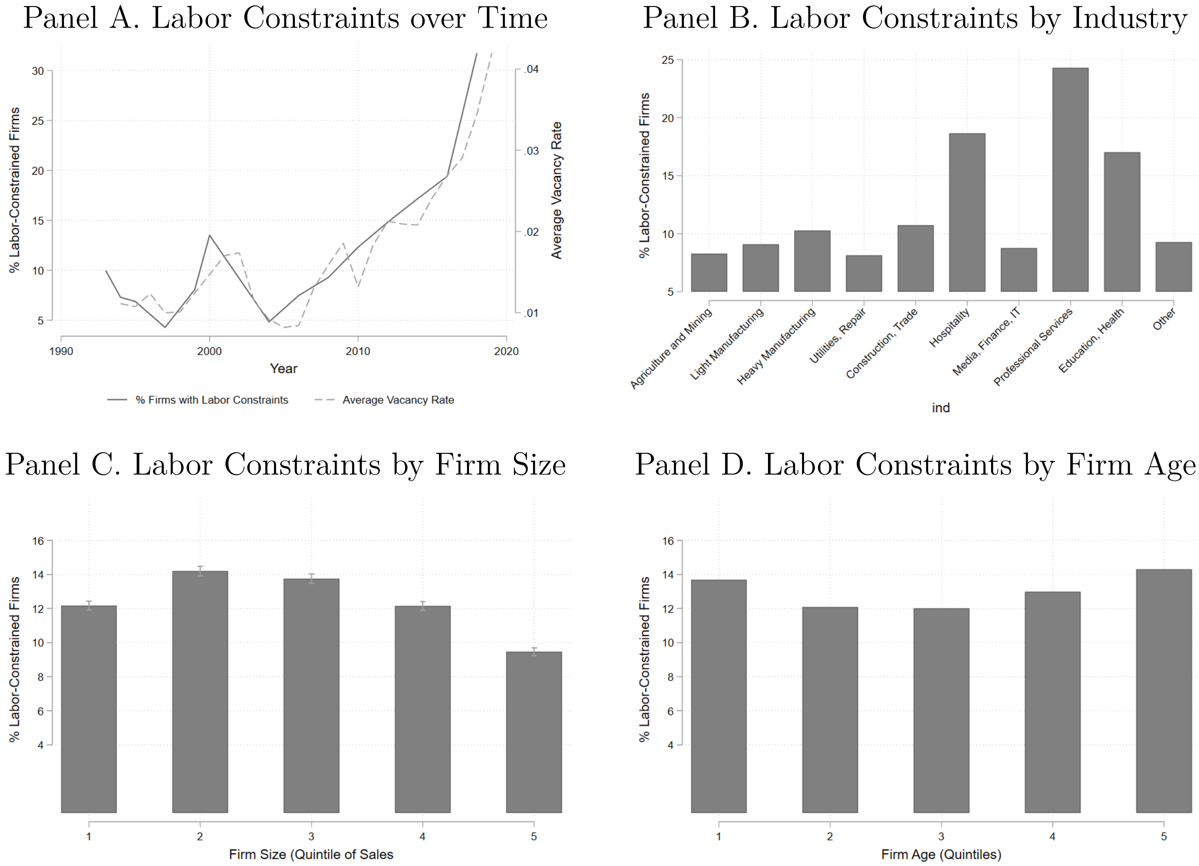
The focus of our analysis is on measures of labor constraints, a unique feature of our data not present in standard firm-level datasets. Figure 1 describes how these measures vary over time and across firm characteristics.

Labor constraints are a longstanding phenomenon but have intensified in recent years (Figure 1, Panel A). In the 1990s and 2000s, the share of labor-constrained firms was often below 10%; after 2010 it always exceeded 10% and reached as much as 30% in 2018. The vacancy rate, measured annually, tracks labor constraints closely, indicating the connection between firms' declarations and their recruiting practices.

Labor constraints vary considerably by industry (Panel B). While all industries display a sizable share of labor-constrained firms, the hospitality sector and professional services, including education and health, are particularly affected. This finding illustrates an interesting subtlety of labor constraints: they may be severe in both low-wage sectors—consistent with Habakkuk's (1962) observation that workers move away from the least desirable jobs—and high-skill sectors that often require scarce, specialized expertise.

In contrast, differences by firm size and age are modest (Panels C and D). The size gradient

Figure 1: Labor Constraints by Time, Industry, and Firm Characteristics



All panels are based on EP data. Panel A shows the evolution of the share of firms reporting labor constraints and of the average vacancy rates over time. Labor constraints are reported only in selected waves of the survey, often every two years. Panel B shows the average share of firms reporting labor constraints in different industries. Industry codes are 1-digit codes based on the WZ 2008 classification of the German Employment Agency, which are close to the International Standard Industrial Classification (ISIC) codes. Panel C shows the share of firms reporting labor constraints by quintile of firm size, computed based on total sales. The average sales level in each quintile in millions of EUR is 0.7, 2.2, 5.7, 18.8, and 737. Panel D shows the average share of firms reporting labor constraints by quintiles of firm age. The average firm age in each quintile in years is 4.3, 10.8, 18.1, 25.4, and 36.0.

is hump-shaped but the variation is relatively small: the smallest firms are less labor constrained than medium-sized ones, but the prevalence of constraints declines again for larger firms. The relationship with firm age is U-shaped, but the differences between quintiles are also limited. Overall, the variation by firm size and age is smaller than the variation by industry or over time.

3 Labor Constraints: Firm Selection and Growth

In this section, we study the link between reported labor constraints and other firm outcomes and characteristics. We first show that labor constraints are associated with higher vacancy rates and longer search duration, which confirms that our measure captures real hiring difficulties. We then study the link between labor constraints and firm growth and profitability. We find that labor constraints are more common among profitable firms that are growing their employment and revenue, but a market-level increase in labor constraints is associated with reduced growth of employment, sales, and profitability.

3.1 Labor Constraints and Other Measures of Hiring Difficulties

One concern with our survey-based measures of "labor constraints" is that they might just reflect managers' complaints with no economic content. However, the evidence in Table 2 rejects this view. Relative to unconstrained firms, labor-constrained firms report over 30% longer search durations for their most recent hire (column 1) and are 45 percentage points more likely to report a failed search in the past year (column 2).

In addition, we find substantially higher vacancy rates (column 3 for SE and column 4 for the EP sample). While the measurements of labor constraints and vacancy rates differ across the two data sources, as discussed in Section 2.2, we find robust differences between constrained and unconstrained firms, with constrained firms reporting 0.35–0.4 standard deviations higher vacancy rates than unconstrained firms. Using the panel dimension in the EP sample, and including establishment fixed effects, we show that vacancy rates are substantially higher in years in which firms report being affected by labor constraints (column 5).

Finally, columns 6 and 7 of Table 2 link individual firms' labor constraint declarations to market-level recruiting difficulties and broader skill shortages. Using occupational employment shares as weights for each firm, we aggregate the average employer-reported recruiting difficulties nationally by 2-digit occupation and year into a firm-specific shift-share measure of labor constraints.[8] Firms with greater exposure to market-level constraints are more likely to report labor constraints, even with firm fixed effects.

Taken together, these patterns show that reported labor constraints consistently track recruiting

[8]Using merged employer-employee data, we assign firms' labor constraints declarations to all their workers. We then calculate average measures of hiring difficulties by collapsing worker-level data to the level of 2-digit occupation-by-year.

Table 2: Cross-Validation of Labor Constraints Measures

Results in columns 1 to 3 are estimated using data from the Vacancy Survey (repeated cross-section of firms); columns 4 to 7 are estimated using data from the IAB Establishment Panel (panel of firm-year observations). The main independent variable in columns 1 to 5 is firms' declaration of labor constraints, which in columns 1 to 3 represents a declaration that the lack of suitable workers is an impediment to the firm's operations, while in columns 4 to 5 it represents a declaration that the firm expects problems with labor shortages. In columns 6 and 7, the main independent variable is the shift-share measure of labor constraints based on firms' occupational structure and national occupation-level constraints intensity. The dependent variables are the logarithm of the length of the last worker search in days (column 1), an indicator for having failed a worker search in the last 12 months (column 2), vacancy rate (columns 3 to 5), defined as the number of reported vacancies divided by the number of workers, and the firm's labor constraints declaration (columns 6 and 7). Each column includes local area-by-year FE, industry-by-year FE and firm size bin FE. Column 1 also includes occupation-by-year FE, where occupation refers to the occupation of the workers in the last successful search. Columns 5 and 7 include establishment FE. Local area is defined as state in columns 1 to 3 and commuting zone in columns 4 to 7. Robust standard errors (columns 1 to 3) and standard errors clustered by establishment (columns 4 to 7) are reported in parentheses. *** represents 1% significance.

	(1) Search Duration	(2) Failed Search Last 12M	(3) Vacancy Rate	(4) Vacancy Rate	(5)	(6) Labor Constraints	(7)
Labor Constraints (SE)	0.296***	0.453***	0.076***				
	(0.013)	(0.005)	(0.001)				
Labor Constraints (EP)				0.014***	0.004***		
				(0.001)	(0.001)		
Labor Constraints (shift-share)						0.827***	0.672***
						(0.044)	(0.055)
N	50,417	50,417	50,417	77,330	77,330	77,310	77,310
Dataset	Vacancy Survey (SE)			IAB-Establishment Panel (EP)			
Baseline FE	✓	✓	✓	✓	✓	✓	✓
Plant FE					✓		✓

difficulties at both the firm and market levels, rather than idiosyncratic complaints. Analyzing self-reported measures may provide important benefits, consistent with the recent discussion by D'Acunto and Weber (2024).

3.2 Labor Constraints and Firm Growth

Even if labor constraints reflect real hiring difficulties, they could be harmless if firms can substitute toward technology, outsourcing, or reorganization, and thereby avoid adverse effects on their business operations. Yet, testing their business impact by using a simple correlation between labor constraints and firm outcomes may be misleading. Labor-constrained firms could be poorly managed and underperforming, which would discourage potential hires; but a constrained firm could also be successful and growing rapidly, thus having an extraordinary labor demand. To shed light on these effects, we now turn to studying the link between labor constraints and firm characteristics and outcomes.

The OLS regression results in columns 1, 3, 5, and 7 of Table 3 show that firms reporting labor constraints are typically expanding: they have faster employment growth and higher capital expenditures today, which also translates into higher sales growth and a higher likelihood of profitability next year. This finding goes against the hypothesis of underperforming firms and suggests that successful, rapidly growing firms are more likely to experience labor constraints.

However, reverse causality concerns complicate the interpretation of these findings: successful

Table 3: Labor Constraints and Real Outcomes

All regressions use data from the IAB Establishment Panel and the observation unit is establishment-year. The main independent variables are firms' own declaration of labor constraints and the shift-share measure of labor constraints based on establishments' occupational structure in a given year and occupation-level measures of labor shortages for the whole economy. The dependent variables are relative changes in the total number of employees, logarithm of capital expenditures in EUR, relative sales growth at t+1, and an indicator of whether the firm is profitable at t+1. Each column includes establishment FE, commuting zone-by-year FE, industry-by-year FE, and firm size bin FE. Standard errors are clustered by establishment. *** represents 1% significance, ** - 5%, * - 10%.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	%ΔEmployment		Log(CAPEX)		%ΔSales [t+1]		Profitable [t+1]	
Labor Constraints (firm reports)	0.009**		0.235***		0.018***		0.011***	
	(0.004)		(0.078)		(0.006)		(0.004)	
Labor Constraints (market-induced)		-0.120**		0.581		-0.104*		-0.067*
		(0.053)		(0.723)		(0.057)		(0.038)
N	77,433	77,433	77,433	77,433	57,747	57,747	58,201	58,201
Baseline FE	✓	✓	✓	✓	✓	✓	✓	✓
Plant FE	✓	✓	✓	✓	✓	✓	✓	✓

firms are likely trying to hire more workers or to fill vacancies more quickly, which generates labor constraints. An alternative test relies on a market-level measure of labor constraints, which combines firms' occupational structure with national occupation-level hiring difficulties. We implement this approach in Columns 6 and 7 of Table 2. Our regression includes industry-time and area-time fixed effects to control for shocks at the market or industry level that might influence both labor constraints and firm outcomes.

The results in even columns of Table 3 show that firms exposed to higher labor constraints because of their occupational mix and outside labor market conditions have significantly lower employment growth today. Firms facing a one-standard-deviation higher level of labor constraints experience around a 1.4 percentage points lower employment growth rate.[9] Those firms do not have significantly higher capital expenditures: while the coefficient is positive, consistent with raising capital expenditures to substitute scarce labor with technology, the estimate is noisy. Overall, lower inputs today translate into slower sales growth next year, with a one-standard-deviation increase in constraints associated with 1.2 percentage points lower sales growth, which is more than half of the average sales growth in our sample, and lower profitability, although we cannot observe the evolution of firm value in the long run. The precision of these two estimates is somewhat limited but they are statistically significant at the 10% level.

Taken together, our results suggest that labor constraints represent real hiring difficulties that are prevalent among growing and profitable firms, yet the exposure to market-level constraints is associated with lower growth and profitability, consistent with the negative effects of labor

[9] The standard deviation of the shift-share measure for labor constraints is 0.113, see Table 1, and we get $-0.12 \cdot 0.113 = -0.0136$.

constraints on firm performance documented by D'Acunto, Weber and Yang (2020) and Le Barbanchon, Ronchi and Sauvagnat (2023).

4 Labor Constraints and Firms' Wages

If labor constraints are a real phenomenon and slow firm growth, why do firms not preempt these situations by setting higher wages? In some cases, regulation prevents wage increases (Friedrich and Hackmann, 2021), but these situations are generally rare. In Germany's private sector, collective bargaining sets wage floors and does not limit firms' ability to pay above-scale wages. It is also possible that labor constraints mainly arise due to non-wage reasons, driven by a lack of amenities that workers value. A massive pay increase may compensate workers, but a reasonable wage increase would not relax the binding constraint, and firms may be better off focusing on non-wage benefits. If so, reported labor constraints should show little relationship with wages—a prediction we test in this section.

4.1 Firms' Own Wages

Firms Reporting Constraints Pay Lower Wages Table 4 shows that a higher average level of wages last year is associated with lower labor constraints today. This result is obtained without including firm fixed effects, and hence should be interpreted as a between-firm comparison. Consistent with basic economic intuition, firms that underpay their workers are more likely to report labor constraints. The relationship is present in both SE and EP, and is stronger in local markets with higher levels of scarcity, as measured by the share of firms reporting constraints (excluding the focal firm), see column (4) of Table 4.

Firms Pay Higher Wages When the Worker Search is Long Figure 2 demonstrates the relationship between search duration and log wage offered and paid to a new hire. We use the information about the last successful hire made by the firm in the SE data and regress the logarithm of the monthly wage paid to the hired candidate on indicators for each decile of the search duration distribution, omitting the first decile as the baseline group. In this first decile, the average search takes less than 10 days, but the duration increases to 21 days on average for decile 3, 73 days for decile 6, and 157 days for decile 9. We find a steep slope in the decile-specific coefficients for the baseline specification (depicted in blue circles in Figure 2), which indicates a strong positive relationship between search duration and wages paid. Part of this relationship is explained by higher-skilled positions requiring longer search and paying higher wages. To account for these differences, we control for occupation-year, industry-year, state-year, and firm-size fixed effects in the specification depicted in red triangles. The magnitudes of the coefficients decline by 30–40%, but the positive slope across deciles of search duration remains. Hence, even comparing similar firms that hire in the same labor market, longer searches go along with higher wages paid. Note

Table 4: Labor Constraints and Initial Pay Level

Columns 1 to 4 are estimated using data from the IAB Establishment Panel (panel of firm-year observations), while column 5 uses data from the Vacancy Survey (repeated cross-section of firms). The main independent variable is the logarithm of average wages in the previous year. In column 4, this variable is also interacted with the local measure of labor constraints, i.e., the leave-one-out average of labor-constraints declarations in a given district. The dependent variable measures declarations of labor constraints, i.e., that the firm expects problems with labor shortages (columns 1 to 4) or that the lack of suitable workers is an impediment to the firm's operations (column 5). Each column includes commuting zone-by-year FE and industry-by-year FE. Columns 2 to 5 further include firm size bin FE. Columns 3 and 4 include additional controls: firm age, existence of a works council, profit situation, sales volume. Standard errors are clustered by establishment. *** represents 1% significance.

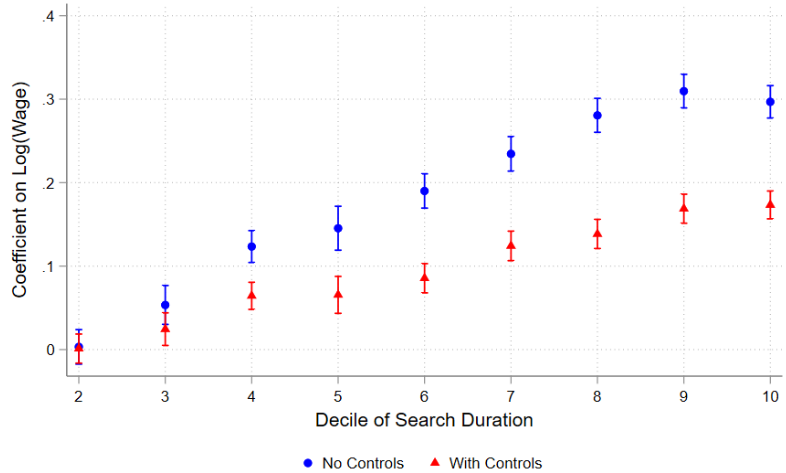
	(1)	(2)	(3)	(4)	(5)
	Labor Constraints				
Log(Wage) [t-1]	-0.029***	-0.052***	-0.046***	-0.023***	-0.067***
	(0.005)	(0.006)	(0.006)	(0.007)	(0.003)
Log(Wage) [t-1] X Local Scarcity				-0.158***	
				(0.041)	
N	71,981	71,981	71,981	71,293	99,334
Dataset	IAB Establishment Panel (EP)				Vacancy Survey (SE)
Baseline FE	✓	✓	✓	✓	✓
Size		✓	✓	✓	✓
Other Controls			✓	✓	

that this finding is not inconsistent with the negative effects of increasing wages on the time to fill a vacancy documented in the literature (Mueller et al., 2023; Bassier, Manning and Petrongolo, 2025) because firms may take a long time to adjust their wages during the search process.

Wages Increase After Labor Constraints Declarations Figure 3 demonstrates the evolution of average wages for new hires around labor constraints declarations. The year in which the firm declared labor constraints is $t = 0$. In the left panel, we present coefficients from a regression of the log average daily wage of workers hired over the last 12 months on lags and leads of the labor constraints declaration and a set of controls, including firm fixed effects. Observations outside the event window from years -3 to +3 are included in the regression and form a reference group with coefficients normalized to zero. The right panel repeats the exercise using residual wages after regressing individual log wages of new hires on observable characteristics (education, experience, occupation, year, and part-time status).

In both panels, coefficients are close to zero in the pre-period, indicating little evidence of an anticipatory wage increase. Residualized wages are completely flat between t=-3 and t=-1, while nominal wages display a small but insignificant increase at t=-1 that is insufficient to prevent constraints from arising. Once firms realize they face labor constraints, they increase wages: at t = 0 and t = 1, both nominal and residualized wages are significantly higher than at baseline. The moderate contemporaneous adjustment in log wages is consistent with the pattern documented by Kölling (2022). Comparing the evolution of nominal and residualized wages, the increase in residual wages from $t = -1$ to $t = 0$ exceeds the initial response of nominal wages, consistent with firms initially keeping their nominal wage stable but temporarily hiring workers with lower skill or experience (as captured by worker observables). In the following period, however, firms do increase

Figure 2: Search Duration and Wages for New Hires



The figure shows the coefficients from regressing the log of the monthly wage offered and paid to the last hire (and thus the sample is limited to firms that hired in the last 12 months) on fixed effects for each decile of the duration of the search for that hire. Blue circles are based on a simple regression without additional controls; red triangles are based on a regression that also controls for fixed effects for occupation-year, industry-year, state-year, and firm size. The whiskers represent 95% confidence intervals. The omitted category is the first decile, in which search duration is around 10 days. Search duration increases to 21 days for decile 3, 73 days for decile 6, and 157 days for decile 9.

their nominal wage so that, relative to their usual wage level, firms pay 4% higher wages one year after declaring that they are labor-constrained.

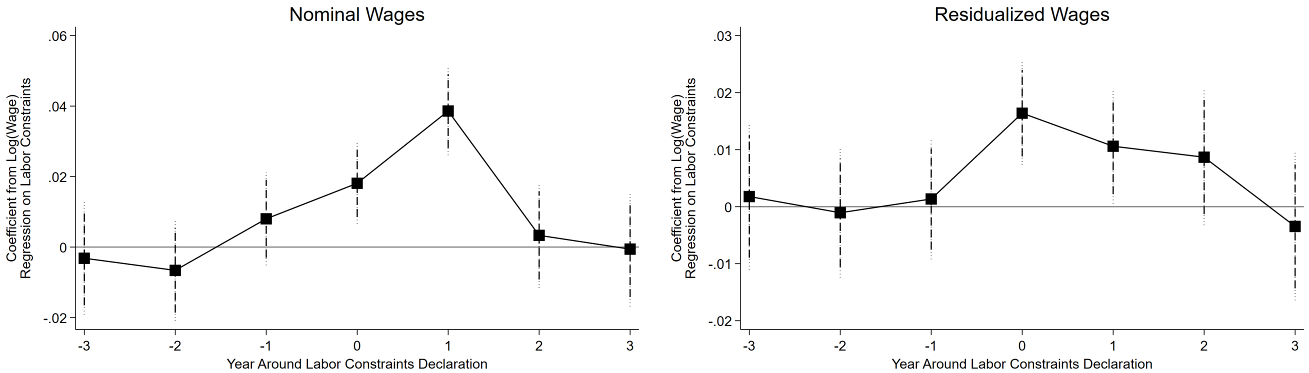
Naturally, the wage response analyzed in Figure 3 is endogenous: firms adjust their wages given their business conditions and find it optimal to increase them in periods during which they also report being labor-constrained. However, the fact that firms reporting labor constraints initially underpay their workers, offer higher wages when their worker search is longer, and labor constraints declarations then coincide with future wage increases suggests that labor constraints are a wage-related phenomenon. This timing between labor constraints declarations and subsequent wage increases also indicates that the pattern does not simply reflect a response to firm-level demand or productivity shocks that raise both desired employment and wages. Instead, the flat wages before the declaration and the delayed adjustment suggest that the wage response is closely tied to the realization of hiring difficulties.

4.2 Central Bargaining-Induced Variation in Wages

Building on the evidence of wage adjustments, we now turn to the question of whether increasing wages helps alleviate labor constraints. Causal identification is challenging because wages and constraints are jointly determined. To mitigate this endogeneity problem, we isolate quasi-exogenous variation in wages coming from central wage bargaining. More than 50 percent of workers in Germany are covered by a collective bargaining agreement, which typically sets baseline wage levels and other provisions through biennial negotiations between labor unions and employer associations. Negotiated wages are floors and firms are free to pay higher wages.

The bargaining coverage is rather complex. The most common form of bargaining happens at

Figure 3: Evolution of Nominal and Residual Wages around Labor Constraints Declarations



In the left panel, the solid line represents the coefficients from regressing the logarithm of average wages for new hires on respective lags and leads of labor constraints declarations by firms. For example, the value for t=-3 represents the coefficient from regressing the level of wages in year t on the 3rd lead of a labor constraints declaration and hence the coefficient represents the level of wages 3 years before the firm declared labor constraints. Each regression includes establishment fixed effects as well as state-by-year FE and industry-by-year FE. Wages are winsorized at the 1st and 99th percentiles. Dashed lines represent 95% confidence intervals. The right panel is analogous, but coefficients come from regressions of residual wages for new hires. Residual wages are residuals from regressing log wages on fixed effects for career tenure, education, occupation, skill level, and year, as well as a part-time-job indicator.

the industry level, e.g., construction or metal manufacturing. The bargaining often takes place by state, but the process in different states is highly correlated. Industry-level agreements set wages for different salary groups, and assignment to a particular group may vary by occupation, seniority, leadership position, and firm characteristics (Jäger, Noy and Schoefer, 2022). Worker-level assignment to specific collective bargaining agreements is not available in our data and deriving ex ante measures of applicable bargaining wages has traditionally been challenging.

To tackle this problem, we propose an ex-post measure of bargaining-induced wages. In the EP data we observe which firms are members of an employer association and subject to collective bargaining. Specifically, we can isolate firms declaring that 1) they are bound by a collective bargaining agreement; and 2) they do not pay wages exceeding those negotiated in the agreement. Based on worker-level administrative wage records for this subsample of firms, we first create average bargaining-induced wages for each occupation in a given year. We then merge these occupation-level average wages into firm-occupation-year-level data using information on each firm's occupational structure. Finally, we compute the occupational-bargaining-induced wage level variable for each firm-year by taking the weighted average of occupation-level wages with weights equal to the share of employees in a given occupation. Bargaining rounds typically last two years. Because bargaining is forward-looking, we define the bargaining-induced wage level based on realized wages in the year after an agreement (year $t+1$) to proxy for wages that the firm offers to new hires today (year t).

Table 5 regresses an indicator for reported labor constraints on bargaining-induced wages. Column 1 demonstrates that when collective bargaining pushes wages higher in a firm's occupational mix, reported labor constraints fall. This relationship is driven by firms that are covered by collective bargaining and whose wage levels are at the average bargained level or at most 10 percent higher (column 3); firms not covered by the bargaining agreement report a noisily estimated reduc-

Figure 4: Bargaining-induced Wage Increases and Labor Constraints

The figure shows the coefficients from regressing leads and lags of an indicator for labor constraints on the log-change in bargaining-induced wages based on occupation-level wage measures and firm occupational structure. Fixed effects for commuting zone-by-year, industry-by-year, and for firm size bins as well as controls for profitability, existence of a works council, and firm age are included. The sample is limited to firms that follow collective bargaining wage agreements and declare that they do not pay more than 10% above the agreed rates. 90% (long-dashed lines) and 95% (dotted lines) confidence intervals are presented based on the standard errors clustered by establishment.

tion in labor constraints (column 2) about half the size of the decrease at covered firms with sufficiently low initial pay levels.[10] In column 4, we interact the bargaining-induced wage increase with occupational employment concentration across industries, measured as a Herfindahl-Hirschman Index (HHI). For each occupation and year, we calculate the HHI of employment shares across industries, then construct a firm-level measure by averaging these occupation-year values weighted by the firm's occupational composition. A high HHI indicates that the firm employs workers in occupations that are typically employed in a small set of industries, suggesting limited outside employment options for workers and thus greater bargaining power for firms. Consistent with this interpretation, we find that firms with high-HHI occupational mixes show a significantly sharper decline in reported labor constraints following a bargaining-induced wage increase, which suggests that these firms may have initially cited labor constraints because they were paying below-market wages, consistent with the behavior of a monopsonist.

Figure 4 shows the dynamics of the labor constraints response. To do so, we run separate regressions of leads and lags of labor constraints declarations on the log-change of bargaining-induced wages. There is no pre-trend: lagged labor constraints declarations are not significantly related to a wage increase today. A wage increase today reduces labor constraints one year (coefficient -0.055, significant at the 5% level) and two years later (coefficient -0.050, significant at the 10% level). The estimates are obtained in the sample of firms that are subject to a collective agreement and pay wages close to (at most 10% above) the bargained wage floor, as in column 3 of Table 5.

In columns 5 and 6 of Table 5, we estimate the effect of firm wages on labor constraints,

[10] Note that about 10,000 firm-year observations have missing information about collective bargaining coverage and are thus excluded from the analysis in columns 2 to 4, and 6. Furthermore, we do not separately analyze ~1,800 firms that follow a collective agreement but voluntarily pay wages more than 10% above the agreed rate. In the subsample of these firms, there is no significant effect of a bargaining-induced wage increase.

Table 5: Central Bargaining-Induced Wage Level and Labor Constraints

All specifications are estimated in the IAB Establishment Panel with the unit of observation being establishment-year. The dependent variable is firms' declaration of labor constraints, i.e., a binary indicator that the firm expects problems with labor shortages. The independent variable in columns 1 to 4 is the logarithm of bargaining-induced wage levels for a given establishment based on its occupational structure and economy-wide measures of recruiting difficulties by occupation. The measure relies on the realized level of wages at time t+1, which proxies for wages offered at time t. In column 4, the measure is further interacted with the firm-level HHI of occupational employment by industry-year, to proxy for firm market power over workers. The level of HHI is included in the regression but not reported to conserve space. In columns 5 and 6, the bargaining-induced wage (independent variable in columns 1-4) is used as an instrument for the logarithm of firms' own average wage. Kleibergen-Paap F-statistics for the first stage regressions are reported at the bottom of the table. The sample is limited to firms following the agreement and paying wages at most 10% above the agreement-required minimum (columns 3, 4, and 6) and firms not formally following a collective wage agreement (column 2). Establishment FE and fixed effects for commuting zone-by-year, industry-by-year, and for firm size bins as well as controls for profitability, existence of a works council, and firm age are included in all columns. The bargaining-induced levels of wages are winsorized at the 2.5th and 97.5th percentiles. Standard errors are clustered by establishment. *** represents 1% statistical significance, ** - 5%, * - 10%.

	(1)	(2)	(3)	(4)	(5)	(6)
	Labor Constraints					
Bargaining-Induced Wage (Occupation)	-0.0332**	-0.0294	-0.0565**	-0.0199		
	(0.0143)	(0.0210)	(0.0255)	(0.0276)		
Barg-Ind Wage X HHI (Occ-Ind)				-0.0863***		
				(0.0281)		
Log(Firm Wage)					-0.456**	-0.768*
					(0.203)	(0.394)
N	62,045	19,847	29,253	29,253	61,969	29,226
Sample	Full	No Barg	Barg, <10%	Barg, <10%	Full	Barg, <10%
Robust F-Stat					43.3	22.1
CZ-Year, Size FE	✓	✓	✓	✓	✓	✓
Ind-Year, Plant FE	✓	✓	✓	✓	✓	✓
Other Controls	✓	✓	✓	✓	✓	✓

instrumenting firm pay using bargaining-induced wages. A 1 percent increase in average pay reduces the probability of being labor constrained by 0.5 percentage points in the full sample and 0.8 percentage points among firms that are directly covered by collective bargaining and pay close to bargained levels. These results are based on a highly significant first stage with robust F-statistics of 43 and 22, respectively.[11] Taken together, these results support the wage-based mechanism behind the labor constraints phenomenon: when a quasi-exogenous increase in wages in a firm's environment occurs, and the firm follows by increasing its own wages, its labor constraints are alleviated.

[11] Results for regressions of firm wages on bargained wages are reported in Appendix Table A.1. We find the largest increase in firm wages in response to bargained wages among covered, initially lower-pay firms. Uncovered firms also adjust, consistent with spillovers to non-union wage setting, as also documented by Demir (2024) in the context of a sectoral minimum wage introduction in Germany, while pass-through is muted at covered high-paying firms.

5 A Conceptual Framework for Labor Constraints

How can we explain the evidence presented so far with economic theory? In this section, we first consider a basic benchmark model of labor market monopsony and show that it can account for some patterns in the data but is harder to reconcile with others. This motivates our dynamic model of hiring, which encompasses the basic insights from the monopsony model but also allows for the impact of imperfect information and future downward wage rigidity.

5.1 Labor Constraints in Basic Monopsony Models

The simplest way to rationalize labor constraints within existing economic theory is through the lens of a basic monopsony model in which a firm hires workers facing an upward-sloping labor supply curve $F(w)$ (Robinson, 1933; Kline, 2025). The firm offers a wage w and receives a marginal revenue y from each worker that it hires, and hence the firm's problem is to choose a wage that maximizes profits $F(w)(y-w)$. Under standard shape restrictions on $F(w)$, the firm's problem has a unique solution w^*. At that wage, the firm hires $F(w^*)$ workers. The firm would still value an additional worker at wage w^*, but cannot hire that worker without also raising the wage paid to all existing workers. Labor constraints therefore arise because the firm does not find it profitable to raise wages to attract more workers.

This logic is not limited to the case in which the firm offers a single profit-maximizing wage w^*. Burdett and Mortensen (1998) consider a richer framework with labor market search frictions, where firms face a labor supply curve $F(w)$ that depends on the distribution of workers' outside options. In this setting, firms' optimal strategy is to randomize wages over a continuous interval $[\underline{w}, \overline{w}]$.[12] Intuitively, firms face a trade-off between high margins (low wage, low hiring probability, high profit) and a high success rate of hiring (high wage, high hiring probability, low profit) but in equilibrium each wage offers the same expected profit. Labor constraints arise in this model because firms that end up offering low wages fail to fill their vacancies, even though ex ante the choice of a low wage was rational.

Depending on the form of $F(w)$, the intuition for labor constraints in the monopsony model differs slightly, but in all cases, firms have wage-setting power, face an upward-sloping effective labor supply schedule, and may therefore optimally choose wages that leave vacancies unfilled. Is this basic monopsony logic consistent with the labor constraints patterns that we have documented in the data? On the one hand, we find clear evidence in favor of such models: firms that offer lower wages are more likely to be labor-constrained (Table 4) and offering higher wages does alleviate labor constraints (Table 5, Figure 4). These findings line up well with the monopsony view that low wages can generate unfilled labor demand. On the other hand, some of the patterns in the data are at odds with the predictions of monopsony models. First, labor search models like Burdett

[12] This mixed strategy of firms follows the results from earlier work by Burdett and Judd (1983). Kline (2025) gives a detailed discussion on the connection between basic monopsony models and these richer search environments.

and Mortensen (1998) predict a negative relationship between search duration and wages: firms posting higher wages should fill vacancies more quickly because higher wages raise acceptance probabilities. This prediction is inconsistent with the patterns in Figure 2, where search duration is actually positively related to the wage paid to the new hire. Second, in these models, more productive firms choose higher wages and have less trouble hiring and retaining workers than less productive firms (Mortensen, 2010).[13] In the data, however, it is fast-growing and more profitable firms that are more likely to report labor constraints (Table 3). Third, a monopsonist that correctly understands the relevant hiring schedule has no systematic reason to revise its wage offer after one unsuccessful hiring attempt. Failure is then an equilibrium outcome of an optimally chosen policy. Yet, Figure 3 demonstrates that firms do increase their wages after reporting labor constraints. This pattern suggests that a hiring failure teaches the firm something about the labor market and hence about the wage required to hire. That mechanism lies outside a complete-information monopsony framework. These observations motivate our full model, to which we turn next.

5.2 A Dynamic Model of Wage Discovery

We now introduce a dynamic wage-posting model that preserves the monopsony mechanism but adds two ingredients to help explain the empirical patterns: firms have incomplete information about recruiting conditions, and those conditions may change over time. The model is meant to formalize a simple idea. Firms may not know the wage needed to hire in the current labor market. A failed hiring attempt is not merely an expected consequence of an optimally chosen wage, but reveals stronger-than-expected labor competition, making labor constraints more salient and leading firms to increase wages in the next period. However, firms may be hesitant to post higher wages when they believe the tight market is less likely to persist, since today's failed hire is then a weaker signal about the conditions they will face going forward.

Consider discrete time periods and an infinite time horizon. In each period t, the economy is in one of two states, $\omega_t \in \{h, l\}$. The high state corresponds to a tight labor market with more firms entering with vacancies: a measure $\lambda^f(\omega_t)$ of firms enters each period, with $\lambda^f(h) > \lambda^f(l)$. The state evolves according to a Markov chain with transition probabilities p_{hh} and p_{ll}, where $p_{hh} = p_{ll}$ and $p_{hh} + p_{ll} > 1$.

Firms do not directly observe ω_t. Instead, an entering firm observes a signal $s \in [0,1]$ drawn from a state-dependent distribution $H(s \mid \omega)$ with density $h(s \mid \omega)$ and forms an initial belief $\beta_0 \in [0,1]$ that the economy is in the high state.[14] The signal s satisfies the monotone likelihood

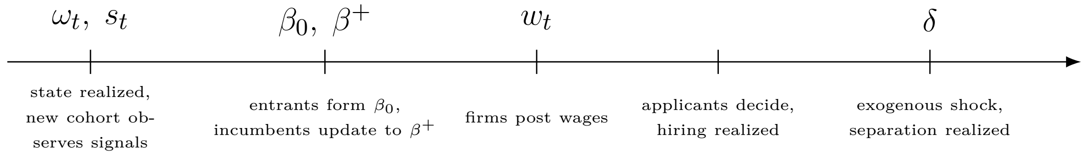
[13]Some subsequent extensions of the canonical monopsony framework can help account for these patterns. For example, Kaas and Kircher (2015) assume convex costs of vacancy posting that can explain more difficulties in hiring for fast-growing firms that aim to fill multiple vacancies.

[14]The firm's initial belief after observing s is

$$\beta_0(s) = \frac{\lambda^f(h)h(s|h)}{\lambda^f(h)h(s|h) + \lambda^f(l)h(s|l)}.$$

This expression reflects both the firm's private signal and the fact that entry itself is more likely in the high state.

Figure 5: Timing of the Stage Game



ratio property, i.e., the likelihood ratio $h(s \mid h)/h(s \mid l)$ is increasing in s. Thus high signals are more common in the high state.

The timing of the stage game is illustrated in Figure 5. Entering firms join any firms that failed to hire in earlier periods and remained in the market; we refer to this combined set as active firms. Every active firm has one vacancy. Entrants form initial beliefs β_0 from their private signals, while firms that remain active after an unsuccessful search carry forward beliefs updated from their hiring experience β^+. A fixed measure λ^a of applicants enters each period and each applicant has an outside option v drawn from a distribution G on $[c, y]$. Active firms post wages and contact applicants through the firm-initiated matching technology described below. Applicants are myopic: they accept the best offer above their outside option; otherwise, they take their outside option. In either case, they exit the market at the end of the period. Firms that successfully hire leave the market, while active firms that do not hire exit with an exogenous probability δ and otherwise continue searching next period.

The matching technology governs contacts between firms and applicants. Let $\xi(\omega)$ denote the equilibrium ratio of active firms to active applicants, and let $W(w \mid \omega)$ denote the steady-state equilibrium distribution of posted wages. Since each active firm contacts one applicant at random, the number of offers received by an applicant follows a Poisson distribution with mean $\xi(\omega)$. The aggregate state continues to follow the Markov chain, but in the stationary equilibrium the relevant distributions depend on the current state rather than calendar time. We therefore write ω for the current realization of the state.

The probability that a firm posting wage w fills its vacancy in state ω is

$$p^f(w|\omega) = G(w)\, e^{-\xi(\omega)(1-W(w|\omega))}. \tag{5.1}$$

The first term is the probability that the wage exceeds the applicant's outside option. The exponential term is the probability that the firm does not lose the applicant to a competing offer above w.

A firm with belief β evaluates its hiring probability as the belief-weighted average

$$p^f(w|\beta) := \beta\, p^f(w|h) + (1-\beta)\, p^f(w|l). \tag{5.2}$$

Hiring failure is informative because failure is more likely in the high state, where competition for workers is stronger. After posting wage w and failing to hire, a firm with prior belief β updates to

$$\tilde{\beta}(w,\beta) := \frac{\beta[1-p^f(w|h)]}{\beta[1-p^f(w|h)]+(1-\beta)[1-p^f(w|l)]}. \tag{5.3}$$

The prior for the next period also incorporates the Markov transition of the aggregate state:

$$\beta^+(w,\beta) := p_{hh}\,\tilde{\beta}(w,\beta)+(1-p_{ll})(1-\tilde{\beta}(w,\beta)). \tag{5.4}$$

Equations (5.3) and (5.4) are the key departure from the complete-information monopsony benchmark. In that benchmark, an unfilled vacancy is simply an equilibrium consequence of the chosen wage. Here, an unfilled vacancy is also a signal that the firm may have underestimated market tightness.

Firms choose wages as a function of their beliefs. The equilibrium object is a wage strategy $w^*(\beta)$ that solves the firm's dynamic problem. If a firm hires at wage w, it receives surplus $y-w$. The firm's Bellman equation is

$$V^f(\beta) = \max_{w\in[c,y]} \Pi(w,\beta), \tag{5.5}$$

where

$$\Pi(w,\beta) := p^f(w|\beta)(y-w)+\alpha(1-\delta)[1-p^f(w|\beta)]\,V^f(\beta^+(w,\beta)). \tag{5.6}$$

The first term is the expected payoff from hiring today. The second term is the continuation value if the firm fails to hire and remains active. The surplus $y-w$ can be read as the present value of an employment relationship formed at wage w; once the match is formed, the wage is not renegotiated downward inside the model. A high wage therefore improves the chance of hiring today, but commits the firm to lower surplus from that match.

Market tightness $\xi(\omega)$ and the wage distribution $W(w|\omega)$ are endogenously determined by the stock of active firms, applicant flows, firms' beliefs and wage strategies, and market clearing. The full equilibrium definition is given in Appendix B. For brevity, the main text keeps only the equations needed to see the economic mechanism: wages affect hiring probabilities, hiring failures update beliefs, and beliefs determine future wage offers.

We impose two regularity conditions. The first ensures that, holding market tightness and the wage distribution fixed, the firm's static trade-off between a higher hiring probability and a lower margin has a well-behaved interior solution.

Assumption 1: For every feasible market tightness ξ and wage distribution W, the static profit function $w \mapsto G(w)e^{-\xi(1-W(w))}(y-w)$ is strictly concave on (c,y). Thus the profit-maximizing wage is unique for any given market tightness and wage distribution.

The second condition restricts attention to equilibria in which firms with higher beliefs about the high state post weakly higher wages, and in which reducing wages implies a larger reduction in hiring chances when the market is tight. This assumption ensures that beliefs translate to wages in the natural direction: a stronger belief that the market is tight leads to firms posting systematically higher wages.

Assumption 2: We focus on wage strategies that are continuous and increasing in beliefs. In addition, the proportional effect of a higher wage on the hiring probability is larger in the high state than in the low state:

$$\frac{\partial}{\partial w} \log p^f(w|h) > \frac{\partial}{\partial w} \log p^f(w|l).$$

Proposition 1. *Under Assumptions (1)-(2), for any probability of firm exit $\delta > 0$ and discount factor satisfying $\gamma := \alpha(1-\delta) < \bar{\gamma}$, where $\bar{\gamma} \in (0,1)$, there exists a Markov-perfect steady-state equilibrium as characterized in Definition 1.*

The proof is in Appendix C.2. The model preserves the monopsony logic discussed above: firms face an upward-sloping labor supply curve because raising the wage (w) increases the hiring probability ($p(w)$), while reducing profits conditional on hiring.[15] If aggregate uncertainty is shut down, the model reduces to a standard wage-posting search model such as those in Burdett and Judd (1983) and Burdett and Mortensen (1998). With aggregate uncertainty, however, hiring outcomes also move the firm's beliefs about recruiting conditions and, through that channel, future wage-setting. This additional learning channel allows the model to generate gradual wage adjustment after labor constraints and to rationalize a positive relationship between search duration and wages. The next subsection derives three implications that guide the empirical analysis.

5.3 Understanding Labor Constraints

We now turn to analyzing how additional frictions beyond the canonical search framework—information frictions about the state of the economy and downward wage rigidities—can help explain labor constraints in our model. When the state of the economy is not publicly observed, firms may underestimate the equilibrium wage level. If the state of the world is persistent, firms learn about the equilibrium wage level over time, but labor constraints may occur in the meantime. We formalize this intuition in a setting where firms with different signal quality compete in the same labor market. A parallel comparison across markets that differ in their average information quality delivers analogous market-level predictions, which we develop in Appendix B.8. In addition, imperfect persistence in the state of the economy itself can give rise to labor constraints: firms may be reluctant to raise wages in today's tight market, worrying that downward wage rigidities will leave them stuck with these high wages even if competition for talent weakens tomorrow. Hence, we compare two markets that differ in state persistence to investigate the role of firms' dynamic considerations.

Throughout, we quantify labor constraints via the firm hiring probability p^f, which measures the likelihood of successful hiring in the current period; similar conclusions can be obtained from considering the search duration, which is a weighted inverse of the hiring probability. For the

[15] See Appendix B.6 for a detailed discussion.

following analysis, we make one additional assumption that ensures that shifting the belief distribution of the whole market upward shifts the posted wage upward at every belief level:

Assumption 3: For $\lambda^f(h)$ sufficiently large, if two markets have belief distributions that satisfy $\Gamma_A^f(\beta \mid h) \leq \Gamma_B^f(\beta \mid h)$, then $w_A^*(\beta) \geq w_B^*(\beta)$.

This assumption introduces a market-level monotonicity condition: when firms expect workers to receive more offers from competitors, they will post higher wages themselves.

5.3.1 The Role of Information

Consider two groups of firms, $g \in \{I, U\}$, that differ in the precision of their information and compete in the same labor market. Each period in state ω, a total measure $\lambda^f(\omega)$ of firms enters: a fraction $\mu \in (0,1)$ are *type I* (informed) and draw their initial signal from $H_I(\cdot|\omega)$, while the remaining fraction $1-\mu$ are *type U* (uninformed) and draw from $H_U(\cdot|\omega)$. We assume a first-order stochastic dominance (FOSD) property for the signal distributions, $H_I(\cdot|h) \succ_{FOSD} H_U(\cdot|h)$ and $H_U(\cdot|l) \succ_{FOSD} H_I(\cdot|l)$, such that informed firms are more likely to receive high signals in the high state and low signals in the low state than uninformed firms. To isolate the role of information, we assume $p_{hh} = 1$, and we focus on the high state ($\omega = h$) when labor constraints are more likely to occur.

Both groups face the same market-level tightness and wage distribution—a firm-stock weighted average of the two groups' wage offers. This implies that for any given firm-specific belief, all firms in the market will choose the same optimal wage and face the same probability of successful hiring. Yet, because of different initial signal distributions, the two groups will have different initial beliefs and learning dynamics, and hence offer different wages, see Appendix C.4 for formal details.

Proposition 2. *Fix $\mu \in (0,1)$. Under Assumptions (1)-(3) and $p_{hh} = 1$, there exists $\bar{\lambda}^f > 0$ such that for all $\lambda^f(h) > \bar{\lambda}^f$: informed firms pay higher average wages $E_I[w \mid h] > E_U[w \mid h]$, and informed firms have a higher hiring probability $\bar{p}_I^f(h) > \bar{p}_U^f(h)$.*

The results in Proposition 2 highlight the core insight for a monopsonistic market with private information: informed firms benefit from information rents through higher hiring probabilities. Better-informed firms recognize earlier that the labor market is tight, workers have plenty of competing offers, and the going wage is high. They bid aggressively from the start. Less-informed firms start with lower offers, calibrated to a market they believe to be looser than it actually is, and revise upward only gradually as repeated hiring failures reveal stronger competition.

Beyond separating better- and worse-informed firms within a market, information also shapes how entire markets perform. When a larger share of firms in a market is well informed, the market posts higher wages and matches workers faster, so even poorly informed firms benefit from the reduced congestion. We state this market-level result formally in Appendix B.8 and present the corresponding evidence in Appendix A.4.

5.3.2 The Role of Dynamic Considerations and State Persistence

We also analyze the role of dynamic considerations by comparing market A with persistence rate $p_{hh}^{A} < 1$, and market B with persistence rate $p_{hh}^{B} < p_{hh}^{A}$. For this analysis, we relax the $p_{hh} = p_{ll}$ requirement and set the transition probability p_{ll} and other parameters to be identical across markets to isolate the effect of p_{hh}. We focus on periods when the economy is in the high state ($\omega = h$), the empirically relevant case for high labor market tightness.

Proposition 3. *Under Assumptions (1)-(3) and $p_{hh}^{B} < p_{hh}^{A} < 1$, there exists $\bar{\lambda}^{f''} > 0$ such that for all $\lambda^{f}(h) > \bar{\lambda}^{f''}$: firms in the more persistent market A pay higher average wages $E_A[w \mid h] > E_B[w \mid h]$ and have a higher hiring probability $\bar{p}_A^{f}(h) > \bar{p}_B^{f}(h)$.*

Proposition 3 captures a different channel of labor constraints: even when firms are equally well informed about today's conditions, they may still hold back on wages today if they expect today's tight market to be temporary. The mechanism is the dynamic counterpart of downward wage rigidity. In our model, hiring at wage w today commits the firm to that wage for the duration of the project and there is no opportunity to renegotiate downward when the cycle turns. This commitment makes wages a long-lived liability, and firms naturally consider not just today's hiring conditions but also tomorrow's when deciding what wage to offer. If the state of the economy is less persistent, firms are willing to offer a lower wage and that makes them hire less often. Slower hiring leaves more vacancies waiting in the market, which raises tightness and makes hiring even harder.

6 Slow Wage Discovery: Empirical Evidence

In this section, we investigate the novel predictions of our model in the data, introducing several specifications that capture the role of different kinds of information and downward wage rigidity. We first analyze wage revisions during the hiring process, which indicate inaccurate initial beliefs. Building on Proposition 2, we then compare labor constraints of firms with different levels of information about market wages by exploring hiring in core vs peripheral occupations. To shed light on the dynamics of information arrival and the wage adjustment process, we trace the reaction to wage changes in other parts of the economy. Finally, we study the role of downward wage rigidity by looking at firm-specific measures of wage stickiness and hiring of temporary workers. We note that previous results (Tables 4 and 5) provide support for the monopsony power mechanism, which is also predicted by our model.

6.1 Wage Revisions During the Hiring Process

We begin with direct evidence that firms revise their wage beliefs upwards during the recruiting process when facing labor constraints. The vacancy survey asks establishments whether the final wage for the last successful hire was higher than initially planned. This measure is useful for our purposes because it captures whether the wage initially viewed as sufficient turned out to be too low.

Table 6 shows that labor-constrained firms were significantly more likely to revise their wage offer upwards for their last hire. This is consistent with the wage dynamics around labor constraints declarations in Figure 3 and supports our interpretation of this upward wage revision: the firm starts the search with a wage plan, faces unexpected hiring difficulties, and adjusts the wage upward to complete the hire.

Consistent with firms' slow and gradual learning, these wage adjustments are more likely for longer searches (column 2), which indicate that the initial wage plan may not be sufficient to attract the desired worker. Upward revisions are also more common when the firm searches for a candidate who is required to have a high level of experience (column 3), a setting in which comparable wage information is likely harder to obtain. Finally, firms are more likely to pay more than planned when hiring in an occupation for which hiring difficulties are generally reported more often, and which has greater dispersion of wages paid to new hires (columns 4 and 5). These results suggest that firms adjust wages upward precisely in situations in which the initial wage belief is more likely to be inaccurate or in which recruiting feedback more clearly reveals that the planned wage is below the market-clearing level.

Starting in 2014, the vacancy survey also asks firms about their plans following a failed hiring attempt. Firms that intend to resume their search are asked how the new search will differ from the previous one. They can report that they will change the job requirements, alter the pool of potential applicants, or offer a higher wage. Firms are substantially more likely to plan a wage increase when their initial search failed because the wage demands of applicants were too high, directly illustrating the learning channel from our model (column 6).

The evidence so far directly demonstrates the occurrence of wage revisions and characterizes market-level conditions and job search attributes that predict a higher likelihood of wage belief updating. A potential critique, however, is that these relationships conflate wage revisions due to firms' learning with recruiting practices for given parts of the labor market. For example, some firms may tend to hire in occupations where wage increases during the recruitment process are more common for other reasons. We therefore turn to a sharper within-occupation test, which exploits variation across occupations hired by the same firm.

6.2 Informed and Uninformed Firms

Consider a construction firm in which machine operators account for a large share of the workforce, while marketing specialists account for only a small share. When hiring another machine operator, the firm is likely to have relatively precise information about the prevailing wage because of its extensive experience employing and recruiting workers in that occupation. Its wage information is likely to be noisier when hiring a marketing specialist. We refer to occupations with larger and

Table 6: Wage Revisions in Hiring Process

All specifications are estimated in the Vacancy Survey (SE) with the unit of observation being establishment-year. The dependent variable is firms' declaration of "having to pay more than initially planned" for the last reported hire (columns 1 to 5) and a declaration that a restarted search will feature a higher wage (column 6) for the last failed search. The independent variables are: an indicator for overall labor constraints being an important impediment to firm operations; and, for the last successful hire: the logarithm of search duration in days, an indicator for high experience being required, as well as occupation-level measures: the average likelihood of recruiting difficulties, and the standard deviation of log monthly wages offered to new hires in this occupation (the sample size is limited due to the availability of monthly wages offered); lastly, for the last failed search, the independent variable is an indicator for too high wage expectations being the reason for the failed search. State-year and industry-year fixed effects are included in all columns, while occupation-year fixed effects are included in all columns except 4 and 5. Robust standard errors in parentheses, except columns 4 and 5, where standard errors are clustered by occupation. *** represents 1% statistical significance, ** - 5%, * - 10%.

	(1)	(2)	(3)	(4)	(5)	(6)
	Wage Revised Upwards					Increased Wage
Labor Constraints	0.121***					
	(0.004)					
Search Duration		0.0363***				
		(0.0012)				
High Experience Was Required			0.0670***			
			(0.0020)			
Average Hiring Difficulties in Occ				0.246***		
				(0.031)		
Wage Dispersion for Occupation					0.0473***	
					(0.0034)	
Wage Demands Too High						0.0469***
						(0.0043)
N	129,642	103,324	128,431	130,182	37,825	36,013
Occupation-Year FE	✓	✓	✓			✓
State-Year FE	✓	✓	✓	✓	✓	✓
Industry-Year FE	✓	✓	✓	✓	✓	✓
Size Control	✓	✓	✓	✓	✓	✓

Figure 6: Wage Revisions and Hiring Difficulties across Deciles of Share of Workforce in Hired Occupation

The figure shows the coefficients from regressing the indicator for having revised the wage upwards during the search process (left panel) and the indicator for experiencing hiring difficulties (right panel) on 10 deciles of the share of workforce in the occupation that the firm hires in. The deciles are defined only for firms for which the share is non-zero; over 30% of firms have zero share and they constitute the reference group. The whiskers represent 95% confidence intervals. The specification includes fixed effects for state, industry-by-year, and occupation-by-year, as well as the logarithm of the total number of workers and the hiring rate in a given occupation (number of workers in a given occupation hired in a given year divided by the total workforce). Standard errors are clustered by occupation.

smaller employment shares within the firm as core and peripheral occupations, respectively. This comparison of core and peripheral occupations is similar to the distinction between informed and noise traders in the finance literature (Kyle, 1985; Glosten and Milgrom, 1985) and allows us to distinguish settings in which firms are more and less well informed. Using an occupation's share of firm employment as a proxy for wage information, we test Proposition 2, which predicts that firms with better information are less likely to experience labor constraints.

Table 7 and Figure 6 analyze the part of the SE data that describes firms' experiences with the last recruited worker. The key independent variable is the share of workers in the firm's current workforce that belong to the same occupational group as the new hire. It therefore provides a continuous measure of how core or peripheral the hired occupation is. Our specifications control for industry-by-year fixed effects, occupational group-by-year fixed effects, and state fixed effects, as well as continuous measures of firm size and hiring intensity in a particular occupational group. We note that the inclusion of occupation-by-year fixed effects means that we are capturing the relative difficulties in hiring for a given occupation by firms that already have many rather than few workers in that occupation.

Hiring in occupations that are more widely represented in the firm's current workforce is associated with a lower likelihood of upward revisions to wage beliefs (left panel of Figure 6, column 1 of Table 7) and with fewer recruiting difficulties overall (Figure 6 and column 2 of Table 7). The effect is roughly monotone and increasing across deciles of the occupation's share of the workforce, and the difference is driven particularly by the wage-setting process: while firms hiring for core occupations are no less likely to adjust qualification or experience requirements (columns 5 and 6), they are less likely to report difficulties in filling the position because applicants' wage expectations are too high (column 3). This is not because firms are more likely to engage in negotiations with candidates or to be more flexible in their wage-setting practices: wage negotiations are actually less likely to occur for firms hiring in core occupations (column 4). These results are consistent

Table 7: Recruitment of Core and Peripheral Occupations

All specifications are estimated in the Vacancy Survey data with the unit of observation being each establishment's last conducted worker search. The dependent variables are an indicator for having revised their wage offer upwards during the search process, experiencing any hiring difficulties, having difficulties in filling the positions because wage expectations were too high, having negotiated the wage, and an indicator that the hired candidate differed from initial expectations due to different qualifications or a different level of experience. The independent variable is the share of the firm's existing workforce in the same occupation as the occupation of the conducted search. All columns include fixed effects for state, industry-by-year, and occupation-by-year, as well as the logarithm of the total number of workers and the hiring rate in a given occupation (number of workers in a given occupation hired in a given year divided by the total workforce). Standard errors are clustered by occupation. *** represents 1% statistical significance, ** - 5%.

	(1) Wage Revised Upwards	(2) Hiring Difficulties	(3) Wage Demands Were Too High	(4) Wage Negotiated	(5) Hire Differs From What Looked For: Qualification	(6) Hire Differs From What Looked For: Experience
%Workforce in Hired Occupation	-0.0330**	-0.0458***	-0.0379***	-0.0898***	-0.00697	-0.0101
	(0.0132)	(0.0166)	(0.0092)	(0.0301)	(0.00969)	(0.00876)
N	43,357	43,357	43,357	26,555	43,357	43,357
Fixed Effects	✓	✓	✓	✓	✓	✓
Hiring Rate	✓	✓	✓	✓	✓	✓
Workforce Size	✓	✓	✓	✓	✓	✓

with firms being better informed about the prevailing market wage in their core occupations, and thus experiencing lower labor constraints.

Market-Level Proxies for Signal Quality The evidence in Table 7 compares firms hiring in the same occupation/labor market but having different levels of information about it. We complement this evidence with alternative results that compare the incidence of labor constraints across markets with different levels of wage-information quality. Results using several proxies for information quality are reported in Appendix A.4. First, we consider measures of wage dispersion. In the EP data, we construct a shift-share measure of wage dispersion in the occupations that make up the firm's workforce. In the SE data, we measure the standard deviation of wages offered to new hires in the occupation of a firm's last hire (as in column 5 of Table 6). When wages in relevant occupations are more dispersed in the labor market, firms may find it more difficult to gather information about the wage needed to hire a marginal worker. Second, we measure how closely local wages move with national wage trends, thereby making national statistics and news more informative for the local market. Consistent with the information mechanism, including the additional market-level predictions derived in Appendix B.8, labor constraints are more common among firms exposed to occupations with greater wage dispersion and less common in local labor markets whose wages are more synchronized with national trends. Since wage dispersion and synchronization may reflect factors beyond information precision, as evidenced by weaker patterns for wage dispersion in the most demanding fixed-effect specifications, we view these results as supportive evidence that complements the more direct results based on firms' own wage revisions and hiring outcomes.

6.3 Wage Dynamics in Response to Wage Shocks in Other Parts of the Economy

Our second test of Proposition 2 aims to explicitly trace out what happens when equilibrium wages in the local labor market change and some firms are more aware of those changes. Firms may update slowly in response to wage shocks that originate outside their own industry. For example, if an industry-specific technology or demand shock raises labor demand in metal manufacturing, metal firms will be relatively well informed about this change and will quickly raise wages to attract workers, drawing some from other sectors (e.g., construction). If the metal industry in a given local area is prominent enough, then its higher wages increase the overall market-clearing level of wages in other industries as well. Yet, firms in other industries may initially be unaware of the rising equilibrium wage and may only learn about it when they face increasing difficulty in recruiting new workers at the old wage level—through labor constraints.

We test this hypothesis in Table 8 by exploring the evolution of labor constraints declarations and wages around variation in the bargaining-induced level of wages in *other industries*. The dependent variable is a firm's labor constraints declaration (columns 1 and 2) or its level of wages (columns 3 and 4) at time T. The independent variables are measured at T, $T-1$, and $T-2$, and hence the regressions reveal whether firms' wage and labor constraints responses are immediate or delayed. Odd columns analyze the full sample and even columns are restricted to firms with up to 200 employees. Across the full sample, we find no contemporaneous response to a bargaining-induced increase in wages in other industries, consistent with worker flows to other industries taking some time. One year later, denoted by the shock at time $T-1$, wages remain unchanged (column 3) but firms now report significantly more labor constraints (column 1). This increase is substantial: 10 percent higher outside wages are associated with 1.5 percentage points higher labor constraints, an increase by 12 percent of the sample average. Another year after the initial shock, denoted $T-2$, we find that firms have increased wages significantly and labor constraints are resolved. These labor constraints and delayed wage adjustment patterns are particularly prevalent among small firms with fewer than 200 employees (columns 2 and 4), which arguably have fewer resources and engage in less hiring to stay informed about labor market conditions. This is also consistent with the more frequent inaccurate wage beliefs among small firms documented by Bertheau and Hoeck (2023) using survey evidence from Denmark.

Overall, the results in Table 8 are consistent with imperfect and delayed information about changing market wages being a key factor behind labor constraints. Firms that face wage increases occurring in other parts of the economy initially do not adjust their wage policies, likely because their beliefs about prevailing market wages did not change. Gradually, however, migration of workers across industries and labor markets leads to difficulties in attracting and retaining employees, and makes firms more likely to report labor constraints. Firms respond to these difficulties by increasing wages, which then alleviates the constraints. The entire process, however, lasts up to two years.

Table 8: Labor Constraints and Wage Shocks in Other Parts of the Economy

All specifications are estimated in the IAB Establishment Panel with the unit of observation being establishment-year. The dependent variable is the firm's declaration of labor constraints (columns 1 and 2) and the logarithm of firms' average wages (winsorized at the 1st and 99th percentiles). Independent variables are logarithms of the central bargaining-induced level of wages in other industries, calculated using local area industrial composition and industry-level bargaining-induced wage levels. The independent variables represent values contemporaneous with the outcome variable, the first annual lag, and the second annual lag. All specifications include establishment FE, commuting zone-year FE, industry-by-year FE, and firm size FE. The sample is limited to firms for whom at least 7 observations are available in the panel. Columns 1 and 3 include all firms; columns 2 and 4 include only firms with fewer than 200 employees. Standard errors are clustered by district. *** represents 1% statistical significance, ** - 5%.

Coefficient on Bargaining-Induced Local Wage in Oth Industries:	(1)	(2)	(3)	(4)
	Labor Constraints		Log(Wage)	
[T]	-0.0474	-0.0723	-0.00316	-0.0207
	(0.0665)	(0.0786)	(0.0235)	(0.0303)
[T-1]	0.153**	0.234***	-0.0179	0.00921
	(0.0712)	(0.0885)	(0.0271)	(0.0355)
[T-2]	-0.0298	-0.0703	0.0510**	0.0636**
	(0.0590)	(0.0704)	(0.0251)	(0.0323)
N	24,669	18,786	24,655	18,772
Sample	Full	Small	Full	Small
CZ-Year, Ind-Year, Size FE	✓	✓	✓	✓
Plant FE	✓	✓	✓	✓
Other Controls	✓	✓	✓	✓

6.4 Downward Rigidity of Nominal Wages

Proposition 3 predicts that the prospects of lower competition for workers in the future make firms hesitant to raise wages today. To investigate this mechanism, we conduct two complementary analyses: we measure the role of wage rigidities at the firm level and compare labor constraints and wage setting for temporary versus regular hires.

Firm-Level Wage Rigidity We first measure the extent to which firms differ in their ability to adjust wages over time. Specifically, we measure wage volatility at the establishment level over time and hypothesize that higher volatility maps into higher flexibility in wage setting, which in turn may help prevent labor constraints. In addition, since overall wage volatility can also arise from compositional changes in the workforce, we construct a firm-specific index of downward wage rigidity by applying the approach developed by Ehrlich and Montes (2024) based on within-firm, within-worker variation. Intuitively, this method measures missing wage cuts based on observed asymmetry in the distribution of wage growth within each firm. Table 9 reports the results of this empirical analysis. Firms with higher wage volatility are less likely to report labor constraints (column 1), while firms with higher wage rigidities face significantly more labor constraints (columns 2 and 3).

Table 9: Expected Downward Rigidity of Nominal Wages

Columns 1 to 4 are estimated using data from the IAB Establishment Panel (panel of firm-year observations), while columns 5 and 6 use data from the Vacancy Survey (repeated cross-section of firms). The dependent variables are an indicator of labor constraints, i.e., that the firm expects problems with labor shortages (columns 1 to 4) or the log wage paid to the most recent hire (columns 5 and 6). Each column includes local area-by-year FE (in columns 1-4, commuting zone; in columns 5-6, state), industry-by-year FE, and firm size bin FE. Columns 1, 3, and 4 further include controls for firm age, profitability, and existence of a works council. Across-time wage volatility is the standard deviation of log average wages across years. Temp workers are workers on temporary employment contracts. Wage Rigidity is the downward wage rigidity index measuring the share of missing wage cuts, similar to Ehrlich and Montes (2024). The local vacancy rate is computed as the state-by-year average vacancy rate defined as the total number of vacancies divided by the total employment of firms responding to the vacancy survey. Local hiring problems are the area-level share of firms reporting any hiring difficulties. Standard errors are clustered by establishment in columns 1-4 and by state in columns 5-6. *** represents 1% statistical significance, ** - 5%.

	(1)	(2)	(3)	(4)	(5)	(6)
	Labor Constraints				Log(Wage)	
Across-Time Wage Volatility	-0.0517***					
	(0.0155)					
Wage Rigidity (Ehrlich, Montes 2024)		0.0107**	0.00973**			
		(0.00470)	(0.00492)			
Share of Temp Workers				-0.00875		
				(0.00993)		
Temp Hire X Local Vacancy					-0.0204	
					(0.228)	
Temp Hire X Local Hiring Problems						0.824**
						(0.362)
N	89,537	48,603	44,587	56,016	39,635	39,635
Area-Year FE	✓	✓	✓	✓	✓	✓
Ind-Year FE	✓	✓	✓	✓	✓	✓
Firm Size FE	✓	✓	✓	✓	✓	✓
Addl. Controls	✓		✓	✓		

Temporary versus Regular Hires Second, the dynamic cost of increasing nominal wages should be less pronounced for workers on temporary contracts with limited future employment commitment. To test for this relationship, we utilize direct measures of temporary worker employment at the establishment level in the EP data. In addition, we also observe the hiring of temporary workers in the vacancy survey and can relate the extent of hiring problems or the vacancy rate at the state level to the wages that these hires receive.

Columns 4 to 6 in Table 9 show that firms with a higher share of temporary workers are somewhat less likely to face labor constraints in the cross-section, but this coefficient is not statistically significant (column 4). We note, however, that this result could also be driven by endogenous firm choices because hiring temporary workers is a strategy to mitigate or prevent constraints. The local vacancy rate is not related to wages paid to temporary hires (column 5), but greater local hiring problems, measured as the share of firms reporting search difficulties, are associated with significantly higher reported wages for temporary workers in the vacancy survey (column 6). This pattern is consistent with firms turning to temporary hires and being more willing to raise wages for these workers in a tight labor market.

Taken together, we find suggestive evidence that dynamic considerations play a role in firms' wage strategies for new hires, and higher wage flexibility is related to lower labor constraints.

7 Conclusion

Labor constraints matter for both firms and policymakers. They are widespread and economically important, especially for growing firms. Higher wages reduce constraints, which is consistent with firms having some labor market power. But monopsony alone cannot explain the evidence: firms often raise wages only after they encounter hiring problems, and they do so with delay. Our results point to slow wage discovery, driven by imperfect information and gradual learning about market wages, together with forward-looking wage rigidity. These frictions can make tight labor markets more persistent and slow the reallocation of workers and jobs after shocks.

This perspective also highlights actionable ways to improve adjustment. Firms can reduce belief errors through more systematic labor-market monitoring and wage benchmarking. Consistent with the rising demand for such signals, recent entrepreneurial efforts have focused on providing high-accuracy, real-time labor market data, which may help firms form more accurate beliefs about market wages. The recent rise in salary benchmarking, using both proprietary data and open online data (Cullen, Li and Perez-Truglia, 2025; Payscale, 2023), is a key example. In addition, a number of new companies, such as Lightcast (formerly Burning Glass) or Revelio Labs, combine information from job postings, government publications, social media, and related sources to capture labor market trends by occupation, education, and industry. An increasing number of firms purchase these data for external benchmarking and to inform staffing and pay strategies.

Public policy can complement these private efforts by providing more timely and comprehensive public wage statistics. This would be especially valuable for smaller firms that do not buy private data. Government regulation can also play an additional role by mandating pay transparency across firms. Indeed, a recent wave of new legislation has introduced requirements for firms to disclose pay ranges in job postings in a number of European countries and US states (Cullen, 2024). In combination with the emergence of labor market data providers, these laws can help disseminate levels and changes in market wages by skill profile and geography. Information policies alone, however, may not address firms' incentives to keep wages low. They can therefore be complemented by policies that strengthen workers' outside options, thereby limiting firms' monopsony power. Such policies include enforcing competition law against wage-fixing and no-poach agreements and incorporating the effects of mergers on workers' employment options into merger review (Prager, 2026).

We note that our analysis studies labor constraints taking the supply of workers as given and focuses on why wages are slow to adjust. A complementary perspective on labor constraints would consider factors affecting labor supply, such as demographics, the education system, immigration policies, or occupational licensing. While these considerations are outside the scope of this paper, we believe they represent an important and promising direction for future research.

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Online Appendix

A Data and Empirical Evidence

A.1 Data Sources

Our main data source is the Institute for Employment Research (IAB) of the Federal Employment Agency in Germany. IAB offers several data products and the two most important datasets in our project are: 1) Establishment Panel (EP), and 2) Vacancy Survey (SE).

Vacancy Survey (SE, *IAB-Stellenerhebung*) The SE is a repeated cross-section covering 2000–2018, with over 230,000 establishment-year observations and 7,500–15,000 unique establishments per year. It reports the number and structure of vacancies, characteristics of the last hire, search duration, and indicators for difficulties and decisions associated with the recruiting process (Gurtzgen et al., 2023).

The SE is composed of two modules: general questions about recruiting and vacancies, and a detailed inquiry into the process for the last successful or failed hiring attempt, including search duration, encountered difficulties, etc. While the core content of the survey remains consistent over time, some variables and classifications change, which is why only selected waves of the survey are used in certain analyses.

Surveyed establishments are sampled from all establishments in Germany with at least one employee subject to social security contributions. Random sampling is stratified to ensure the survey is representative of East and West Germany, seven establishment size categories, and approximately 20 industries. The average response rate is 18.5%. Selected establishments are contacted by mail in October, receive the questionnaire, and are invited to participate in the survey. A second mailing with a reminder is sent out a few weeks later. Most establishments submit responses between late October and early December, but responses are typically collected until early January. The SE is merged with establishment-year administrative records (Establishment History Panel (BHP)), which contain information on employment and wages for the establishment by worker age, sex, education, and occupational group.

Establishment Panel (EP, *IAB-Betriebspanel*) The EP is a longitudinal survey of establishments covering 1993–2019. While fewer than 10,000 establishments were included in the initial waves, the sample contains around 15,000 establishments annually after 2000. The data combine the establishment survey and administrative employment and wage records. The data are collected via interviews with a person who is knowledgeable about the establishment's operations. The interviews are usually conducted face-to-face, and the average interview lasts 36 minutes. The average response rate is close to 60%, but it varies by the type of establishment: among establishments that were surveyed in the past, the response rate is usually higher than 80%, while new establishments that are added to the sample over time usually have a ~25% response rate. The interviews are checked for quality and consistency, and a large fraction of establishments is contacted by phone with a follow-up request to clarify or correct some information. The survey is composed of a core module with the key characteristics of the establishment and additional rotating modules about various areas of business operations. We rely heavily on the "Staffing Challenges" module, which is included in the survey approximately every two years. We focus on establishments with at least 10 employees, and our main sample includes 77,330 establishment-year observations across 25,190 unique establishments.

A.2 Question Text for Labor Constraints Variables

Our key variables are measures of labor constraints derived from firms' survey responses. Below we present the exact questionnaire text for these measures:

Vacancy Survey

Every year, the vacancy survey contains a question that asks about the relevance of "not having suitable workers" as an impediment to firm operations. The exact formulation of this question changes slightly over the years, but the changes were minor, and we use the responses without further adjustment. Below, we present the text of the question for the 2014 wave of the survey, as well as for the 2018 wave.

9. When you look back on the **last 12 months**: Were there any **external** reasons which prevented

– your establishment from making full use of its economic opportunities in the past 12 months?

– your administrative post from fully completing all its tasks in the past 12 months?

Yes ☐ No ☐ ➡ *Please continue with Question 10*

If yes, what were these external reasons?

- Not enough orders, not enough revenue ☐
- Not enough suitable workers ☐
- Financing problems, budget difficulties ☐
- Other ☐

2014 Vacancy Survey

17. Did you suffer one of the following economic **difficulties** in your establishment during the **past 12 months in your establishment/administrative post**?

Multiple answers possible

Not enough orders, not enough turnover	Yes ☐	No ☐
Too many outstanding bills	Yes ☐	No ☐
Not enough suitable workers	Yes ☐	No ☐
Financing difficulties	Yes ☐	No ☐

Other difficulty

2018 Vacancy Survey

Establishment Panel The relevant question in the Establishment Panel is presented below. The available answers to that question remained consistent until 2019, which is when our sample ends. We use respondents' selection of option D as a proxy for labor constraints, which most directly captures general staff shortages and hiring difficulties. However, many empirical patterns would remain similar if the selection of option C were used as a measure of labor constraints.

5. What kind of problems with human resources management do you expect for your establishment/office during the next two years? Please tick where applicable in the list!

Interviewer: Present list 2 and tick where applicable!

A Overstaffing .... ☐

B High turnover .... ☐

C Difficulties in finding the required specialized personnel on the labor market .... ☐

D Staff shortage .... ☐

E High percentage of older employed people .... ☐

F High demand for further training and qualification .... ☐

G Lack of motivation in the workplace .... ☐

H High rate of absenteeism/high rate of sickness absence .... ☐

I High financial burden on wage costs .... ☐

J Other problems with human resources management .... ☐

No problems with human resources management .... ☐

A.3 Additional Evidence

Table A.1: Central Bargaining-Induced Wage Level and Firms' Wages

All specifications are estimated in the IAB Establishment Panel with the unit of observation being establishment-year. The dependent variable is the logarithm of firms' average wage and the independent variable is the logarithm of bargaining-induced wage levels for a given establishment based on its occupational structure and economy-wide measures of recruiting difficulties by occupation. In columns 2 and 3, the sample is limited to firms following the agreement and paying wages at most 10% above the agreement-required minimum (column 2) and firms not formally following a collective wage agreement (column 3). Establishment FE and fixed effects for commuting zone-by-year, industry-by-year, and for firm size bins as well as controls for profitability, existence of a works council, and firm age are included in all columns. The bargaining-induced levels of wages are winsorized at the 2.5th and 97.5th percentiles. Standard errors are clustered by establishment. *** represents 1% statistical significance, ** - 5%, * - 10%.

	(1)	(2)	(3)
	Log(Firm Wage)		
Bargaining-Induced Wage (Occupation)	0.087***	0.091***	0.0700***
	(0.011)	(0.0152)	(0.0141)
N	149,014	69,358	49,462
Sample	Full	Barg, <10%	No Barg
CZ-Year, Size FE	✓	✓	✓
Ind-Year, Plant FE	✓	✓	✓
Other Controls	✓	✓	✓

Table A.1 shows that firm wages rise when bargaining-induced wages increase (column 1), with the largest pass-through among covered, initially lower-pay firms (column 2).[16] Uncovered firms also adjust, consistent with spillovers to non-union wage setting, as documented, e.g., in recent work by Green, Sand and Snoddy (2022) and Bassier (2022).

[16] Note that the sample size is larger than in Table 5 for the same subsamples of firms, because the firm wage analysis is not limited to years in which firms report labor constraints.

A.4 Markets with Different Signal Precision

Proposition 4 (stated formally with the additional model analysis in Appendix B.8) demonstrates that labor constraints are more common when the quality of wage information in the market is lower. Empirically, wage information is more difficult and costly to gather when a firm competes in a labor market with higher wage dispersion or where wages evolve differently than in the rest of the economy. In this subsection, we investigate the relationship between these two factors and declarations of labor constraints.

Dispersion of Information. We measure information dispersion with the variance of wages by occupation-year. To construct firm-specific measures of wage dispersion, we compute a weighted average for each firm based on its occupational mix. We then regress declarations of labor constraints on these shift-share dispersion measures at the firm level, controlling for a rich set of fixed effects and time-varying firm characteristics. Intuitively, firms that employ occupations with greater wage dispersion in the labor market as a whole face more difficulties in determining the correct equilibrium wage.

Table A.2, column 1 shows that firms facing a higher level of wage dispersion in the labor market for their occupational mix are more likely to report labor constraints. However, this relationship gradually loses its statistical significance when including establishment fixed effects (column 2) and industry-by-year fixed effects (column 3). This weaker evidence suggests that dispersion in wages by occupation may change little over time and is largely concentrated within industry.

Similar patterns can be observed in the vacancy survey data (column 4), where we can measure difficulty hiring a given worker as a function of the wage dispersion in this worker's occupation. Workers in occupations where wages offered to new hires have a higher standard deviation across firms and workers are more likely to be difficult to hire.

Market Synchronicity. National public news and wage statistics are cheaper sources of information but their informativeness may vary across firms whose local markets are more or less in sync with national trends. Hence, labor constraints may be more pronounced in labor markets that are less synchronized with the whole economy. To analyze this hypothesis, we measure synchronicity by the temporal correlation between the average wage level in a given county/district and the average wage in the entire German economy. This approach builds on ideas about stock price informativeness from the market microstructure literature (Roll, 1988; Durnev et al., 2003).[17]

Columns 5 and 6 of Table A.2 present the link between labor constraints declarations and labor market synchronicity in the data. Consistent with the role of information, firms are less

[17] In this literature, the relationship between synchronicity and information is reversed, since the focus is on firm-specific information: stock prices that have the highest firm-specific variance and lowest correlation with the aggregate market are more informative about firms' fundamentals. If one does not observe the stock price of equities with high firm-specific variance and low correlation with the market, it is difficult to have accurate beliefs about the prevailing stock price.

Table A.2: Dispersion of Information

Columns 1-3 and 5-6 are estimated in the IAB Establishment Panel data, while column 4 comes from SE data. The dependent variable is an indicator for labor constraints (columns 1-3 and 5-6) and search difficulties (column 4). The independent variables are the establishment-level average variance of wages within a given occupation-year in the entire economy, calculated as a shift-share measure with shares based on a firm's occupational structure and shifts equal to occupation-year-level variance of wages in the economy; the standard deviation of wages offered to workers hired in a given occupation in the same year by all firms; and market synchronicity, measured as the temporal correlation between average wage changes in a given district and in the German economy as a whole; high synchronicity is an indicator for above-median synchronicity. All IAB-EP columns include fixed effects for area-year, firm size, profitability, firm age, and the existence of a works council. Columns 2-3 include establishment FE; columns 5-6 further include industry-year fixed effects. Column 4 includes industry-year and area-year FE and a control for log(employment). Standard errors are clustered by establishment in all columns except column 4, where they are clustered by occupation. *** represents 1% statistical significance, ** - 5%, * - 10%.

	(1)	(2)	(3)	(4)	(5)	(6)
	Labor Constraints					
Variance of Wages (Shift-Share)	0.0644***	0.0279*	0.0109			
	(0.00979)	(0.0141)	(0.0150)			
Variance of Wages (by Occupation)				0.0224***		
				(0.0032)		
Synchronicity of Local Labor Market					-0.0246***	
					(0.0078)	
High Synchronicity						-0.0114***
						(0.00325)
N	92,114	72,397	72,300	216,559	93,380	93,380
Controls	✓	✓	✓	✓	✓	✓
Establishment FE		✓	✓			
Industry-Year FE			✓	✓	✓	✓

likely to report labor constraints in areas with high synchronicity. This is true when using either a continuous measure of synchronicity or a binary indicator for markets with an above-median degree of synchronicity. We note that this effect may be driven not just by firms' beliefs being more accurate, but also by workers' beliefs: if all the agents in a given market are better informed, successful job matches are more likely to occur, and labor constraints are less likely to arise. In any case, the results support the importance of the accuracy of beliefs in avoiding labor constraints.

B Model Appendix

This appendix presents the complete formal model underlying Section 5. The main text states the equations needed to see the economic mechanism. Here we collect the full environment, timing, equilibrium conditions, and benchmark cases. The proofs of the theoretical results are in the subsequent proof appendix.

The appendix follows the logic of the main text but spells out the additional objects needed to characterize equilibrium. We begin with the environment and timing, then introduce the hiring and learning equations, and finally define the steady-state system. The last part of the appendix characterizes benchmark cases.

B.1 Environment and Timing

Time is discrete and the horizon is infinite. The economy is in one of two states, $\omega_t \in \{h, l\}$, which evolves according to a Markov chain with transition probabilities p_{hh} and p_{ll}. The high state corresponds to a tight labor market. We assume $p_{hh} + p_{ll} > 1$, so labor market conditions are persistent.

In the baseline model, we set $p_{hh} = p_{ll}$. This implies the stationary distribution

$$\pi(h) = \frac{1 - p_{ll}}{2 - p_{hh} - p_{ll}} = \frac{1}{2}, \qquad \pi(l) = 1 - \pi(h) = \frac{1}{2}.$$

All players are risk neutral and share a common discount factor $\alpha \in [0, 1)$.

Each period, a measure $\lambda^f(\omega_t)$ of firms enters the labor market, where $\lambda^f(h) > \lambda^f(l)$. Entering firms join firms that failed to hire in earlier periods but remained in the market, and we refer to this combined set as active firms. Each active firm has one vacancy.

Firms do not observe the aggregate state directly. Instead, each entering firm receives an independent private signal $s \in [0, 1]$ drawn from $H(s \mid \omega)$, with continuous density $h(s \mid \omega)$. The signal distribution satisfies the monotone likelihood ratio property, so high signals are more common in the high state. Because entry is state-dependent, the firm's posterior belief after observing signal s is

$$\beta_0(s) = \frac{\pi(h)\lambda^f(h)h(s|h)}{\pi(h)\lambda^f(h)h(s|h) + \pi(l)\lambda^f(l)h(s|l)} = \frac{\lambda^f(h)h(s|h)}{\lambda^f(h)h(s|h) + \lambda^f(l)h(s|l)}.$$

This expression reflects the symmetric priors, the private signal, and the fact that the firm's own entry is more likely in the high state.

With a slight abuse of notation, let $H(\beta \mid \omega)$ denote the state-dependent distribution of initial beliefs induced by the signal distribution and the mapping $s \mapsto \beta_0(s)$.

A fixed measure λ^a of applicants enters each period. Each applicant has a privately known outside option v, drawn from a state-independent distribution G with support $[c, y]$, density $g > 0$, and $g' < 0$. We assume that G has a strictly increasing hazard rate $g(\cdot)/(1 - G(\cdot))$.

The timing distinguishes between two sources of beliefs. Entrants form initial beliefs from their private signals, while firms that remain active after an unsuccessful search carry forward beliefs updated from their hiring experience. The timeline below summarizes the order of events within a period; the belief-updating equations are introduced in Section B.3.

1. **Entry and signals.** The state ω_t realizes. A new cohort of firms with measure $\lambda^f(\omega_t)$ enters, and each entering firm receives a private signal. A new cohort of applicants with measure λ^a enters, and each applicant draws an outside option v.

2. **Belief formation/updating**. Entrants form their beliefs $\beta_0(s)$ from their signals, while remaining incumbent firms update beliefs to $\beta^+(w, \beta)$.

3. **Meetings.** Each firm randomly contacts one applicant. Each applicant may be contacted by multiple firms.

4. **Wage posting and acceptance.** Firms simultaneously post wages. Each applicant accepts the highest wage offer that exceeds the outside option. If no offer exceeds the outside option, the applicant takes the outside option.

5. **Exit and continuation.** Matched firms and all applicants exit the market. Unmatched firms exit with probability δ and otherwise continue searching next period.

Applicants are not dynamic decision makers in the model. They choose the best current-period option and then leave the market. This keeps the focus on firm-side learning. Allowing applicants to stay in the market and optimize reservation wages would turn our environment into a two-sided dynamic search model. With private outside options v and unobserved states, applicants must form beliefs about future market tightness $\xi(\omega)$ and wage distributions $W(w|\omega)$, creating a dynamic value function $V^a(\beta)$ that shifts with selection (e.g., high-v types wait longer). Firms must then track the time-varying mix of applicant types, expanding the state space and yielding a coupled fixed-point problem for both sides' strategies and beliefs—which could feature search externalities, bargaining hold-ups, and potentially multiple or non-existent equilibria. For tractability, we focus on firm-side learning in a one-sided search model with heterogeneous reservation values, which suffices for rich wage dynamics and our testable predictions.

B.2 Matching, Wages, and Trading Probabilities

The matching technology is firm initiated. Each active firm contacts one applicant. If the measure of active firms in state ω is $\Lambda^f(\omega)$ and the measure of active applicants is $\Lambda^a(\omega)$, then market tightness is $\xi(\omega) = \frac{\Lambda^f(\omega)}{\Lambda^a(\omega)}$. The number of firms contacting a given applicant is Poisson with parameter $\xi(\omega)$. Denote the corresponding contact-count probabilities by $m_k(\xi(\omega)) = \frac{\xi(\omega)^k}{k! e^{\xi(\omega)}}$.

Let $W(w \mid \omega)$ be the steady-state distribution of wage offers in state ω. As in the main text, the probability that a firm posting wage w hires in state ω is

$$p^f(w|\omega) = G(w)\, e^{-\xi(\omega)(1-W(w|\omega))}.$$

The term $G(w)$ is the probability that the applicant's outside option is below the offer. The exponential term is the probability that no other firm contacting the applicant posts a wage above w. Higher wages therefore raise the probability of hiring while lowering surplus conditional on a hire.

The applicant-side trading probability will be useful below when we characterize firm stocks and market clearing. The probability that an applicant receives at least one acceptable wage offer that exceeds the outside option v, given that the market is in state ω, is

$$p^a(v|\omega) = 1 - e^{-\xi(\omega)[1-W(v|\omega)]}. \tag{B.1}$$

B.3 Beliefs and the Firm Problem

A firm with belief β evaluates its hiring probability as $p^f(w|\beta) = \beta p^f(w|h) + (1-\beta)p^f(w|l)$. If the firm fails to hire after posting wage w, it updates its belief about the current state according to Bayes' rule:

$$\tilde{\beta}(w,\beta) = \frac{\beta[1-p^f(w|h)]}{\beta[1-p^f(w|h)] + (1-\beta)[1-p^f(w|l)]}.$$

The belief entering the next period is

$$\beta^+(w,\beta) = p_{hh}\tilde{\beta}(w,\beta) + (1-p_{ll})(1-\tilde{\beta}(w,\beta)).$$

A failed hiring attempt is therefore informative because failure is more likely in the high state, where competition for workers is stronger.

Firms choose wages as a function of their beliefs. The equilibrium object is a wage strategy $w^*(\beta)$ that solves the firm's dynamic problem. If a firm hires at wage w, it receives surplus $y - w$. The firm chooses w to maximize

$$V^f(\beta) = \max_{w \in [c,y]} \Pi(w,\beta),$$

where

$$\Pi(w,\beta) = p^f(w|\beta)(y-w) + \alpha(1-\delta)[1-p^f(w|\beta)]V^f(\beta^+(w,\beta)).$$

The first term is the expected payoff from hiring today. The second term is the continuation value if the firm remains unmatched and does not exit. A higher wage improves the chance of hiring today, but commits the firm to lower surplus from a successful match.

B.4 Steady-State System

The steady-state stock of applicants is equal to the measure of newly entering applicants, since all applicants exit each period:

$$\Lambda^a(\omega) = \lambda^a. \tag{B.2}$$

To describe survivor flows, let $\omega^- \in \{h, l\}$ denote the previous period's aggregate state. In steady state, $\pi(\omega^- \mid \omega)$ denotes the probability that the previous state was ω^- conditional on the current state being ω.

The firm stock satisfies the flow-balance equation

$$\Lambda^f(\omega) = \lambda^f(\omega) + (1-\delta) \sum_{\omega^- \in \{h,l\}} \pi(\omega^-|\omega)\, S(\omega^-), \tag{B.3}$$

where

$$S(\omega^-) = \Lambda^f(\omega^-) - \lambda^a(1 - \Phi(\omega^-))$$

is the mass of unmatched firms in state ω^-, and

$$\Phi(\omega) := \int_c^y e^{-\xi(\omega)(1-W(v|\omega))}\, dG(v) \tag{B.4}$$

is the probability that a random applicant in state ω receives no acceptable wage offer.

The steady-state belief distribution $\Gamma^f(\cdot \mid \omega)$ satisfies

$$\begin{aligned} \Lambda^f(\omega)\,\Gamma^f(\beta|\omega) &= \lambda^f(\omega)\, H(\beta|\omega) \\ &\quad + (1-\delta) \sum_{\omega^- \in \{h,l\}} \pi(\omega^-|\omega) \int_{\{\beta' : \beta^+(w^*(\beta'),\beta') \leq \beta\}} \Lambda^f(\omega^-)\Big(1 - p^f(w^*(\beta')|\omega^-)\Big)\, d\Gamma^f(\beta'|\omega^-). \end{aligned} \tag{B.5}$$

The first term is the inflow of entering firms with initial beliefs below β. The second term is the inflow of surviving firms whose updated beliefs are below β.

The wage distribution is induced by the belief distribution and the wage strategy. If $w^*(\cdot)$ is increasing, then

$$W(w|\omega) = \Gamma^f(w^{*-1}(w)|\omega),$$

where w^{*-1} is the generalized inverse of the wage strategy.

Market clearing requires the total measure of applicants accepting jobs to equal the total measure of firms that hire:

$$\lambda^a \int p^a(v|\omega)dG(v) = \Lambda^f(\omega) \int p^f(w|\omega)dW(w|\omega). \tag{B.6}$$

B.5 Equilibrium Definition

Definition 1. *A Markov-perfect steady-state equilibrium consists of:*

1. *State-dependent stocks of firms and applicants $\Lambda^f(\omega)$, $\Lambda^a(\omega)$ satisfying (B.2) and (B.3).*
2. *A steady-state belief distribution for firms $\Gamma^f(\cdot|\omega)$ satisfying equation (B.5).*
3. *A Bayesian-consistent belief update (5.3)–(5.4).*
4. *An optimal wage strategy $w^* : [0,1] \to [c,y]$ solving (5.5) for every β, with $W(w|\omega) = \Gamma^f(w^{*-1}(w)|\omega)$.*
5. *Trading probabilities (5.1) and (B.1) consistent with W.*
6. *A market clearing condition (B.6).*
7. *The state ω_t follows the stationary distribution $(\pi_h, \pi_l) = (1/2, 1/2)$.*

Proposition 1 establishes existence under Assumptions 1 and 2 in the main text. These assumptions guarantee a well-behaved static wage choice and the monotonicity needed for the existence argument; the proof in Appendix C.2 restates them in the technical form used below.

B.6 Connection to Monopsony Models

In monopsony models, a firm faces an upward-sloping effective labor supply curve: raising the wage increases the amount of labor the firm can hire, but lowers profits conditional on hiring. In our environment, this force arises endogenously from workers' heterogeneous outside options and from competition among firms through the matching process.

To see how the model preserves the basic monopsony logic, fix a state ω and an input wage distribution $W(\cdot \mid \omega)$. Differentiating a firm's hiring probability with respect to w gives

$$\frac{\partial p^f(w \mid \omega)}{\partial w} = \exp\{-\xi(\omega)[1 - W(w \mid \omega)]\} \left[g(w) + \xi(\omega)G(w)f_W(w \mid \omega)\right],$$

where $f_W(\cdot \mid \omega)$ is the density of the wage distribution. At interior points of the support, this derivative is positive. Higher wages therefore increase the probability of hiring but lower profits conditional on a hire. The hiring probability $p^f(w \mid \omega)$ plays the role of an upward-sloping effective labor supply schedule.

B.7 No-Uncertainty Benchmark

It is useful to see how the model relates to a standard wage-posting benchmark. Shut down aggregate uncertainty by assuming that the state $\omega \in \{h, l\}$ is perfectly observed and constant ($p_{hh} = p_{ll} = 1$). Firms no longer differ in beliefs, and there is no learning from hiring outcomes. Applicants still draw heterogeneous outside options and firms still compete through the same urn-ball matching process.

In this benchmark, firms randomize wages over a continuous interval $[\underline{w}(\omega), \overline{w}(\omega)] \subseteq [c, y]$, as in Burdett and Judd (1983) and Burdett and Mortensen (1998). A point-mass wage distribution cannot be an equilibrium in an active market because a firm could raise its wage by an arbitrarily small amount and win all ties with workers whose outside option lies below the common wage. Thus the equilibrium wage distribution is pinned down by an indifference condition: all wages in the support yield the same value.

For a firm offering wage w in state ω, the no-uncertainty value satisfies

$$V^f(\omega) = p^f(w|\omega)(y - w) + \alpha(1-\delta)[1 - p^f(w|\omega)]V^f(\omega).$$

Rearranging over the support gives

$$p^f(w|\omega)\left[y - w - \alpha(1-\delta)V^f(\omega)\right] = [1 - \alpha(1-\delta)]\, V^f(\omega).$$

Substituting the matching probability yields the wage-distribution condition

$$G(w)e^{-\xi(\omega)[1-W(w|\omega)]} = \frac{[1-\alpha(1-\delta)]\, V^f(\omega)}{y - w - \alpha(1-\delta)V^f(\omega)}.$$

Solving for the wage CDF yields

$$W(w|\omega) = 1 - \frac{1}{\xi(\omega)} \log\left(\frac{G(w)\left[y - w - \alpha(1-\delta)V^f(\omega)\right]}{[1-\alpha(1-\delta)]\, V^f(\omega)}\right).$$

The stock of firms closes the benchmark:

$$\Lambda^f(\omega) = \frac{\lambda^f(\omega)}{\delta + (1-\delta)\overline{p}^f(\omega)}, \qquad \overline{p}^f(\omega) = \int p^f(w|\omega)\, dW(w|\omega).$$

This no-uncertainty case clarifies what the learning model adds. In the benchmark, unfilled vacancies are equilibrium outcomes of wage posting, but they do not change what firms know. In the full model, failed hiring attempts update beliefs about labor market tightness and therefore affect subsequent wage offers.

B.8 Additional Model Analysis: Information Quality Across Markets

We extend the setting in Proposition 2 to two separate markets. Consider two markets $g \in \{IM, UM\}$ that differ only in their shares of informed firms entering the market, $0 \leq \mu^{UM} < \mu^{IM} \leq 1$. The informed firms still receive more precise signals than the uninformed firms in the same market: $H_I(\cdot|h) \succ_{FOSD} H_U(\cdot|h)$ and $H_U(\cdot|l) \succ_{FOSD} H_I(\cdot|l)$. We keep assuming $p_{hh} = 1$, and focus on the high state ($\omega = h$) for the analysis.

Proposition 4. Under Assumptions (1)-(3) and $p_{hh} = 1$, there exists $\bar{\lambda}^{f'} > 0$ such that for all $\lambda^f(h) > \bar{\lambda}^{f'}$: the more informed market IM has higher aggregate wages $E_{IM}[w \mid h] > E_{UM}[w \mid h]$, and higher aggregate hiring probability $\bar{p}^f_{IM}(h) > \bar{p}^f_{UM}(h)$.

Proposition 4 analyzes the role of information at the aggregate level. The proof of the result reveals that both the direct effect from the behavior of informed firms and a positive informational externality contribute to the aggregate outcome. When more firms in the market are well informed, the average firm posts higher wages. This pulls up the aggregate wage distribution, which means workers are matched faster, more matches form each period, and the stock of vacant firms left waiting for workers shrinks. The thinner pool of competing vacancies, in turn, reduces market tightness and makes hiring easier even for the firms that are still poorly informed. The uninformed therefore benefit from the presence of informed firms because the market is less congested, which improves their own hiring probability at any given wage.

C Proofs of Model Propositions

C.1 Main Assumptions

For reference, we restate the two maintained assumptions in the form used in the proofs below. Assumption 1 ensures that, for fixed market tightness and wage distribution, the firm's static wage problem is strictly concave. Assumption 2 restricts attention to continuous, increasing wage strategies and imposes the monotonicity needed for the best-response and comparative-statics arguments.

Assumption 1. For every $\xi \in [0, \bar{\lambda}/(\delta\lambda^a)]$ and every absolutely continuous CDF W on $[c, y]$ with density $W' \geq 0$, the static profit function $w \mapsto G(w)e^{-\xi(1-W(w))}(y-w)$ is strictly concave on (c, y).

Assumption 2. We restrict attention to wage strategies $w : [0,1] \to [c, y]$ that are continuous and increasing. In addition, the hiring probability satisfies $\frac{\partial}{\partial w} \log p^f(w|h) > \frac{\partial}{\partial w} \log p^f(w|l)$. Using the hiring probability formula, this can be written as $\frac{\partial}{\partial w} \log p^f(w|\omega) = r_G(w) + \xi(\omega)W'(w|\omega)$ where $r_G(w) := \frac{g(w)}{G(w)}$, so the condition requires a marginal wage increase to have a larger proportional effect on hiring in the high state.

C.2 Proof of Proposition 1

The proof builds the equilibrium through a sequence of lemmas. Lemma 1 defines the strategy space $\mathcal{W}$ as a Lipschitz class of wage strategies parameterized by two constants ℓ, L that are derived from primitives in Lemmas 2 and 12; these in turn rest on the static-payoff bounds established in Lemmas 10 and 11. For each $w_{\text{in}} \in \mathcal{W}$, Lemmas 4–5 construct the unique aggregates, Lemma 3 establishes that the belief operator preserves atom-freeness so the equilibrium belief and wage distributions are atomless, and Lemma 6 shows these aggregates vary continuously with w_{in}. With the aggregates in hand, the Bellman equation has a unique $C^{1,1}$ solution by Lemmas 7–9, the full payoff Π is strictly concave in w and has increasing differences in (w,β) by Lemmas 10–11, and the best-response operator T maps $\mathcal{W}$ into itself and is sup-norm continuous by Lemmas 12–14. Schauder's fixed-point theorem then yields a fixed point of T, whose regularity is verified in Lemma 15.

We work in the Banach space $(C([0,1]), \|\cdot\|_\infty)$. Two constants determine the strategy space: a uniform Lipschitz *upper* bound $L > 0$ derived in Lemma 12 and a uniform Lipschitz *lower* bound $\ell \in (0, L]$ derived in Lemma 2. Both depend only on the model primitives, and we impose throughout the parameter restriction $\ell \le y - c$, which is automatic when the constants are chosen as in Lemma 2.

Define

$$\mathcal{W} := \Big\{ w \in C([0,1]) : w(\beta) \in [c,y]\ \forall \beta,\ \ \ell(\beta' - \beta) \le w(\beta') - w(\beta) \le L(\beta' - \beta)\ \forall\, 0 \le \beta < \beta' \le 1 \Big\}. \quad \text{(C.1)}$$

The lower bound $\ell > 0$ ensures every $w_{\text{in}} \in \mathcal{W}$ is strictly increasing on $[0,1]$. Hence w_{in} is a continuous strictly monotone bijection from $[0,1]$ onto its image $[w_{\text{in}}(0), w_{\text{in}}(1)] \subset [c,y]$. The induced wage CDF $W(\cdot|\omega) := \Gamma^f(w_{\text{in}}^{-1}(\cdot)|\omega)$ is the pushforward of the belief CDF $\Gamma^f(\cdot|\omega)$ through the continuous bijection w_{in}^{-1}. We will prove (Lemma 3) that $\Gamma^f(\cdot|\omega)$ is atomless at every iterate $w_{\text{in}} \in \mathcal{W}$, so W is also atomless throughout the iteration. Consequently, the formula $p^f(w|\omega) = G(w)e^{-\xi(\omega)(1-W(w|\omega))}$ is valid at every iterate, the static profit $\pi_s(w) = p^f(w|\beta)(y-w)$ is continuous in w, and Assumption 1 applies.

Lemma 1. *$\mathcal{W}$ is a non-empty, compact, convex subset of $(C([0,1]), \|\cdot\|_\infty)$.*

Proof. Non-emptiness. Define $w_0(\beta) := c + \ell\beta \min\{1, (y-c)/\ell\}$. By the standing assumption $\ell \le y - c$, this simplifies to $w_0(\beta) = c + \ell\beta$ for $\beta \in [0,1]$, and $w_0(1) = c + \ell \le y$. Then $w_0 \in C([0,1])$, $w_0(\beta) \in [c, c+\ell] \subset [c,y]$, and $w_0(\beta') - w_0(\beta) = \ell(\beta' - \beta)$, satisfying the lower Lipschitz bound with equality and the upper Lipschitz bound because $\ell \le L$. Hence $w_0 \in \mathcal{W}$.

Convexity. Let $w_1, w_2 \in \mathcal{W}$ and $\theta \in [0,1]$. Set $w_\theta := \theta w_1 + (1-\theta)w_2$. Then w_θ is continuous as a linear combination of continuous functions. For every β, $w_\theta(\beta) = \theta w_1(\beta) + (1-\theta)w_2(\beta) \in [c,y]$ since $[c,y]$ is convex. For $0 \le \beta < \beta' \le 1$:

$$w_\theta(\beta') - w_\theta(\beta) = \theta[w_1(\beta') - w_1(\beta)] + (1-\theta)[w_2(\beta') - w_2(\beta)].$$

Each bracketed term lies in $[\ell(\beta'-\beta), L(\beta'-\beta)]$ since $w_1, w_2 \in \mathcal{W}$. A convex combination of numbers in this interval again lies in the interval. Hence $w_\theta \in \mathcal{W}$.

Closedness. Let $\{w_n\}_{n=1}^{\infty} \subset \mathcal{W}$ with $\|w_n - w\|_\infty \to 0$ for some $w \in C([0,1])$. Since $w_n(\beta) \in [c, y]$ for all β and n, and $w_n(\beta) \to w(\beta)$ pointwise, we have $w(\beta) \in [c, y]$ for all β. For any $0 \le \beta < \beta' \le 1$, define $d_n := w_n(\beta') - w_n(\beta)$ and $d := w(\beta') - w(\beta)$. Then $|d_n - d| \le 2\|w_n - w\|_\infty \to 0$, so $d_n \to d$. Since $\ell(\beta'-\beta) \le d_n \le L(\beta'-\beta)$ for all n, the limit satisfies $\ell(\beta'-\beta) \le d \le L(\beta'-\beta)$. Therefore $w \in \mathcal{W}$, confirming closedness.

Compactness. $\mathcal{W}$ is uniformly bounded: $\|w\|_\infty \le y$ for all $w \in \mathcal{W}$. $\mathcal{W}$ is equicontinuous: for all $w \in \mathcal{W}$ and all $\beta, \beta' \in [0,1]$, $|w(\beta') - w(\beta)| \le L|\beta' - \beta|$. By the Arzelà–Ascoli theorem, $\mathcal{W}$ is relatively compact in $C([0,1])$. Since $\mathcal{W}$ is closed, it is compact. □

We now derive the two Lipschitz constants ℓ, L, and establish the inductive atom-freeness of belief distributions at every iterate.

Lemma 2. *There exist constants $0 < \ell \le L < \infty$ such that for every $w_{\text{in}} \in \mathcal{W}$ and every $\beta \in [0,1]$, the best response $w_{\text{out}}(\beta) := \operatorname{argmax}_{w \in [c,y]} \Pi(w, \beta; w_{\text{in}})$ is differentiable a.e. on $(0,1)$ with*

$$\ell \;\le\; \frac{dw_{\text{out}}}{d\beta}(\beta) \;\le\; L \qquad \text{at every interior optimum.}$$

Equivalently, $\ell(\beta'-\beta) \le w_{\text{out}}(\beta') - w_{\text{out}}(\beta) \le L(\beta'-\beta)$ for all $0 \le \beta < \beta' \le 1$.

Proof. The upper bound $L = 2M/\kappa$, where $M = \sup|\Pi_{w\beta}| < \infty$ is the uniform upper bound on the cross-partial, is established in Lemma 12.

For the lower bound ℓ, define $\kappa_{\max} := \sup_{w_{\text{in}} \in \mathcal{W}} \sup_{w \in [c+\epsilon_0, y], \beta \in [0,1]} |\Pi_{ww}(w, \beta; w_{\text{in}})|$, where $\epsilon_0 > 0$ is the gap from Lemma 10. This supremum is finite because $|(\pi_s)_{ww}|$ is bounded on $[c+\epsilon_0, y]$ by a constant depending on $\sup_{(c,y)} r_G$, $\sup r'_G$, Ξ, and $y - c$, and $|(\pi_c)_{ww}| \le A\gamma/(1-\gamma)$ uniformly (Lemma 10). Hence $\kappa_{\max} < \infty$.

At any interior optimum $w^*(\beta) \in [c+\epsilon_0, y]$, the first-order condition $\Pi_w(w^*(\beta), \beta) = 0$ holds. Differentiating in β using the implicit function theorem:

$$\frac{dw^*}{d\beta} = -\frac{\Pi_{w\beta}(w^*, \beta)}{\Pi_{ww}(w^*, \beta)} = \frac{\Pi_{w\beta}(w^*, \beta)}{|\Pi_{ww}(w^*, \beta)|} \ge \frac{\sigma}{\kappa_{\max}} =: \ell,$$

where we used $\Pi_{w\beta}(w^*, \beta) \ge \sigma > 0$ (Lemma 11) and $|\Pi_{ww}(w^*, \beta)| \le \kappa_{\max}$. Hence $\ell > 0$.

The integrated form $\ell(\beta'-\beta) \le w^*(\beta') - w^*(\beta) \le L(\beta'-\beta)$ follows by integrating the pointwise bound over $[\beta, \beta']$. □

Lemma 3. *Let $\mathcal{S}$ denote the belief operator defined in equation (C.2) in the proof of Lemma 4. The following propagation property holds: for every $w_{\text{in}} \in \mathcal{W}$ and every pair of stocks $\Lambda^f(h), \Lambda^f(l) > 0$, if $\Gamma \in \mathcal{D}$ is atomless in both states, so is $\mathcal{S}[\Gamma]$. Iterating from any atomless seed therefore produces a sequence of atomless CDFs, whose uniform limit—the unique fixed point $\Gamma^f(\cdot|\omega)$ in Lemma 4—is* *also atomless on* $[0,1]$ *for both* $\omega\in\{h,l\}$*. The induced wage CDF* $W(w|\omega)=\Gamma^f(w_{\text{in}}^{-1}(w)|\omega)$ *is atomless and* $p^f(w|\omega)=G(w)e^{-\xi(\omega)(1-W(w|\omega))}$ *is the hiring probability.*

Proof. Fix $w_{\text{in}}\in\mathcal{W}$. By the definition of $\mathcal{W}$, w_{in} is strictly increasing on $[0,1]$ with slope at least $\ell>0$, hence is a bi-Lipschitz bijection from $[0,1]$ onto $[w_{\text{in}}(0),w_{\text{in}}(1)]$.

Recall the operator equation (B.5) for Γ^f:

$$\Gamma^f(\beta|\omega) \;=\; \alpha_\omega\, H(\beta|\omega) \;+\; (1-\alpha_\omega)\, R[\Gamma^f](\beta|\omega),$$

where $\alpha_\omega=\lambda^f(\omega)/\Lambda^f(\omega)\in(0,1)$ and $R[\Gamma^f]$ is the normalized survivor CDF. The entry component $\alpha_\omega H(\cdot|\omega)$ is atomless since $H(\cdot|\omega)$ has continuous density $h(\cdot|\omega)>0$ by MLRP.

We show the survivor component $R[\Gamma^f]$ inherits atom-freeness from Γ^f, hence atom-freeness propagates through the contraction iteration. The survivor CDF is, up to normalization, the integral $\int_{\{\beta':\beta^+(\beta',w_{\text{in}}(\beta'))\le\beta\}}\Big(1-p^f(w_{\text{in}}(\beta')|\omega^-)\Big)\,d\Gamma^f(\beta'|\omega^-)$.

Define $\varphi_{w_{\text{in}}}(\beta'):=\beta^+(\beta',w_{\text{in}}(\beta'))$. We claim $\varphi_{w_{\text{in}}}$ is strictly increasing and continuous on $[0,1]$. Continuity is clear due to the composition of continuous functions. For monotonicity: at fixed w, $\tilde\beta(\beta',w)=\beta'\eta(w)/(\beta'\eta(w)+1-\beta')$ is strictly increasing in β'. The chain $\beta'\mapsto w_{\text{in}}(\beta')$ is strictly increasing by the definition of $\mathcal{W}$, so the map $\beta'\mapsto\tilde\beta(\beta',w_{\text{in}}(\beta'))$ inherits monotonicity provided $\tilde\beta$ is non-decreasing in w at non-atom wages. Then $\beta^+=(p_{hh}+p_{ll}-1)\tilde\beta+(1-p_{ll})$, an increasing affine function of $\tilde\beta$, gives strict monotonicity of $\varphi_{w_{\text{in}}}$.

Since $\varphi_{w_{\text{in}}}$ is a strictly monotone continuous bijection from $[0,1]$ onto its image, the integration set $\{\beta':\varphi_{w_{\text{in}}}(\beta')\le\beta\}=\{\beta':\beta'\le\varphi_{w_{\text{in}}}^{-1}(\beta)\}=[0,\varphi_{w_{\text{in}}}^{-1}(\beta)]$ is an interval whose right endpoint is a continuous function of β. The unnormalized survivor CDF $\beta\mapsto\int_0^{\varphi_{w_{\text{in}}}^{-1}(\beta)}\Big(1-p^f(w_{\text{in}}(\beta')|\omega^-)\Big)\,d\Gamma^f(\beta'|\omega^-)$ is then a continuous function of β whenever $\Gamma^f(\cdot|\omega^-)$ is continuous (atomless), since the integral up to a moving continuous endpoint of a Γ^f-integrable function is continuous in the endpoint.

Hence $R[\Gamma^f](\cdot|\omega)$ is atomless whenever $\Gamma^f(\cdot|\omega^-)$ is atomless for both $\omega^-\in\{h,l\}$. The contraction iteration $\Gamma^f\mapsto\mathcal{S}\Gamma^f$ thus preserves atom-freeness: if the iterate Γ_n is atomless, so is $\Gamma_{n+1}=\mathcal{S}\Gamma_n=\alpha_\omega H+(1-\alpha_\omega)R[\Gamma_n]$. Starting the iteration from any atomless seed Γ_0, every iterate is atomless, and the contraction limit Γ^* is the uniform limit of atomless CDFs, hence atomless.

$W(w|\omega)=\Gamma^f(w_{\text{in}}^{-1}(w)|\omega)$ with w_{in}^{-1} a continuous bijection, so $W(\cdot|\omega)$ is the composition of two continuous functions, hence continuous, hence atomless. With W atomless, the formula $p^f(w|\omega)=G(w)e^{-\xi(\omega)(1-W(w|\omega))}$ has no atomic correction and gives the hiring probability. □

We now establish existence and uniqueness of belief distributions for *any* given pair of firm stocks, without assuming these stocks arise from a fixed-point equation.

Lemma 4. *Fix* $w_{\text{in}}\in\mathcal{W}$ *and any pair of firm stocks* $\Lambda^f(h),\Lambda^f(l)>0$ *with* $\Lambda^f(\omega)\le\bar\lambda/\delta$ *where* $\bar\lambda:=\max\{\lambda^f(h),\lambda^f(l)\}$*. Define* $\xi(\omega):=\Lambda^f(\omega)/\lambda^a$*. Then there exist unique steady-state belief distributions* $\Gamma^f(\cdot|h),\Gamma^f(\cdot|l)$ *satisfying (B.5). These are CDFs on* $[0,1]$ *with* $\Gamma^f(0|\omega)=0$ *and*

$\Gamma^f(1|\omega) = 1$. *Moreover, the map* $(\Lambda^f(h), \Lambda^f(l)) \mapsto (\Gamma^f(\cdot|h), \Gamma^f(\cdot|l))$ *is continuous from* $(0, \bar{\lambda}/\delta]^2$ *to* $(C([0,1])^2, \|\cdot\|_\infty)$.

Proof. Fix $w_{\text{in}} \in \mathcal{W}$ and $\Lambda^f(h), \Lambda^f(l) > 0$. The tightness $\xi(\omega) = \Lambda^f(\omega)/\lambda^a$ and the wage strategy w_{in} are now all fixed parameters.

Let $\mathcal{D}$ denote the set of pairs of CDFs (Γ_h, Γ_l) on $[0,1]$, with the metric

$$d\Big((\Gamma_h, \Gamma_l), (\Gamma'_h, \Gamma'_l)\Big) := \max\Big\{\|\Gamma_h - \Gamma'_h\|_\infty,\ \|\Gamma_l - \Gamma'_l\|_\infty\Big\}.$$

The space $\mathcal{D}$ is convex and compact in the topology of pointwise convergence by Helly's selection theorem: every sequence of CDF pairs in $\mathcal{D}$ has a subsequence converging pointwise (at every continuity point of the limit) to a CDF pair. However, $\mathcal{D}$ is *not* compact in the sup-norm topology in general, because monotone functions on $[0,1]$ need not be equicontinuous. We therefore work in two stages: first, we establish existence using the Schauder–Tychonoff theorem in the pointwise topology; then, we show the fixed point is continuous.

Define the operator $\mathcal{S} : \mathcal{D} \to \mathcal{D}$ componentwise by

$$(\mathcal{S}\Gamma)(\beta|\omega) := \frac{\lambda^f(\omega)}{\Lambda^f(\omega)} H(\beta|\omega) + \frac{(1-\delta)}{\Lambda^f(\omega)} \sum_{\omega^-} \pi(\omega^-|\omega) \int_{\{\beta' : \beta^+(\beta', w_{\text{in}}(\beta')) \le \beta\}} \Lambda^f(\omega^-)\, q[\Gamma](\beta', \omega^-)\, d\Gamma(\beta'|\omega^-), \quad \text{(C.2)}$$

where

$$q[\Gamma](\beta', \omega^-) := 1 - p^f[\Gamma](w_{\text{in}}(\beta')|\omega^-) = 1 - G(w_{\text{in}}(\beta'))\, e^{-\xi(\omega^-)(1-\Gamma(\beta'|\omega^-))}$$

is the non-hiring probability, which depends on Γ through the wage distribution $W(w|\omega) = \Gamma(w_{\text{in}}^{-1}(w)|\omega)$. A fixed point $\Gamma^* = \mathcal{S}(\Gamma^*)$ satisfies (B.5).

Let $\lambda_{\min} := \min\{\lambda^f(h), \lambda^f(l)\}$ and $\lambda_{\max} := \bar{\lambda}/\delta$. Define, for any pair of stocks $\Lambda = (\Lambda_h, \Lambda_l) \in K := [\lambda_{\min}, \lambda_{\max}]^2$ and any $\Gamma \in \mathcal{D}$:

$$N(\beta|\omega; \Lambda, \Gamma) := \lambda^f(\omega)\, H(\beta|\omega) + (1-\delta) \sum_{\omega^-} \pi(\omega^-|\omega) \int_{\{\beta' : \beta^+(\beta') \le \beta\}} \Lambda(\omega^-)\, q[\Gamma](\beta', \omega^-)\, d\Gamma(\beta'|\omega^-). \quad \text{(C.3)}$$

This is the *unnormalized* cumulative mass of firms with beliefs $\le \beta$ in state ω. The total mass is $m(\omega) := N(1|\omega; \Lambda, \Gamma)$, which defines the *new stock*. The normalized CDF is $\Gamma'(\beta|\omega) := N(\beta|\omega)/m(\omega)$, which is a CDF by construction: N is non-negative and non-decreasing in β, $N(0|\omega) = 0$ since $H(0|\omega) = 0$ and $\beta^+ \ge 1 - p_{ll} > 0$, and $\Gamma'(1|\omega) = 1$ trivially.

The combined operator $\mathcal{T} : K \times \mathcal{D} \to K \times \mathcal{D}$ is defined by $\mathcal{T}(\Lambda, \Gamma) := (m, \Gamma')$ where $m(\omega) := N(1|\omega; \Lambda, \Gamma)$ and $\Gamma'(\beta|\omega) := N(\beta|\omega)/m(\omega)$. The map $m(\omega)$ is bounded: $m(\omega) \ge \lambda^f(\omega) \ge \lambda_{\min}$ and $m(\omega) \le \lambda^f(\omega) + (1-\delta) \max_\omega \Lambda(\omega) \le \bar{\lambda} + (1-\delta)\lambda_{\max} = \lambda_{\max}$. Hence $\mathcal{T}$ maps $K \times \mathcal{D}$ to itself.

We show that $\mathcal{S} : \mathcal{D} \to \mathcal{D}$ is continuous in the sup-norm metric. For two pairs $\Gamma, \tilde{\Gamma} \in \mathcal{D}$, decompose the operator as $(\mathcal{S}\Gamma)(\beta|\omega) = \alpha_\omega H(\beta|\omega) + (1-\alpha_\omega) R[\Gamma](\beta|\omega)$, where $\alpha_\omega := \lambda^f(\omega)/\Lambda^f(\omega) \in (0,1]$

is the "refreshment rate" and $R[\Gamma](\beta|\omega)$ is the normalized survivor CDF. The first term is independent of Γ; only $R[\Gamma]$ depends on Γ. The functional $R[\Gamma]$ depends on Γ through the following channels:

Channel (i): sensitivity of q to Γ. At a specific (β', ω^-):

$$|q[\Gamma](\beta',\omega^-) - q[\tilde{\Gamma}](\beta',\omega^-)| = G(w_{\text{in}}(\beta')) \left| e^{-\xi(\omega^-)(1-\Gamma(\beta'|\omega^-))} - e^{-\xi(\omega^-)(1-\tilde{\Gamma}(\beta'|\omega^-))} \right|.$$

By the mean value theorem applied to $t \mapsto e^{-\xi t}$ on $[0,1]$:

$$|e^{-\xi(1-\Gamma)} - e^{-\xi(1-\tilde{\Gamma})}| \leq \xi(\omega^-)\,|\Gamma(\beta'|\omega^-) - \tilde{\Gamma}(\beta'|\omega^-)| \leq \Xi\, d(\Gamma, \tilde{\Gamma}),$$

where $\Xi := \max_\omega \xi(\omega) \leq \bar{\lambda}/(\delta\lambda^a)$. Since $G \leq 1$:

$$|q[\Gamma] - q[\tilde{\Gamma}]| \leq \Xi\, d(\Gamma, \tilde{\Gamma}). \tag{C.4}$$

Channel (ii): sensitivity through the measure. For a fixed bounded integrand $f : [0,1] \to [0, \Lambda^f(\omega^-)]$ and two CDFs $\Gamma_{[\omega^-]}, \tilde{\Gamma}_{[\omega^-]}$ on $[0,1]$, integration by parts gives

$$\left| \int_0^a f\, d(\Gamma_{[\omega^-]} - \tilde{\Gamma}_{[\omega^-]}) \right| \leq 2\, \|f\|_\infty\, \|\Gamma_{[\omega^-]} - \tilde{\Gamma}_{[\omega^-]}\|_\infty,$$

where the factor of 2 arises from the boundary term and the interior integral $|\int (\Gamma - \tilde{\Gamma})\, df| \leq \|\Gamma - \tilde{\Gamma}\|_\infty \cdot TV(f) \leq \|\Gamma - \tilde{\Gamma}\|_\infty \cdot \|f\|_\infty$.

Channel (iii): sensitivity of the integration set. The integration set $\{\beta' : \beta^+(\beta', w_{\text{in}}(\beta')) \leq \beta\}$ depends on Γ through β^+, since $\beta^+(\beta', w)$ is a function of the trading probabilities $p^f(w|\omega)$, which themselves depend on $W = \Gamma \circ w_{\text{in}}^{-1}$. Define $\varphi[\Gamma](\beta') := \beta^+[\Gamma](\beta', w_{\text{in}}(\beta'))$. Because β^+ is a smooth rational function of p^f with bounded denominator, and p^f depends smoothly on Γ through $\Gamma \to W \to p^f$, the map $\varphi[\Gamma]$ is Lipschitz in Γ:

$$\left| \varphi[\Gamma](\beta') - \varphi[\tilde{\Gamma}](\beta') \right| \;\leq\; C_3'\, d(\Gamma, \tilde{\Gamma}) \qquad \forall \beta' \in [0,1],$$

where $C_3' > 0$ depends only on Ξ, G, and the slopes of $\tilde{\beta}$ as a function of $p^f(w|\omega)$, all bounded uniformly over $\mathcal{D}$ and $\mathcal{W}$. Lemma 3 ensures that $\varphi[\Gamma]$ is invertible with derivative bounded below by some $\rho_* > 0$, so the inverse $\varphi[\Gamma]^{-1}(\beta)$ is Lipschitz in Γ with constant $C_3 := C_3'/\rho_*$. The symmetric difference of the two integration sets $\{\beta' : \varphi[\Gamma](\beta') \leq \beta\}$ and $\{\beta' : \varphi[\tilde{\Gamma}](\beta') \leq \beta\}$ is then the interval $[\min, \max]$ of $\varphi[\Gamma]^{-1}(\beta)$ and $\varphi[\tilde{\Gamma}]^{-1}(\beta)$, whose Lebesgue length is at most $C_3\, d(\Gamma, \tilde{\Gamma})$. The integrand is bounded above by $\Lambda^f(\omega^-)$, so the contribution from the symmetric difference is bounded by $\Lambda^f(\omega^-) \cdot C_3 \cdot d(\Gamma, \tilde{\Gamma})$.

The unnormalized survivor mass is

$$I[\Gamma](\beta, \omega^-) := \int_{\{\beta' : \beta^+[\Gamma](\beta', w_{\text{in}}(\beta')) \leq \beta\}} \Lambda^f(\omega^-)\, q[\Gamma](\beta', \omega^-)\, d\Gamma(\beta'|\omega^-).$$

Decomposing this into all three channels:

$$
\begin{aligned}
|I[\Gamma]-I[\tilde{\Gamma}]| \leq & \underbrace{\left|\int \Lambda^f q[\Gamma]\, d(\Gamma-\tilde{\Gamma})\right|}_{\text{measure channel (ii)}} + \underbrace{\left|\int \Lambda^f (q[\Gamma]-q[\tilde{\Gamma}])\, d\tilde{\Gamma}\right|}_{\text{integrand channel (i)}} \\
& + \underbrace{\Lambda^f(\omega^-)\cdot C_3 \cdot d(\Gamma,\tilde{\Gamma})}_{\text{integration-set channel (iii)}} \\
& \leq 2\Lambda^f(\omega^-)\, d(\Gamma,\tilde{\Gamma}) + \Lambda^f(\omega^-)\,\Xi\, d(\Gamma,\tilde{\Gamma}) + \Lambda^f(\omega^-)\, C_3\, d(\Gamma,\tilde{\Gamma}) \\
& = \Lambda^f(\omega^-)(2+\Xi+C_3)\, d(\Gamma,\tilde{\Gamma}).
\end{aligned} \tag{C.5}
$$

This establishes the Lipschitz continuity of $\mathcal{S}$: for every $\Gamma,\tilde{\Gamma}\in\mathcal{D}$,

$$
d(\mathcal{S}\Gamma,\mathcal{S}\tilde{\Gamma}) \leq (1-\underline{\alpha})(2+\Xi+C_3)\, d(\Gamma,\tilde{\Gamma}). \tag{C.6}
$$

More importantly, $\mathcal{S}$ is continuous on $\mathcal{D}$.

We must show that if $(\Lambda_n,\Gamma_n)\to(\Lambda,\Gamma)$ in $K\times\mathcal{D}$, then $\mathcal{T}(\Lambda_n,\Gamma_n)\to\mathcal{T}(\Lambda,\Gamma)$. The stock component $m_n(\omega)=N(1|\omega;\Lambda_n,\Gamma_n)\to N(1|\omega;\Lambda,\Gamma)=m(\omega)$ by the argument below applied at $\beta=1$. For the CDF component, we use the *Portmanteau theorem*. The integrals in the operator involve $\int f(\beta')\, d\Gamma_n(\beta'|\omega^-)$ where Γ_n varies. Since CDFs on $[0,1]$ are uniformly bounded, Helly's selection theorem gives $\Gamma_n\to\Gamma$ weakly along subsequences; by the Portmanteau theorem, $\int f\, d\Gamma_n\to\int f\, d\Gamma$ for any bounded function f whose set of discontinuities has zero Γ-measure. The relevant integrand is $q[\Gamma_n](\beta',\omega^-)\cdot\mathbf{1}_{\beta^+(\beta')\leq\beta}$, which is bounded (in $[0,1]$) and discontinuous only at the single point $\beta'=(\beta^+)^{-1}(\beta)$. This point has zero Γ-measure for Lebesgue-a.e. β, since Γ has at most countably many atoms. Since the output of the operator is a CDF, and CDFs are determined by their values on a dense set, the operator is continuous in the weak topology on CDFs.

The set $K\times\mathcal{D}$ is non-empty, compact, and convex in the locally convex space $R^2\times(R^{[0,1]})^2$. Since $\mathcal{T}\colon K\times\mathcal{D}\to K\times\mathcal{D}$ is continuous, Schauder–Tychonoff gives a fixed point $(\Lambda^*,\Gamma^*)\in K\times\mathcal{D}$. At this fixed point, $\Lambda^*(\omega)=m(\omega)=N(1|\omega;\Lambda^*,\Gamma^*)$, which is the stock equation (B.3), and $\Gamma^*(\beta|\omega)=N(\beta|\omega;\Lambda^*,\Gamma^*)/\Lambda^*(\omega)$, which is the belief equation (B.5). Any fixed point (Λ^*,Γ^*) of $\mathcal{T}$ yields belief CDFs $\Gamma^*(\cdot|\omega)$ that are well-defined on $[0,1]$ with $\Gamma^*(0|\omega)=0$ and $\Gamma^*(1|\omega)=1$. The atom-freeness and smoothness of Γ^* are deferred to Lemma 15.

Define $\underline{\alpha}:=\min_\omega\alpha_\omega=\min_\omega\lambda^f(\omega)/\Lambda^f(\omega)>0$. Let $\Gamma^*,\tilde{\Gamma}^*$ be any two fixed points. Since $\Gamma^*=\mathcal{S}(\Gamma^*)$ and $\tilde{\Gamma}^*=\mathcal{S}(\tilde{\Gamma}^*)$, and the entrant component $\alpha_\omega H$ cancels:

$$
d(\Gamma^*,\tilde{\Gamma}^*)=(1-\alpha_\omega)\,\|R[\Gamma^*]-R[\tilde{\Gamma}^*]\|_\infty\leq(1-\underline{\alpha})\,\|R[\Gamma^*]-R[\tilde{\Gamma}^*]\|_\infty.
$$

The survivor CDF $R[\Gamma]$ depends on Γ through three channels: (i) the non-trading probability $q[\Gamma]$; (ii) the measure $d\Gamma$; and (iii) the integration set $\{\beta':\varphi[\Gamma](\beta')\leq\beta\}$, with sensitivity $\leq C_3$ via the Lipschitz constant of $\varphi[\Gamma]^{-1}$ in Γ. Combining all three and normalizing by the survivor mass: $\|R[\Gamma^*]-R[\tilde{\Gamma}^*]\|_\infty\leq(2+\Xi+C_3)\, d(\Gamma^*,\tilde{\Gamma}^*)$. Therefore: $d(\Gamma^*,\tilde{\Gamma}^*)\leq(1-\underline{\alpha})(2+\Xi+C_3)\, d(\Gamma^*,\tilde{\Gamma}^*)$.

If $(1-\underline{\alpha})(2+\Xi+C_3)<1$, then $d(\Gamma^*,\tilde{\Gamma}^*)=0$, establishing uniqueness. This condition is equivalent to $\underline{\alpha}>(1+\Xi+C_3)/(2+\Xi+C_3)$, i.e., the refreshment rate exceeds the feedback sensitivity. Since $\underline{\alpha}\geq\underline{\lambda}\delta/\bar{\lambda}$ and $\Xi\leq\bar{\lambda}/(\delta\lambda^a)$, the condition holds when

$$\frac{\underline{\lambda}\delta}{\bar{\lambda}}>\frac{1+\bar{\lambda}/(\delta\lambda^a)+C_3}{2+\bar{\lambda}/(\delta\lambda^a)+C_3}, \tag{C.7}$$

which is satisfied whenever δ is bounded away from zero and λ^a is sufficiently large relative to $\bar{\lambda}$. The constant C_3 from the integration-set channel is bounded uniformly by primitives. We impose (C.7) as a maintained parameter restriction throughout.

Let $(\Lambda_n^f(h),\Lambda_n^f(l))\to(\Lambda^f(h),\Lambda^f(l))$ in $(0,\bar{\lambda}/\delta]^2$. The corresponding operators $\mathcal{S}_n$ converge to $\mathcal{S}$ as $\sup_{\Gamma\in\mathcal{D}}d(\mathcal{S}_n\Gamma,\mathcal{S}\Gamma)\to 0$. Denote the unique fixed points by Γ_n^* and Γ^*. For any n and any $m\geq 1$:

$$d(\Gamma_n^*,\Gamma^*)\leq d(\mathcal{S}_n^m(\Gamma_n^*),\mathcal{S}^m(\Gamma^*))\leq(1-\underline{\alpha}/2)^m+C_m\cdot\sup_{\Gamma}d(\mathcal{S}_n\Gamma,\mathcal{S}\Gamma),$$

where C_m is a constant depending on m. Choosing m large enough that $(1-\underline{\alpha}/2)^m<\epsilon/2$, and then n large enough that the second term is less than $\epsilon/2$, we obtain $d(\Gamma_n^*,\Gamma^*)<\epsilon$. □

Using Lemma 4, for any pair $(\Lambda^f(h),\Lambda^f(l))\in(0,\bar{\lambda}/\delta]^2$ and any $w_{\text{in}}\in\mathcal{W}$, the belief distributions $\Gamma^f(\cdot|\omega)$ are uniquely determined. This allows us to define the wage distribution $W(w|\omega)=\Gamma^f(w_{\text{in}}^{-1}(w)|\omega)$, the unmatched probability $\Phi(\omega)$ (B.4), and the survivor mass $S(\omega)=\Lambda^f(\omega)-\lambda^a(1-\Phi(\omega))$ as well-defined functions of $(\Lambda^f(h),\Lambda^f(l))$.

Lemma 5. *For any $w_{\text{in}}\in\mathcal{W}$, there exist unique steady-state firm stocks $\Lambda^f(h),\Lambda^f(l)>0$ satisfying (B.3), with $\Lambda^f(\omega)\leq\bar{\lambda}/\delta$. Moreover, $\Lambda^f(h)>\Lambda^f(l)$, and hence $\xi(h)>\xi(l)$.*

Proof. Fix $w_{\text{in}}\in\mathcal{W}$. For each candidate $(\Lambda^f(h),\Lambda^f(l))\in(0,\bar{\lambda}/\delta]^2$, Lemma 4 provides unique belief distributions $\Gamma^f(\cdot|\omega)$, and hence the wage distribution $W(w|\omega)=\Gamma^f(w_{\text{in}}^{-1}(w)|\omega)$, the tightness $\xi(\omega)=\Lambda^f(\omega)/\lambda^a$, and the unmatched probability $\Phi(\omega)$ (B.4) are all well-defined continuous functions of $(\Lambda^f(h),\Lambda^f(l))$; continuity follows from the last claim of Lemma 4.

The survivor mass is $S(\omega)=\Lambda^f(\omega)-\lambda^a(1-\Phi(\omega))$. The stock equation (B.3) becomes

$$F_h(\Lambda^f(h),\Lambda^f(l)):=\Lambda^f(h)-(1-\delta)\Big[p_{hh}S(h)+(1-p_{hh})S(l)\Big]-\lambda^f(h)=0, \tag{C.8}$$

$$F_l(\Lambda^f(h),\Lambda^f(l)):=\Lambda^f(l)-(1-\delta)\Big[(1-p_{ll})S(h)+p_{ll}S(l)\Big]-\lambda^f(l)=0. \tag{C.9}$$

The survivor mass $S(\omega)=\Lambda^f(\omega)-\lambda^a(1-\Phi(\omega))$ depends on both stocks $(\Lambda^f(h),\Lambda^f(l))$ through the belief distributions. We bound both the own-state and cross-state partial derivatives. Define the marginal match term $M(\omega):=\int_c^y(1-W(v|\omega))\,e^{-\xi(\omega)(1-W(v|\omega))}\,dG(v)$. Since $1-W(v|\omega)\in[0,1]$, $e^{-\xi(1-W)}\in(0,1]$, and G has positive density on (c,y):

- $M(\omega)>0$: the integrand is strictly positive on any subinterval of (c,y) where $W(v|\omega)<1$, and such a subinterval exists because $W(y|\omega)=1$ but $W(c|\omega)<1$, since $w_{\text{in}}(\beta)>c$ for β sufficiently large.

- $M(\omega) < 1$: since total hires $\lambda^a(1-\Phi(\omega))$ is a concave function of $\Lambda^f(\omega)$, and $\lambda^a(1-\Phi) \leq \Lambda^f(\omega)$, while $\lambda^a(1-\Phi)|_{\Lambda^f=0} = 0$, concavity implies $\frac{d[\lambda^a(1-\Phi)]}{d\Lambda^f(\omega)} \leq \frac{\lambda^a(1-\Phi(\omega))}{\Lambda^f(\omega)} \leq 1$. The strict inequality $M(\omega) < 1$ follows from $e^{-\xi(1-W)} < 1$ for $W < 1$.

Given $S(\omega)$, the total own-state derivative includes both the direct effect through $\xi(\omega) = \Lambda^f(\omega)/\lambda^a$ and the indirect effect through the belief-induced wage distribution $W(\cdot|\omega)$. The direct effect gives $M(\omega) \in (0,1)$. The indirect effect is non-negative, hence:

$$\frac{\partial S(\omega)}{\partial \Lambda^f(\omega)} \in [0, 1-\underline{M}), \tag{C.10}$$

where $\underline{M} := \inf_{w_{\text{in}} \in \mathcal{W}, \omega} M(\omega) > 0$.

Cross-state derivative $\partial S(\omega)/\partial \Lambda^f(\omega')$ for $\omega' \neq \omega$. The stock $\Lambda^f(\omega')$ affects $S(\omega)$ only through the belief distribution $\Gamma^f(\cdot|\omega)$, which depends on $\Lambda^f(\omega')$ via the cross-state survivor flow in (B.5). By the implicit function theorem applied to the contraction mapping of Lemma 4, whose contraction modulus $\rho = (1-\underline{\alpha})(2+\Xi+C_3) < 1$ holds under the standing parameter restriction (C.7), the sensitivity of Γ^f to $\Lambda^f(\omega')$ is bounded:

$$\left\| \frac{\partial \Gamma^f}{\partial \Lambda^f(\omega')} \right\|_\infty \leq \frac{1}{1-\rho} \cdot \left\| \frac{\partial (\mathcal{S}\Gamma)}{\partial \Lambda^f(\omega')} \Big|_{\Gamma=\Gamma^*} \right\|_\infty.$$

The derivative $\partial(\mathcal{S}\Gamma)/\partial \Lambda^f(\omega')$ involves the cross-state transition with coefficient $\pi(\omega'|\omega) = (1-p_{hh})$, bounded by $(1-p_{hh})/\underline{\alpha}$. The perturbation of $\Phi(\omega)$ through $W(\cdot|\omega)$ gives:

$$\left| \frac{\partial S(\omega)}{\partial \Lambda^f(\omega')} \right| \leq \frac{\Xi\,(1-p_{hh})}{\underline{\alpha}\,(1-\rho)} =: \eta, \tag{C.11}$$

where $\Xi = \max_\omega \xi(\omega)$ enters through the sensitivity of Φ to W. The constant η is small when p_{hh} is close to 1 (high persistence) or $\underline{\alpha}$ is large (high refreshment rate).

Existence via Brouwer. Any solution satisfies $\Lambda^f(\omega) \leq \bar{\lambda}/\delta$: taking the maximum over ω in the stock equation and using $S(\omega) \leq \Lambda^f(\omega)$ and $\sum_{\omega^-} \pi(\omega^-|\omega) = 1$: $\bar{\Lambda} \leq \bar{\lambda} + (1-\delta)\bar{\Lambda} \implies \bar{\Lambda} \leq \frac{\bar{\lambda}}{\delta}$. Define the compact rectangle $K := [\underline{\lambda}, \bar{\lambda}/\delta]^2$ (with $\underline{\lambda} := \min\{\lambda^f(h), \lambda^f(l)\} > 0$).

Define $T_\Lambda : K \to R^2$ by the right-hand sides of the stock equations. Each component maps into $[\underline{\lambda}, \bar{\lambda}/\delta]$, so $T_\Lambda : K \to K$. The map T_Λ is continuous because $S(\omega)$ depends continuously on $(\Lambda^f(h), \Lambda^f(l))$ through the chain established in Lemma 4. By Brouwer's theorem, T_Λ has a fixed point with $\Lambda^f(\omega) \geq \underline{\lambda} > 0$.

Uniqueness via contraction. Suppose (Λ_h, Λ_l) and (Λ'_h, Λ'_l) are two solutions. The 2×2 Jacobian of the stock map T_Λ has entries bounded by the survivor derivatives (C.10)–(C.11). Explicitly, the Jacobian matrix is

$$J \;=\; (1-\delta) \begin{pmatrix} p_{hh}\,\partial S_h/\partial \Lambda_h + (1-p_{hh})\,\partial S_l/\partial \Lambda_h & p_{hh}\,\partial S_h/\partial \Lambda_l + (1-p_{hh})\,\partial S_l/\partial \Lambda_l \\ (1-p_{ll})\,\partial S_h/\partial \Lambda_h + p_{ll}\,\partial S_l/\partial \Lambda_h & (1-p_{ll})\,\partial S_h/\partial \Lambda_l + p_{ll}\,\partial S_l/\partial \Lambda_l \end{pmatrix}.$$

Using the bounds $\partial S_\omega/\partial\Lambda_\omega \le 1-\underline{M}$ and $|\partial S_\omega/\partial\Lambda_{\omega'}| \le \eta$, each row of J has ℓ^∞-norm at most $(1-\delta)[(1-\underline{M})+\eta]$. By the mean value theorem applied to each $S(\omega)$:

$$|S_\omega - S'_\omega| \le (1-\underline{M})\,|\Lambda_\omega - \Lambda'_\omega| + \eta\,|\Lambda_{\omega'} - \Lambda'_{\omega'}|,$$

where $\omega' \neq \omega$. Subtracting the stock equations:

$$\begin{aligned}
|\Lambda_h - \Lambda'_h| &\le (1-\delta)\Big[p_{hh}\,|S_h - S'_h| + (1-p_{hh})\,|S_l - S'_l|\Big],\\
|\Lambda_l - \Lambda'_l| &\le (1-\delta)\Big[(1-p_{ll})\,|S_h - S'_h| + p_{ll}\,|S_l - S'_l|\Big].
\end{aligned}$$

Substituting the S bounds and using $p_{hh} = p_{ll}$, let $\Delta := \max\{|\Lambda_h - \Lambda'_h|, |\Lambda_l - \Lambda'_l|\}$:

$$\begin{aligned}
|\Lambda_h - \Lambda'_h| &\le (1-\delta)\Big[(1-\underline{M})+\eta\Big]\,\Delta,\\
|\Lambda_l - \Lambda'_l| &\le (1-\delta)\Big[(1-\underline{M})+\eta\Big]\,\Delta.
\end{aligned}$$

Hence $\Delta \le (1-\delta)[(1-\underline{M})+\eta]\,\Delta$. The spectral radius condition $\rho(J) < 1$ requires $(1-\delta)[(1-\underline{M})+\eta] < 1$, i.e., the contraction condition

$$(1-\delta)\Big[(1-\underline{M})+\eta\Big] < 1 \tag{C.12}$$

holds under (C.7) and $\delta > 0$. This is satisfied when δ is not too small and $\underline{M} > 0$. When $\eta = 0$—the limiting case $p_{hh} \to 1$—the condition reduces to $\delta > 0$; more generally, it requires $\eta < \delta/(1-\delta) + \underline{M}$. Under (C.12), $\Delta = 0$, establishing uniqueness.

Ordering: Define $D(\Lambda_h, \Lambda_l) := F_h - F_l$. Computing directly from (C.8)–(C.9):

$$\begin{aligned}
D(\Lambda_h, \Lambda_l) = (\Lambda_h - \Lambda_l)&\\
&- (1-\delta)\Big\{[p_{hh}\,S(h) + (1-p_{hh})\,S(l)] - [(1-p_{ll})\,S(h) + p_{ll}\,S(l)]\Big\}\\
&- (\lambda^f(h) - \lambda^f(l))\\
&= (\Lambda_h - \Lambda_l) \;-\; (1-\delta)(p_{hh}+p_{ll}-1)[S(h) - S(l)] \;-\; (\lambda^f(h) - \lambda^f(l)),
\end{aligned}$$

where the second equality uses $p_{hh} - (1-p_{ll}) = p_{hh} + p_{ll} - 1 =: q$ and $(1-p_{hh}) - p_{ll} = -q$. On the diagonal $\Lambda_h = \Lambda_l =: \Lambda_s$, the first term vanishes:

$$D(\Lambda_s, \Lambda_s) \;=\; -(1-\delta)\,(p_{hh}+p_{ll}-1)\,[S(h) - S(l)] \;-\; (\lambda^f(h) - \lambda^f(l)).$$

On the diagonal, $\Lambda^f(h) = \Lambda^f(l)$, so the sign of $S(h) - S(l)$ is the sign of $\Phi(h) - \Phi(l)$. Now $\Phi(\omega) = \int_c^y e^{-\xi(\omega)(1-W(v|\omega))}\,dG(v)$ is the applicant-side unmatched probability. With $\xi(h) = \xi(l) =: \xi$ at the diagonal and $W(\cdot|h)$ FOSD-dominating $W(\cdot|l)$, so $1 - W(v|h) \le 1 - W(v|l)$ and $e^{-\xi(1-W(v|h))} \ge e^{-\xi(1-W(v|l))}$, so $\Phi(h) \ge \Phi(l)$ pointwise in v, hence after integration. Therefore $S(h) \ge S(l)$ on the diagonal.

Substituting into $D(\Lambda_s, \Lambda_s)$:

$$D(\Lambda_s, \Lambda_s) \;=\; -\underbrace{(1-\delta)(p_{hh}+p_{ll}-1)[S(h)-S(l)]}_{\geq 0} - \underbrace{(\lambda^f(h)-\lambda^f(l))}_{>0} \;<\; 0.$$

Both terms are non-positive and the second is strictly negative, so $D(\Lambda_s, \Lambda_s) < 0$ on the entire diagonal.

We compute $\partial D/\partial \Lambda_h = 1-(1-\delta)(p_{hh}+p_{ll}-1)\Big[\partial S(h)/\partial \Lambda_h - \partial S(l)/\partial \Lambda_h\Big]$. Using the bounds (C.10) and (C.11), $|\partial S(h)/\partial \Lambda_h - \partial S(l)/\partial \Lambda_h| \leq (1-\underline{M})+\eta$, so $\partial D/\partial \Lambda_h \;\geq\; 1-(1-\delta)(p_{hh}+p_{ll}-1)[(1-\underline{M})+\eta] \;>\; 0$ under the contraction condition (C.12). Similarly, $\partial D/\partial \Lambda_l < 0$.

Since $D < 0$ on the diagonal and the unique equilibrium satisfies $D(\Lambda_h^*, \Lambda_l^*) = 0$, moving from a diagonal point to the equilibrium requires increasing the difference D from a negative value to zero. With $\partial D/\partial \Lambda_h > 0$ and $\partial D/\partial \Lambda_l < 0$, the equilibrium must satisfy $\Lambda_h^* > \Lambda_l^*$. Division by $\lambda^a > 0$ gives $\xi(h) > \xi(l)$. □

Lemma 6. *The maps $w_{\text{in}} \mapsto \Lambda^f(\omega)$, $w_{\text{in}} \mapsto \Gamma^f(\cdot|\omega)$, $w_{\text{in}} \mapsto W(\cdot|\omega)$, $w_{\text{in}} \mapsto p^f(\cdot|\omega)$ are continuous from $(\mathcal{W}, \|\cdot\|_\infty)$ to the appropriate spaces (with sup-norm topology).*

Proof. Let $\{w_n\} \subset \mathcal{W}$ with $\|w_n - w\|_\infty \to 0$. We establish convergence of all aggregates in sequence.

Since each w_n is continuous and weakly increasing on $[0,1]$, its generalized inverse $w_n^{-1}(w) := \inf\{\beta : w_n(\beta) \geq w\}$ is well-defined and right-continuous. For any continuity point w of w^{-1}: $\|w_n - w\|_\infty \to 0$ implies $w_n^{-1}(w) \to w^{-1}(w)$. To see this: for any $\epsilon > 0$, for large n, $w_n(\beta) \geq w(\beta) - \epsilon$ and $w_n(\beta) \leq w(\beta) + \epsilon$ for all β. If $w^{-1}(w) = \beta_0$, then $w(\beta_0) \geq w$, so $w_n(\beta_0) \geq w - \epsilon$, which implies $w_n^{-1}(w-\epsilon) \leq \beta_0$. A symmetric argument gives $w_n^{-1}(w+\epsilon) \geq \beta_0 - \delta(\epsilon)$ for some $\delta(\epsilon) \to 0$. At continuity points of w^{-1}, this gives $w_n^{-1}(w) \to w^{-1}(w)$.

The stocks $\Lambda_n^f(\omega)$ lie in the compact set $[0, \bar{\lambda}/\delta]$. Extract a convergent subsequence $\Lambda_{n_k}^f(\omega) \to \Lambda^*(\omega)$ for both $\omega \in \{h, l\}$. Along this subsequence, $\xi_{n_k}(\omega) \to \xi^*(\omega) := \Lambda^*(\omega)/\lambda^a$.

For each n_k, the belief distribution $\Gamma_{n_k}^f$ is the unique fixed point of the operator $\mathcal{S}_{n_k}$. Each $\Gamma_{n_k}^f$ is a CDF on $[0,1]$. By Helly's selection theorem, extract a further subsequence with $\Gamma_{n_k}^f \to \tilde{\Gamma}$ pointwise at all continuity points of $\tilde{\Gamma}$.

The limit $\tilde{\Gamma}$ is continuous. To see this: by the flow-balance structure, each $\Gamma_{n_k}^f$ satisfies $\Gamma_{n_k}^f = \alpha_{\omega,n_k} H_{n_k} + (1-\alpha_{\omega,n_k}) R_{n_k}[\Gamma_{n_k}^f]$, where $\alpha_{\omega,n_k} = \lambda^f(\omega)/\Lambda_{n_k}^f(\omega) \geq \underline{\alpha} > 0$ and H is continuous. For any $\beta \in [0,1]$ and $\varepsilon > 0$: $\Gamma_{n_k}^f(\beta+\varepsilon) - \Gamma_{n_k}^f(\beta) \leq \alpha_{\omega,n_k}[H(\beta+\varepsilon) - H(\beta)] + (1-\alpha_{\omega,n_k}) \cdot 1$. Taking $\varepsilon \to 0$ first (using continuity of H) and then $n_k \to \infty$: $\tilde{\Gamma}(\beta+\varepsilon) - \tilde{\Gamma}(\beta) \leq \underline{\alpha} \cdot o(1) + (1-\underline{\alpha}) \cdot [\tilde{\Gamma}(\beta+\varepsilon) - \tilde{\Gamma}(\beta)]$, which gives $\underline{\alpha}[\tilde{\Gamma}(\beta+\varepsilon) - \tilde{\Gamma}(\beta)] \leq \underline{\alpha} \cdot o(1)$, hence $\tilde{\Gamma}$ is continuous.

Since $\tilde{\Gamma}$ is a continuous CDF on $[0,1]$ and $\Gamma_{n_k}^f \to \tilde{\Gamma}$ pointwise at all points: $\|\Gamma_{n_k}^f - \tilde{\Gamma}\|_\infty \to 0$. We claim $\tilde{\Gamma}$ is a fixed point of $\mathcal{S}^*$, the operator constructed from w and Λ^*. To see this, write

$\tilde{\Gamma} = \lim_k \Gamma^f_{n_k} = \lim_k \mathcal{S}_{n_k}(\Gamma^f_{n_k})$. Decompose:

$$|\mathcal{S}_{n_k}(\Gamma^f_{n_k}) - \mathcal{S}^*(\tilde{\Gamma})| \leq \underbrace{|\mathcal{S}_{n_k}(\Gamma^f_{n_k}) - \mathcal{S}^*(\Gamma^f_{n_k})|}_{\text{operator convergence}} + \underbrace{|\mathcal{S}^*(\Gamma^f_{n_k}) - \mathcal{S}^*(\tilde{\Gamma})|}_{\text{continuity of } \mathcal{S}^*}.$$

The first term vanishes as $k \to \infty$ because $w_{n_k} \to w$ uniformly, $w^{-1}_{n_k} \to w^{-1}$ at continuity points, $\xi_{n_k} \to \xi^*$, and the bounded convergence theorem applies to the integrals defining $\mathcal{S}$. The second term vanishes because $\mathcal{S}^*$ is Lipschitz continuous on $\mathcal{D}$ by the bound (C.6), and $\Gamma^f_{n_k} \to \tilde{\Gamma}$ uniformly. Hence $\tilde{\Gamma} = \mathcal{S}^*(\tilde{\Gamma})$. By uniqueness of the fixed point, $\tilde{\Gamma} = \Gamma^{f*}$, the unique belief distribution for stocks Λ^*. Since every convergent subsequence of $\{\Gamma^f_{n_k}\}$ has the same limit, the full subsequence converges: $\Gamma^f_{n_k} \to \Gamma^{f*}$.

The limit (Λ^*, Γ^{f*}) satisfies the stock equation (B.3) and the belief equation (B.5), since $\Gamma^{f*} = \mathcal{S}^*(\Gamma^{f*})$. By uniqueness, $\Lambda^*(\omega)$ must equal the unique stocks associated with w, i.e., $\Lambda^*(\omega) = \Lambda^f(\omega)$. Similarly, $\Gamma^{f*} = \Gamma^f(\cdot|\omega)$.

Since every convergent subsequence of $\Lambda^f_n(\omega)$ converges to the same limit $\Lambda^f(\omega)$, and the sequence lies in a compact set, the entire sequence converges: $\Lambda^f_n(\omega) \to \Lambda^f(\omega)$. The same argument applies to Γ^f_n, W_n, p^f_n. For each w at which $W(\cdot|\omega)$ is continuous: $W_n(w|\omega) = \Gamma^f_n(w^{-1}_n(w)|\omega) \to \Gamma^f(w^{-1}(w)|\omega) = W(w|\omega)$. Since every admissible $w_{\text{in}} \in \mathcal{W}$ satisfies $w_{\text{in}}(\beta') - w_{\text{in}}(\beta) \geq \ell(\beta' - \beta)$ with $\ell > 0$, each w_{in} is strictly increasing and the induced wage distributions are atomless by Lemma 3. Hence the limit CDF $W(\cdot|\omega)$ is continuous, and Pólya's theorem upgrades the pointwise convergence $W_n \to W$ at continuity points to uniform convergence on $[c, y]$. For the hiring probability: $p^f_n(w|\omega) = G(w)e^{-\xi_n(\omega)(1-W_n(w|\omega))}$. Since $\xi_n \to \xi$ and $W_n(w|\omega) \to W(w|\omega)$ at every continuity point of W, we have $p^f_n(w|\omega) \to p^f(w|\omega)$ at every such point. Hence $p^f_n(w^*(\beta)|\omega) \to p^f(w^*(\beta)|\omega)$ and $\Pi_n(w^*(\beta), \beta) \to \Pi(w^*(\beta), \beta)$ for every β. □

Lemma 7. *For any $w_{\text{in}} \in \mathcal{W}$, the Bellman equation (5.5) has a unique solution $V^f \in C([0,1])$ with $\|V^f\|_\infty \leq (y-c)/(1-\gamma)$, where $\gamma := \alpha(1-\delta) < 1$. Moreover, V^f depends continuously on w_{in}.*

Proof. Fix $w_{\text{in}} \in \mathcal{W}$ and the corresponding aggregates from Lemmas 5–4. We use formula (5.1) as the *definition* of $p^f(w|\omega)$ for all w, and build the Bellman operator from it.

Define $\mathcal{B} : C([0,1]) \to C([0,1])$ by $(\mathcal{B}V)(\beta) := \sup_{w \in [c,y]} \Pi(w, \beta; V)$, where $\Pi(w, \beta; V) := p^f(w|\beta)(y - w) + \gamma[1 - p^f(w|\beta)]V(\beta^+(w, \beta))$.

The objective $\Pi(\cdot, \beta; V)$ is continuous in w and continuous in β. Since $[c, y]$ is compact and Π is jointly continuous, the supremum is attained and the maximum is well-defined.

Continuity of $(\mathcal{B}V)$ in β: by Berge's maximum theorem, since $\Pi(w, \beta; V)$ is jointly continuous in (w, β) on the compact set $[c, y] \times [0, 1]$ and the constraint correspondence $\beta \rightrightarrows [c, y]$ is constant (hence continuous), the value function $\beta \mapsto (\mathcal{B}V)(\beta) = \max_w \Pi(w, \beta; V)$ is continuous, and the argmax correspondence is upper hemicontinuous. Hence $\mathcal{B}V \in C([0,1])$.

We verify two Blackwell conditions: monotonicity and discounting.

Monotonicity: If $V_1(\beta) \geq V_2(\beta)$ for all $\beta \in [0, 1]$, then for each (w, β), the continuation coefficient $\alpha(1-\delta)[1 - p^f(w|\beta)] \geq 0$ since all factors are non-negative and $p^f \leq 1$. Hence the objective under

V_1 exceeds that under V_2 for every (w,β). Taking the maximum over w preserves the inequality: $(\mathcal{B}V_1)(\beta) \geq (\mathcal{B}V_2)(\beta)$ for all β.

Discounting: For any $V \in C([0,1])$ and constant $a > 0$, define $V_a := V + a$. For any (w,β):

$$\begin{aligned}\Pi(w,\beta;V_a) - \Pi(w,\beta;V) &= \alpha(1-\delta)[1-p^f(w|\beta)] \cdot a \\ &\leq \alpha(1-\delta) \cdot a = \gamma \cdot a,\end{aligned}$$

since $1 - p^f(w|\beta) \leq 1$. Taking the maximum over w: $(\mathcal{B}V_a)(\beta) \leq (\mathcal{B}V)(\beta) + \gamma \cdot a$. Therefore $\|\mathcal{B}V_a - \mathcal{B}V\|_\infty \leq \gamma \cdot a$ with $\gamma = \alpha(1-\delta) < 1$.

By Blackwell's theorem, $\mathcal{B}$ is a contraction with modulus $\gamma < 1$ on the complete metric space $(C([0,1]), \|\cdot\|_\infty)$. The Banach contraction mapping theorem yields a unique fixed point $V^f = \mathcal{B}V^f$.

At the fixed point, $V^f(\beta) \leq \max_{w\in[c,y]}\{(y-w) + \gamma\|V^f\|_\infty\} \leq (y-c) + \gamma\|V^f\|_\infty$. Solving this gives us $\|V^f\|_\infty \leq (y-c)/(1-\gamma)$.

If $w_n \to w$ in $\mathcal{W}$, then by Lemma 6, the stocks $\Lambda_n^f \to \Lambda^f$, tightness $\xi_n \to \xi$, and belief CDFs $\Gamma_n^f \to \Gamma^f$ uniformly. For any fixed $V \in C([0,1])$ and $\beta \in [0,1]$, $(\mathcal{B}_n V)(\beta) = \max_w \Pi_n(w,\beta;V)$. By Lemma 3, all wage CDFs W_n and W are atomless, so $p_n^f \to p^f$ uniformly in w and $\beta_n^+ \to \beta^+$ uniformly. Hence $\Pi_n \to \Pi$ uniformly on $[c,y] \times [0,1]$. By the maximum theorem, $(\mathcal{B}_n V)(\beta) \to (\mathcal{B}V)(\beta)$ uniformly in β. The standard perturbation bound for contractions gives

$$\|V_n^f - V^f\|_\infty \leq \frac{1}{1-\gamma}\|\mathcal{B}_n V^f - \mathcal{B}V^f\|_\infty \to 0. \qquad \square$$

By Lemma 3, for every $w_{\text{in}} \in \mathcal{W}$, the wage CDF $W(\cdot|\omega)$ is atomless for both $\omega \in \{h,l\}$. Consequently the hiring probability $p^f(w|\omega) = G(w)\,e^{-\xi(\omega)(1-W(w|\omega))}$ is continuous in w, the static profit $\pi_s(w,\beta)$ is continuous, and the full payoff $\Pi(w,\beta;V)$ is continuous in w. The maximizer of $\Pi(\cdot,\beta;V)$ on $[c,y]$ exists by the extreme value theorem.

Lemma 8. *For any $w_{\text{in}} \in \mathcal{W}$, $V \in C([0,1])$, and $\beta \in [0,1]$: (i) The static profit $\pi_s(w,\beta) = p^f(w|\beta)(y-w)$ is continuous in w on $[c,y]$. (ii) The full payoff $\Pi(w,\beta;V)$ is continuous in w on $[c,y]$. (iii) A maximizer $w^*(\beta) \in argmax_{w\in[c,y]}\Pi(w,\beta;V)$ exists.*

Proof. Part (i). By Lemma 3, $W(\cdot|\omega)$ is atomless for both $\omega \in \{h,l\}$. Therefore $1 - W(\cdot|\omega)$ is continuous in w, and so is $e^{-\xi(\omega)(1-W(w|\omega))}$. The factor $G(w)$ is C^1. Therefore $p^f(w|\omega) = G(w)e^{-\xi(\omega)(1-W(w|\omega))}$ is continuous in w, and so is the belief-weighted average $p^f(w|\beta) = \beta\, p^f(w|h) + (1-\beta)\,p^f(w|l)$. Multiplying by the continuous function $(y-w)$, the static profit $\pi_s(w,\beta)$ is continuous in w on $[c,y]$.

Part (ii). The continuation value $\pi_c(w,\beta;V) = \gamma[1-p^f(w|\beta)]V(\beta^+(w,\beta))$ is the product of $1-p^f(w|\beta)$ with the composition $V \circ \beta^+(\cdot,\beta)$. The next-period belief $\beta^+(w,\beta) = p_{hh}\tilde{\beta}(w,\beta) + (1-p_{ll})(1-\tilde{\beta}(w,\beta))$ is a smooth function of $p^f(w|h)$ and $p^f(w|l)$, hence continuous in w. Since V is continuous, $V \circ \beta^+$ is continuous in w. Therefore π_c is continuous in w, and so is $\Pi = \pi_s + \pi_c$.

Part (iii). A continuous function on the compact set $[c,y]$ attains its supremum (extreme value theorem). Hence $argmax_{w\in[c,y]}\Pi(w,\beta;V)\neq\emptyset$. The claim $w^*(\beta)>c$ (uniformly in β and w_{in}) is established in Lemma 10: posting $w=c$ yields hiring probability $p^f(c|\beta)=G(c)\cdot e^{-\xi(\cdot)}=0$, so the full payoff at $w=c$ reduces to the pure continuation value, which is strictly dominated by any nearby $w>c$ for γ sufficiently small. □

Lemma 9. *For any $w_{\text{in}}\in\mathcal{W}$, the value function V^f from Lemma 7 has a Lipschitz-continuous derivative: $V^f\in C^{1,1}([0,1])$ with $\|(V^f)'\|_{\text{Lip}}\le\frac{C_0}{1-\gamma}$, where $C_0>0$.*

Proof. We use a bootstrap argument through the Bellman iteration. Define $V_0^f\equiv 0$ and $V_{n+1}^f:=\mathcal{B}V_n^f$. The Banach fixed-point theorem gives $V_n^f\to V^f$ uniformly.

Base case. $V_0^f\equiv 0$ is C^∞, hence $C^{1,1}$ with $(V_0^f)'\equiv 0$.

Inductive step. Suppose $V_n^f\in C^{1,1}([0,1])$ with $\|(V_n^f)'\|_{\text{Lip}}\le K_n$. Define $\Pi_n(w,\beta):=p^f(w|\beta)(y-w)+\gamma[1-p^f(w|\beta)]V_n^f(\beta^+(w,\beta))$. By Lemma 3, $W(\cdot|\omega)$ is atomless, so p^f is continuous in w and the unique maximizer $w_n^*(\beta)$ from Lemma 8 is well-defined.

Regularity at the optimum. We establish $(V_{n+1}^f)'$ and its Lipschitz bound *without* assuming W is differentiable. The partial derivative Π_β exists everywhere: $\Pi_\beta(w,\beta)=[p^f(w|h)-p^f(w|l)](y-w)+\gamma[(p^f(w|l)-p^f(w|h))V_n^f(\beta^+)+(1-p^f(w|\beta))(V_n^f)'(\beta^+)\,\partial\beta^+/\partial\beta]$. Here $p^f(w|\omega)$ depends on w but not on derivatives of W, and $(V_n^f)'$ exists everywhere (inductive hypothesis). Hence Π_β is well-defined at every (w,β) and is Lipschitz in β with a constant depending on $\|(V_n^f)'\|_{\text{Lip}}$.

By the envelope theorem: $(V_{n+1}^f)'(\beta)=\Pi_\beta(w_n^*(\beta),\beta)$ a.e. on $[0,1]$. Since w_n^* is increasing and Π_β is Lipschitz in β (uniformly in w), the composition $\beta\mapsto\Pi_\beta(w_n^*(\beta),\beta)$ is measurable and Lipschitz: $\|(V_{n+1}^f)'\|_{\text{Lip}}\le C_1+\gamma C_2K_n$, where C_1 bounds the Lipschitz constant of Π_β in β at fixed w, and C_2 bounds the chain through $w_n^*(\beta)$. Moreover, since V_{n+1}^f is Lipschitz with Lipschitz derivative, $V_{n+1}^f\in C^{1,1}$.

Choosing $\bar\gamma$ small enough that $\gamma C_2<1$ for all $\gamma<\bar\gamma$, the sequence K_n satisfies $K_{n+1}\le C_1+\gamma C_2K_n$ with $K_0=0$. Iterating: $K_n\le C_1/(1-\gamma C_2)\le C_0/(1-\gamma)$ for $C_0:=C_1/(1-C_2\bar\gamma/(1-\bar\gamma))$. The constant C_1 is itself uniform in n by an a priori bound: the first derivative satisfies $\|(V_n^f)'\|_\infty\le(y-c)/(1-\gamma)$ independently of K_n, so the iterate cross-partial bound $M_n=\sup|\Pi_{w\beta}^n|$ is bounded by primitives; and on the inductive hypothesis $K_n\le\bar K:=C_1/(1-\gamma C_2)$ the concavity modulus obeys $\kappa_n\ge\kappa_s/2$ for γ small, whence $L_n=2M_n/\kappa_n\le 4M_n/\kappa_s$ and C_1 are bounded by primitives. The bound $K_{n+1}\le\bar K$ then closes the induction.

The functions $(V_n^f)'$ are uniformly bounded and equi-Lipschitz. By Arzelà–Ascoli, a subsequence converges uniformly to a Lipschitz function. Since $V_n^f\to V^f$ uniformly and each V_n^f is C^1, the limit V^f is C^1 with $(V^f)'=\lim(V_n^f)'$, and inherits the Lipschitz bound $\|(V^f)'\|_{\text{Lip}}\le C_0/(1-\gamma)$. □

Lemma 10. *Under Assumptions 1 and 2, for any $w_{\text{in}}\in\mathcal{W}$ and the corresponding aggregates, there exists $\bar\gamma\in(0,1)$ such that if $\gamma:=\alpha(1-\delta)<\bar\gamma$, then $\Pi(\cdot,\beta;w_{\text{in}})$ is strictly concave in w on $[c,y]$ for every $\beta\in[0,1]$.*

Proof. Decompose Π as $\Pi(w,\beta) = \pi_s(w,\beta) + \pi_c(w,\beta)$ where

$$\pi_s(w,\beta) := p^f(w|\beta)(y-w), \qquad \text{(static profit)}$$
$$\pi_c(w,\beta) := \gamma[1-p^f(w|\beta)]\,V^f(\beta^+(w,\beta)). \qquad \text{(continuation value)}$$

Strict concavity of the static profit. The belief-weighted hiring probability is $p^f(w|\beta) = \beta\, p^f(w|h) + (1-\beta)\,p^f(w|l)$, and each state-specific component is $p^f(w|\omega) = G(w)e^{-\xi(\omega)(1-W(w|\omega))}$. Define $\mu(w|\omega) := r_G(w) + \xi(\omega)W'(w|\omega) > 0$ where $r_G(w) := g(w)/G(w)$ is the reversed hazard rate of G and $W'(w|\omega) \geq 0$ is the wage density.

The first and second derivatives of each state-specific profit with respect to w are:

$$\frac{\partial}{\partial w}[p^f(w|\omega)(y-w)] = p^f(w|\omega)\Big[\mu(w|\omega)(y-w) - 1\Big], \tag{C.13}$$
$$\frac{\partial^2}{\partial w^2}[p^f(w|\omega)(y-w)] = p^f(w|\omega)\Big[\mu^2(y-w) + \mu'(y-w) - 2\mu\Big]. \tag{C.14}$$

Assumption (1) states that $w \mapsto G(w)e^{-\xi(1-W(w))}(y-w)$ is strictly concave on (c,y) for every feasible (ξ, W). When W is C^2, this is equivalent to the bracket in (C.14) being strictly negative; when W is only C^1, Assumption (1) is imposed directly as a concavity condition. In either case, $p^f(w|\omega)(y-w)$ is strictly concave in w for each ω. Since $\pi_s(w,\beta) = \beta \cdot [p^f(w|h)(y-w)] + (1-\beta) \cdot [p^f(w|l)(y-w)]$ is a convex combination of strictly concave functions, π_s is strictly concave in w for every $\beta \in [0,1]$.

We claim the best-response wage $w_{\text{out}}(\beta)$ is bounded away from c uniformly. Since $G(c) = 0$, posting $w = c$ yields zero hiring probability $p^f(c|\beta) = 0$, so the full payoff at $w = c$ reduces to the pure continuation value: $\Pi(c,\beta) = \gamma V^f(\beta^+(c,\beta))$. This is positive, but we show that any nearby $w > c$ strictly improves upon it. For $w = c+\eta$ with small $\eta > 0$, the payoff decomposes as

$$\Pi(c+\eta,\beta) \;=\; \underbrace{p^f(c+\eta|\beta)\Big[(y-c-\eta) - \gamma V^f(\beta^+(c+\eta,\beta))\Big]}_{\text{net gain from hiring}} + \gamma V^f(\beta^+(c+\eta,\beta)).$$

Since $V^f(\beta) \leq (y-c)/(1-\gamma)$ (Lemma 7), we have $(y-c-\eta) - \gamma V^f \geq (y-c)(1-2\gamma)/(1-\gamma) - \eta$. Choosing $\bar{\gamma} \leq 1/2$ ensures $\gamma < 1/2$, so this expression is strictly positive for η sufficiently small. Combined with $p^f(c+\eta|\beta) > 0$ and the continuity of β^+ in w, we obtain $\Pi(c+\eta,\beta) > \Pi(c,\beta)$ for all small $\eta > 0$. Hence $w_{\text{out}}(\beta) > c$ for all β. Since G has bounded density $g > 0$ on $[c,y]$ with $G(c) = 0$, the reverse hazard rate $r_G(w) = g(w)/G(w)$ diverges as $w \to c^+$: indeed $G(w) \sim g(c)(w-c)$ so $r_G(w)(y-w) \to \infty$ as $w \to c^+$. Therefore there exists $\varepsilon_0 > 0$, depending only on $g(c)$ and $y-c$, such that $r_G(w)(y-w) > 1$ for all $w \in [c, c+\varepsilon_0]$. At any such w the static derivative satisfies $(\pi_s)_w(w,\beta) = \beta f(w,h) + (1-\beta) f(w,l) > 0$ because $f(w,\omega) = p^f(w|\omega)[\mu(w|\omega)(y-w)-1] \geq p^f(w|\omega)[r_G(w)(y-w)-1] > 0$ in both states. The full payoff satisfies $\Pi_w(w,\beta) = (\pi_s)_w(w,\beta) + (\pi_c)_w(w,\beta)$ with $|(\pi_c)_w| \leq B_0\gamma/(1-\gamma)$ uniformly, and for γ sufficiently small the static contribution dominates, so $\Pi_w(w,\beta)>0$ on $[c,c+\varepsilon_0/2]$. Hence the full-FOC maximizer satisfies $w_{\text{out}}(\beta)\geq c+\varepsilon$ with $\varepsilon:=\varepsilon_0/2$.

Define $\kappa_s := \inf_{w_{\text{in}}\in\mathcal{W}}\inf_{w\in[c+\varepsilon,y],\beta\in[0,1]}|(\pi_s)_{ww}(w,\beta)|>0$. This infimum is strictly positive because: $G(w)\geq G(c+\varepsilon)>0$ for $w\geq c+\varepsilon$ by strict monotonicity of G, so $p^f(w|\beta)>0$; and the concavity modulus from Assumption 1 is bounded away from zero on $[c+\varepsilon,y]$. The infimum over $w_{\text{in}}\in\mathcal{W}$ is achieved because $\xi(\omega)\leq\bar{\lambda}/(\delta\lambda^a)$ and W' are uniformly bounded.

Write $\pi_c(w,\beta)=\gamma[1-p^f(w|\beta)]V^f(\beta^+(w,\beta))$. By Lemma 9, $V^f\in C^{1,1}$ with $\|(V^f)'\|_{\text{Lip}}\leq C_0/(1-\gamma)$. Hence $(V^f)''$ exists a.e. and $|(V^f)''|\leq C_0/(1-\gamma)$ a.e. Differentiating π_c twice with respect to w by the product rule produces terms involving p^f_w, p^f_{ww}, V^f, $(V^f)'$, $(V^f)''$, β^+_w, and β^+_{ww}. We bound each: (1) $|p^f_w(w|\beta)|\leq\mu_{\max}$ where $\mu_{\max}:=\sup_{w,\omega}\mu(w|\omega)<\infty$; (2) $|p^f_{ww}(w|\beta)|$ is bounded by a constant depending on $\mu_{\max}$ and $y-c$; (3) $\|V^f\|_\infty\leq(y-c)/(1-\gamma)$ and $\|(V^f)'\|_\infty\leq(y-c)/(1-\gamma)$; (4) $|(V^f)''|\leq C_0/(1-\gamma)$ at Lebesgue-a.e. β; (5) $|\beta^+_w|$ and $|\beta^+_{ww}|$ are bounded because the posterior (5.3) is a smooth function of $p^f(w|\omega)$.

The full second derivative is:

$$(\pi_c)_{ww}=\gamma\Big[-p^f_{ww}\,V^f(\beta^+)-2p^f_w\,(V^f)'(\beta^+)\,\beta^+_w+(1-p^f)\Big((V^f)''(\beta^+)(\beta^+_w)^2+(V^f)'(\beta^+)\,\beta^+_{ww}\Big)\Big].$$

Each term carries the explicit factor γ, and the V^f-dependent factors are bounded by $O(1/(1-\gamma))$. Collecting the terms: $|(\pi_c)_{ww}(w,\beta)|\leq\frac{A\gamma}{1-\gamma}$.

Combining these factors. For $w\in[c+\varepsilon,y]$, the full second derivative satisfies

$$\Pi_{ww}(w,\beta)=\underbrace{(\pi_s)_{ww}(w,\beta)}_{\leq-\kappa_s<0}+\underbrace{(\pi_c)_{ww}(w,\beta)}_{|\cdot|\leq A\gamma/(1-\gamma)}.$$

For $\gamma<\bar{\gamma}$ where $\bar{\gamma}\in(0,1)$ is given by $\bar{\gamma}=\kappa_s/(\kappa_s+A)$, we have $\Pi_{ww}\leq-\kappa_s+A\gamma/(1-\gamma)<0$, establishing strict concavity of Π in w uniformly over $w_{\text{in}}\in\mathcal{W}$ and $\beta\in[0,1]$. ☐

Lemma 11. *Under Assumptions 1 and 2, for any $w_{\text{in}}\in\mathcal{W}$ and the corresponding aggregates, the best-response wage $w_{\text{out}}(\beta):=argmax_{w\in[c,y]}\Pi(w,\beta)$ is weakly increasing in β, provided $\gamma<\bar{\gamma}$ with the same threshold as in Lemma 10.*

Proof. We prove monotonicity by establishing $\Pi_{w\beta}\geq0$ at the best-response wage, then applying the implicit function theorem. Full-domain increasing differences are *not* needed: it suffices that the cross-partial is non-negative at the optimum.

Define, for each state ω,

$$f(w,\omega):=p^f(w|\omega)\Big[\mu(w|\omega)(y-w)-1\Big], \tag{C.15}$$

where $\mu(w|\omega):=r_G(w)+\xi(\omega)W'(w|\omega)>0$ as in Lemma 10. By (C.13), $f(w,\omega)=\frac{\partial}{\partial w}[p^f(w|\omega)(y-w)]$. The static component of Π_w is $(\pi_s)_w(w,\beta)=\beta\,f(w,h)+(1-\beta)\,f(w,l)$. The cross-partial of the static profit is

$$(\pi_s)_{w\beta}(w,\beta) = f(w,h) - f(w,l). \tag{C.16}$$

At any w^* where $(\pi_s)_w(w^*,\beta) = 0$, we have $\beta f(w^*,h) + (1-\beta) f(w^*,l) = 0$. Since $\beta \in (0,1)$, this means $f(w^*,h)$ and $f(w^*,l)$ have opposite signs. We now use Assumption 2 to determine which is positive.

By Lemma 5, $\xi(h) > \xi(l)$. Assumption (2) gives $\mu(w|h) > \mu(w|l)$ for all $w \in (c,y)$. At the static FOC, $\mu(w^*|\omega)(y-w^*) - 1$ has the same sign as $f(w^*,\omega)$. Because $\mu(w^*|h) > \mu(w^*|l)$: $\mu(w^*|h)(y - w^*) - 1 \geq \mu(w^*|l)(y-w^*) - 1$. Combined with the FOC condition $\beta f(h) + (1-\beta) f(l) = 0$, the ordering $\mu(h) \geq \mu(l)$ forces

$$f(w^*,h) \geq 0 \geq f(w^*,l). \tag{C.17}$$

To see this: suppose toward a contradiction that $f(w^*,h) < 0$. Then $\mu(w^*|h)(y-w^*) < 1$, and since $\mu(w^*|h) \geq \mu(w^*|l)$, also $\mu(w^*|l)(y-w^*) < 1$, so $f(w^*,l) < 0$. But then $\beta f(h) + (1-\beta) f(l) < 0$, contradicting the FOC. Similarly, $f(w^*,l) > 0$ leads to a contradiction.

From (C.17) and (C.16):

$$(\pi_s)_{w\beta}(w^*,\beta) = f(w^*,h) - f(w^*,l) \geq 0. \tag{C.18}$$

The inequality is strict whenever $\mu(w^*|h) > \mu(w^*|l)$.

The continuation term $\pi_c(w,\beta) = \gamma[1 - p^f(w|\beta)]V^f(\beta^+(w,\beta))$ contributes $(\pi_c)_{w\beta}$ to the cross-partial. By the same bounding argument as for $(\pi_c)_{ww}$ above, all terms in $(\pi_c)_{w\beta}$ carry at least one factor of γ, so $|(\pi_c)_{w\beta}| \leq B\gamma/(1-\gamma)$ for a constant B.

At the full FOC $\Pi_w(w^*,\beta) = 0$, the static FOC is perturbed: $(\pi_s)_w(w^*,\beta) = -(\pi_c)_w(w^*,\beta)$, where $|(\pi_c)_w| \leq B_0\gamma/(1-\gamma)$ with $B_0 > 0$. The static cross-partial at this perturbed point satisfies $(\pi_s)_{w\beta}(w^*,\beta) \geq \kappa_{ID} - B_1\gamma/(1-\gamma)$, where $B_1 > 0$ and $\kappa_{ID} > 0$ is a lower bound on (C.18) at exact static optima. The infimum κ_{ID} is strictly positive because $(\pi_s)_{w\beta} = f(w,h) - f(w,l) > 0$ at the static optimum by Assumption 2, and this lower bound is continuous on $[0,1] \times \mathcal{W}$.

Combining:

$$\Pi_{w\beta}(w^*,\beta) = \underbrace{(\pi_s)_{w\beta}}_{\geq \kappa_{ID} - B_1\gamma/(1-\gamma)} + \underbrace{(\pi_c)_{w\beta}}_{|\cdot| \leq B\gamma/(1-\gamma)} \geq \kappa_{ID} - \frac{(B+B_1)\gamma}{1-\gamma} > 0$$

for $\gamma < \bar{\gamma}$, where B_1 accounts for the perturbation of the FOC point and $\bar{\gamma}$ is adjusted from Lemma 10 to also ensure this inequality.

The argument above establishes that, after reducing $\bar{\gamma}$, there exists $\sigma > 0$ such that $\Pi_{w\beta}(w^*(\beta),\beta) \geq \sigma$ at every interior best-response wage $w^*(\beta)$, uniformly over $\beta \in [0,1]$ and $w_{\text{in}} \in \mathcal{W}$. Combined with strict concavity $\Pi_{ww} < 0$ from Lemma 10, the implicit function theorem at interior optima gives

$$\frac{dw_{\text{out}}}{d\beta} = -\frac{\Pi_{w\beta}(w^*(\beta),\beta)}{\Pi_{ww}(w^*(\beta),\beta)} \geq \frac{\sigma}{\sup_{w_{\text{in}},w,\beta}|\Pi_{ww}|} > 0,$$

so the best-response wage is strictly increasing in beliefs at interior optima with a uniform positive lower slope. $\square$

Lemma 12. *Suppose Π is strictly concave in w (Lemma 10) and has increasing differences in (w, β) (Lemma 11). For each $w_{\text{in}} \in \mathcal{W}$, define $w_{\text{out}}(\beta) := \mathit{argmax}_{w \in [c,y]} \Pi(w, \beta; w_{\text{in}})$. Then: (a) $w_{\text{out}}(\beta)$ exists and is unique for each $\beta \in [0, 1]$. (b) w_{out} is weakly increasing on $[0, 1]$. (c) w_{out} is continuous on $[0, 1]$. (d) There exists a constant $L > 0$, such that $|w_{\text{out}}(\beta') - w_{\text{out}}(\beta)| \leq L|\beta' - \beta|$ for all $\beta, \beta' \in [0, 1]$ and all $w_{\text{in}} \in \mathcal{W}$.*

Proof. Part (a). For each β, $\Pi(\cdot, \beta; w_{\text{in}})$ is continuous on the compact set $[c, y]$. By the extreme value theorem, a maximizer exists. By strict concavity, the maximizer is unique.

Part (b). By Lemma 11, $\Pi_{w\beta}(w_{\text{out}}(\beta), \beta) \geq 0$ at every interior optimum. Combined with $\Pi_{ww} < 0$, the implicit function theorem gives $dw_{\text{out}}/d\beta = -\Pi_{w\beta}/\Pi_{ww} \geq 0$ at every interior β. At corner solutions ($w_{\text{out}} = c$ or $w_{\text{out}} = y$), monotonicity holds trivially. Hence w_{out} is weakly increasing on $[0, 1]$.

Part (c). Let $\beta_n \to \beta$ in $[0, 1]$. Since $w_{\text{out}}(\beta_n) \in [c, y]$ for all n, every subsequence has a further convergent sub-subsequence $w_{\text{out}}(\beta_{n_k}) \to \bar{w} \in [c, y]$. For each k, optimality gives $\Pi(w_{\text{out}}(\beta_{n_k}), \beta_{n_k}) \geq \Pi(w, \beta_{n_k})$ for all $w \in [c, y]$. By the continuity of Π in both arguments, passing to the limit: $\Pi(\bar{w}, \beta) \geq \Pi(w, \beta)$ for all w. By uniqueness (Part (a)), $\bar{w} = w_{\text{out}}(\beta)$. Since every convergent subsequence has the same limit, $w_{\text{out}}(\beta_n) \to w_{\text{out}}(\beta)$.

Part (d): By Lemma 10, $\Pi(\cdot, \beta; w_{\text{in}})$ is strictly concave on $[c, y]$ with a quantitative modulus: for all $w_1, w_2 \in [c + \varepsilon, y]$ and all $\beta \in [0, 1]$,

$$\Pi(w_2, \beta) \ \leq \ \Pi(w_1, \beta) + \Pi_w^+(w_1, \beta)(w_2 - w_1) - \frac{\kappa}{2}(w_2 - w_1)^2, \tag{C.19}$$

where $\kappa := \kappa_s - A\gamma/(1 - \gamma) > 0$, and Π_w^+ denotes the right-derivative in w, which exists by concavity. In particular, at any maximizer $w^*(\beta) \in [c + \varepsilon, y]$:

$$\Pi(w, \beta) \ \leq \ \Pi(w^*(\beta), \beta) - \frac{\kappa}{2}\Big(w - w^*(\beta)\Big)^2 \qquad \forall\, w \in [c + \varepsilon, y]. \tag{C.20}$$

By Lemma 11 and the quantitative bound from the increasing-differences analysis, the payoff Π has *quantitative increasing differences* in (w, β): there exists $\sigma > 0$ such that for all $w_2 > w_1$ in $[c + \varepsilon, y]$ and $\beta_2 > \beta_1$ in $[0, 1]$:

$$\Big[\Pi(w_2, \beta_2) - \Pi(w_1, \beta_2)\Big] - \Big[\Pi(w_2, \beta_1) - \Pi(w_1, \beta_1)\Big] \ \geq \ \sigma\,(w_2 - w_1)(\beta_2 - \beta_1). \tag{C.21}$$

This follows because the left side equals $\int_{w_1}^{w_2} \int_{\beta_1}^{\beta_2} \Pi_{w\beta}\, d\beta\, dw$ when the cross-partial exists, and Lemma 3 guarantees that W is atomless, so the cross-partial is well-defined throughout.

Bounded cross-partial. By the uniform derivative bounds in Lemmas 10–11, the cross-partial is bounded: $|\Pi_{w\beta}(w, \beta)| \leq M < \infty$ for all $w \in [c + \varepsilon, y]$ and $\beta \in [0, 1]$. Integrating over the rectangle and using W atomless (Lemma 3),

$$\left|\Big[\Pi(w_2,\beta_2)-\Pi(w_1,\beta_2)\Big]-\Big[\Pi(w_2,\beta_1)-\Pi(w_1,\beta_1)\Big]\right| \le M\,(w_2-w_1)(\beta_2-\beta_1). \tag{C.22}$$

Fix $\beta_1<\beta_2$. By Part (b), $w_1:=w_{\text{out}}(\beta_1)\le w_2:=w_{\text{out}}(\beta_2)$. By optimality at β_2 and strong concavity (C.20): $\Pi(w_1,\beta_2)\le\Pi(w_2,\beta_2)-\frac{\kappa}{2}(w_2-w_1)^2$. By optimality at β_1: $\Pi(w_1,\beta_1)\ge\Pi(w_2,\beta_1)$. By the bounded-cross-partial inequality (C.22), the cross-difference is bounded *above* by $M(w_2-w_1)(\beta_2-\beta_1)$, so $\Big[\Pi(w_2,\beta_2)-\Pi(w_1,\beta_2)\Big]\le\Big[\Pi(w_2,\beta_1)-\Pi(w_1,\beta_1)\Big]+M(w_2-w_1)(\beta_2-\beta_1)$. Combining with the strong-concavity lower bound at β_2 and optimality at β_1:

$$\frac{\kappa}{2}(w_2-w_1)^2 \le \Pi(w_2,\beta_2)-\Pi(w_1,\beta_2) \le \underbrace{\Pi(w_2,\beta_1)-\Pi(w_1,\beta_1)}_{\le 0}+M(w_2-w_1)(\beta_2-\beta_1).$$

Hence $(\kappa/2)(w_2-w_1)^2\le M(w_2-w_1)(\beta_2-\beta_1)$, giving

$$w_{\text{out}}(\beta_2)-w_{\text{out}}(\beta_1) \le \frac{2M}{\kappa}(\beta_2-\beta_1) =: L\,(\beta_2-\beta_1). \tag{C.23}$$

□

Lemma 13. *The best-response operator $T:\mathcal{W}\to C([0,1])$ defined by $T(w_{\text{in}}):=w_{\text{out}}$ maps $\mathcal{W}$ into itself: $T(\mathcal{W})\subset\mathcal{W}$.*

Proof. Fix $w_{\text{in}}\in\mathcal{W}$ and let $w_{\text{out}}:=T(w_{\text{in}})$. By Lemma 12: (i) $w_{\text{out}}\in C([0,1])$. (ii) $w_{\text{out}}(\beta)\in[c,y]$ for all β. (iii) $w_{\text{out}}(\beta')-w_{\text{out}}(\beta)\le L(\beta'-\beta)$ for all $0\le\beta<\beta'\le 1$. By Lemma 2, the same best response also satisfies the lower Lipschitz bound $\ell(\beta'-\beta)\le w_{\text{out}}(\beta')-w_{\text{out}}(\beta)$. Hence $w_{\text{out}}\in\mathcal{W}$. □

Lemma 14. *The operator $T:\mathcal{W}\to\mathcal{W}$ is continuous in the sup-norm topology.*

Proof. Let $\{w_n\}\subset\mathcal{W}$ with $\|w_n-w\|_\infty\to 0$. We must show $\|T(w_n)-T(w)\|_\infty\to 0$.

By Lemma 6, as $w_n\to w$ uniformly: $\Lambda_n^f(\omega)\to\Lambda^f(\omega)$, $\xi_n(\omega)\to\xi(\omega)$, and $\Gamma_n^f\to\Gamma^f$ uniformly (in the sup-norm on CDFs). By Lemma 3, all the $W_n(\cdot|\omega)$ and the limit $W(\cdot|\omega)$ are atomless, so $W_n\to W$ uniformly (uniform limit of continuous CDFs is continuous, and Pólya's theorem upgrades pointwise convergence to uniform). Hence $p_n^f(w|\omega)\to p^f(w|\omega)$ uniformly in w, and $\beta_n^+\to\beta^+$ uniformly. By Lemma 7, $V_n^f\to V^f$ uniformly. Therefore the objective $\Pi_n(w,\beta;V_n^f)$ converges to $\Pi(w,\beta;V^f)$ *uniformly* on $[c,y]\times[0,1]$. Define the correspondence $\Phi(\beta):=argmax_{w\in[c,y]}\Pi(w,\beta;V^f)$. By Lemma 10, Π is strictly concave in w, so $\Phi(\beta)=\{T(w)(\beta)\}$ is a singleton, and $T(w)$ is the unique optimal selection. Similarly $T(w_n)(\beta)$ is the unique selection from the n-th argmax correspondence Φ_n. By Berge's maximum theorem applied to the uniformly convergent objective $\Pi_n\to\Pi$ on the compact constraint set $[c,y]$: the value functions $(\mathcal{B}_nV_n^f)(\beta)=\max_w\Pi_n(w,\beta;V_n^f)$ converge to $(\mathcal{B}V^f)(\beta)$ uniformly in β, and the argmax correspondences are upper hemicontinuous in n. Combined with the single-valuedness of Φ, we get pointwise convergence $T(w_n)(\beta)\to T(w)(\beta)$ for every β. The functions $T(w_n)$ and $T(w)$ both lie in $\mathcal{W}$, hence are L-Lipschitz. For any $\epsilon > 0$, cover $[0,1]$ by finitely many intervals of length $\epsilon/(3L)$. On each interval, the functions vary by at most $\epsilon/3$. At the interval endpoints, pointwise convergence gives $|T(w_n)(\beta_j) - T(w)(\beta_j)| < \epsilon/3$ for n sufficiently large. By the triangle inequality, $\|T(w_n) - T(w)\|_\infty < \epsilon$ for n sufficiently large. Hence $T(w_n) \to T(w)$ uniformly.

By Lemma 1, $\mathcal{W}$ is non-empty, compact, and convex in $C([0,1])$. By Lemma 13, $T : \mathcal{W} \to \mathcal{W}$. By Lemma 14, T is continuous. Applying Schauder's theorem, there exists $w^* \in \mathcal{W}$ such that $T(w^*) = w^*$. This fixed point defines the equilibrium in Definition 1. □

Lemma 15. *At any equilibrium fixed point $w^* = T(w^*)$ under Assumptions (1)–(2): (a) w^* is strictly increasing on $(0,1)$. (b) $\Gamma^f(\cdot|\omega)$ is atomless. (c) $W(w|\omega)$ is atomless, and the hiring rate is exactly given by formula (5.1) in equilibrium.*

Proof. Part (a): We show that w^* is strictly increasing on $(0,1)$ via the cross-partial argument. Suppose, for contradiction, that $w^*(\beta_1) = w^*(\beta_2) =: \bar{w}$ for some $0 < \beta_1 < \beta_2 < 1$. By Lemma 10, $\bar{w} \geq c + \epsilon_0$ uniformly in β, so $\bar{w}$ is in the strict-concavity region $[c+\epsilon_0, y]$.

By the strong-concavity inequality applied to $\Pi(\cdot, \beta_i)$ at the maximizer $\bar{w}$, for any $w' \in [c+\epsilon_0, y]$ with $w' \neq \bar{w}$:

$$\Pi(w', \beta_i) \;\leq\; \Pi(\bar{w}, \beta_i) \;-\; \frac{\kappa}{2}(w' - \bar{w})^2 \qquad i = 1, 2. \tag{C.24}$$

By the strict version of Assumption (2),

$$[\Pi(w', \beta_2) - \Pi(\bar{w}, \beta_2)] - [\Pi(w', \beta_1) - \Pi(\bar{w}, \beta_1)] \;\geq\; \sigma\,(w' - \bar{w})\,(\beta_2 - \beta_1). \tag{C.25}$$

Optimality of $\bar{w}$ at β_1 gives $\Pi(w', \beta_1) - \Pi(\bar{w}, \beta_1) \leq 0$, so by strict concavity (C.24), $\Pi(w', \beta_1) - \Pi(\bar{w}, \beta_1) \leq -\frac{\kappa}{2}(w' - \bar{w})^2$. Combining with (C.25): $\Pi(w', \beta_2) - \Pi(\bar{w}, \beta_2) \;\geq\; -\frac{\kappa}{2}(w' - \bar{w})^2 + \sigma(w' - \bar{w})(\beta_2 - \beta_1)$. For $w' = \bar{w} + \eta$ with $\eta > 0$ chosen small enough that $\sigma(\beta_2 - \beta_1) > \frac{\kappa}{2}\eta$, the right-hand side is strictly positive, contradicting optimality of $\bar{w}$ at β_2. Hence $w^*(\beta_1) \neq w^*(\beta_2)$, and combined with weak monotonicity from Lemma 12(b), $w^*(\beta_2) > w^*(\beta_1)$.

Part (b): The flow-balance gives $\Gamma^*(\cdot|\omega) = \alpha_\omega H(\cdot|\omega) + (1-\alpha_\omega) R[\Gamma^*](\cdot|\omega)$, where $\alpha_\omega := \lambda^f(\omega)/\Lambda^f(\omega) > 0$ and $R[\Gamma^*]$ is the normalized survivor CDF. The entry component $\alpha_\omega H$ is atomless since $h > 0$ is a continuous density by MLRP. An atom of Γ^* at β_0 must therefore come from the survivor component.

Define $\varphi(\beta) := \beta^+(\beta, w^*(\beta))$. By Part (a), w^* is strictly increasing. By the structure of $\tilde{\beta}$, the map $\beta \mapsto \tilde{\beta}(\beta, w^*(\beta))$ is also strictly increasing: $\partial\tilde{\beta}/\partial\beta = \eta/[\beta\eta + (1-\beta)]^2 > 0$ at fixed w, and the chain through $w^*(\beta)$ contributes a non-negative term $(\partial\tilde{\beta}/\partial w) \cdot (dw^*/d\beta)$, where the factor $dw^*/d\beta \geq 0$ by Part (a) and $\partial\tilde{\beta}/\partial w \geq 0$ at non-atom wages. Hence $\tilde{\beta} \circ w^*$ is strictly increasing. Since $\beta^+ = (p_{hh} + p_{ll} - 1)\tilde{\beta} + (1 - p_{ll})$ is an increasing affine function of $\tilde{\beta}$, φ is strictly increasing and continuous.

In particular, φ is a strictly monotone, continuous bijection from $(0,1)$ onto its image, and its pre-image $\varphi^{-1}(\beta_0)$ of any point $\beta_0 \in \varphi((0,1))$ is a *single point* β_0'.

Suppose Γ^* has an atom of mass $a_0 > 0$ at some $\beta_0 \in (0,1)$. The flow-balance restricted to $\{\beta_0\}$ gives

$$a_0 \;=\; \Gamma^*(\{\beta_0\}) \;\le\; (1-\alpha_\omega)\int_{\varphi^{-1}(\beta_0)}(1-\delta)\Big(1-p^f(w^*(\beta')|\omega)\Big)\,d\Gamma^*(\beta') \;\le\; (1-\alpha_\omega)(1-\delta)\,\Gamma^*(\{\beta_0'\}).$$

Hence $\Gamma^*(\{\beta_0'\}) \ge a_0/[(1-\alpha_\omega)(1-\delta)]$. Since $\alpha_\omega > 0$ and $\delta > 0$ imply $(1-\alpha_\omega)(1-\delta) < 1$, the multiplier is strictly greater than 1, so $\Gamma^*(\{\beta_0'\}) > a_0$. Iterating: $\Gamma^*(\{\varphi^{-n}(\beta_0)\}) \ge a_0/[(1-\alpha_\omega)(1-\delta)]^n$, which diverges to ∞ as $n \to \infty$. But Γ^* is a probability measure on $[0,1]$, so $\Gamma^*(\{\beta\}) \le 1$ for every β, a contradiction.

If at some iterate the pre-image $\varphi^{-n}(\beta_0)$ exits the domain $(0,1)$ where φ^{-1} is defined, the argument terminates with $\Gamma^*(\{\varphi^{-n+1}(\beta_0)\}) \le 1$ as a finite upper bound; choosing n such that $a_0 \cdot [(1-\alpha_\omega)(1-\delta)]^{-n} > 1$ produces a contradiction at that iterate. Hence Γ^* has no atoms in $(0,1)$. At the boundary points $\{0,1\}$, the entry term $H(\cdot|\omega)$ has zero mass at $\{0,1\}$, and the survivor term cannot create boundary atoms because $\beta^+(\beta,w) \in [1-p_{ll}, p_{hh}] \subset (0,1)$ for w in the wage support.

Part (c): Since w^* is strictly increasing and continuous (Part (a) and Lemma 12(c)), and Γ^* is atomless (Part (b)), the wage distribution $W(w|\omega) = \Gamma^*(w^{*-1}(w)|\omega)$ is atomless. □

C.3 Preliminary Lemma for Propositions 2–3

The following lemma on hiring probability bounds is used in the proofs of Propositions 2–3.

Lemma 16. *Consider a market with tightness $\xi = \Lambda^f/\lambda^a$, wage strategy $w^*(\cdot) \in \mathcal{W}$, steady-state belief CDF $\Gamma^f(\cdot|\omega)$, and signal CDF $H(\cdot|\omega)$. Condition on state $\omega = h$. Then: (i) The average hiring probability satisfies*

$$\bar{p}^f(h) \;\le\; \phi(\xi(h)) \;:=\; \frac{1-e^{-\xi(h)}}{\xi(h)}. \tag{C.26}$$

In particular, $\bar{p}^f(h) \to 0$ as $\lambda^f(h) \to \infty$. (ii) For any $\epsilon > 0$, the truncated hiring probability $\bar{p}^\epsilon(h) := \sup_{\beta' \in [0,1-\epsilon]} p^f\Big(w^*(\beta') \mid h\Big)$ satisfies

$$\bar{p}^\epsilon(h) \;\le\; \exp\!\Big(-\frac{\lambda^f(h)}{\lambda^a}\Big(1-H(1-\epsilon \mid h)\Big)\Big) \;\xrightarrow{\lambda^f(h)\to\infty}\; 0. \tag{C.27}$$

Proof. Part (i). Since $G(w) \le 1$, we have $p^f(w \mid h) \le e^{-\xi(h)(1-W(w|h))}$. Averaging over the wage distribution and substituting $u = W(w \mid h)$: $\bar{p}^f(h) \;\le\; \int_0^1 e^{-\xi(h)(1-u)}\,du \;=\; \frac{1-e^{-\xi(h)}}{\xi(h)}$. From the stock equation, $\Lambda^f(h) = \lambda^f(h)/[\delta + (1-\delta)\bar{p}^f(h)] \ge \lambda^f(h)$, hence $\xi(h) = \Lambda^f(h)/\lambda^a \ge \lambda^f(h)/\lambda^a \to \infty$, and $\phi(\xi(h)) \to 0$.

Part (ii). For any $\beta' \le 1-\epsilon$, the wage $w^*(\beta')$ satisfies $W(w^*(\beta') \mid h) = \Gamma^f(\beta' \mid h) \le \Gamma^f(1-\epsilon \mid h)$. Hence

$$p^f\Big(w^*(\beta') \mid h\Big) \;\le\; \exp\!\Big(-\xi(h)\,[1-\Gamma^f(1-\epsilon \mid h)]\Big). \tag{C.28}$$

We lower-bound the tail mass $1-\Gamma^f(1-\epsilon\mid h)$ using the flow-balance equation (B.5) evaluated at $\beta=1-\epsilon$. With $p_{hh}=1$, only state-h predecessors contribute:

$$\Lambda^f(h)\Big[1-\Gamma^f(1-\epsilon\mid h)\Big]=\lambda^f(h)\Big[1-H(1-\epsilon\mid h)\Big] \\ +(1-\delta)\int_{\{\beta':\beta^+(\beta')>1-\epsilon\}}\Big(1-p^f(w^*(\beta')\mid h)\Big)\,\Lambda^f(h)\,d\Gamma^f(\beta'\mid h).$$

The second term is non-negative. Hence $1-\Gamma^f(1-\epsilon\mid h)\ \geq\ \frac{\lambda^f(h)}{\Lambda^f(h)}\Big(1-H(1-\epsilon\mid h)\Big)$. Multiplying by $\xi(h)=\Lambda^f(h)/\lambda^a$: $\xi(h)\Big[1-\Gamma^f(1-\epsilon\mid h)\Big]\ \geq\ \frac{\lambda^f(h)}{\lambda^a}\Big(1-H(1-\epsilon\mid h)\Big)$. Since $H(1-\epsilon\mid h)<1$ (by full support of H), the right side diverges as $\lambda^f(h)\to\infty$. Substituting into (C.28) yields (C.27). $\square$

C.4 Proofs of Propositions 2 and 4

Consider a single market where both groups of firms match in the same pool. The total firm stock is $\Lambda^f(\omega):=\Lambda^f_I(\omega)+\Lambda^f_U(\omega)$, market tightness is $\xi(\omega):=\Lambda^f(\omega)/\lambda^a$, and the aggregate wage CDF is

$$W(w\mid\omega)\ =\ \frac{\Lambda^f_I(\omega)\,W_I(w\mid\omega)+\Lambda^f_U(\omega)\,W_U(w\mid\omega)}{\Lambda^f(\omega)},\tag{C.29}$$

where $W_g(w\mid\omega):=\Gamma^f_g(w^{*-1}(w)\mid\omega)$ is the group-g wage CDF. Trading probabilities (5.1) use the *common* $\xi(\omega)$ and $W(w\mid\omega)$.

A firm of either type with belief β solves $V^f(\beta)\ =\ \max_{w\in[c,y]}\Pi(w,\beta)$, where Π uses the common aggregates (ξ,W). Since the Bellman equation does not depend on the firm's group identity g and only on its belief β, the optimal wage function $w^*(\beta)$ and value function $V^f(\beta)$ are common to both groups.

The firm stock of group g satisfies:

$$\Lambda^f_g(h)\ =\ \frac{\mu_g\,\lambda^f(h)}{\delta+(1-\delta)\,\bar p^f_g(h)},\tag{C.30}$$

where $\mu_I:=\mu$, $\mu_U:=1-\mu$, and $\bar p^f_g(h):=\int p^f(w^*(\beta)\mid h)\,d\Gamma^f_g(\beta\mid h)$ is the group-g average hiring probability. The belief CDF satisfies

$$\Gamma^f_g(\beta\mid h)=\frac{\mu_g\,\lambda^f(h)}{\Lambda^f_g(h)}\,H_g(\beta\mid h)+(1-\delta)\int_{\{\beta':\beta^+(\beta')\leq\beta\}}\Big(1-p^f(w^*(\beta')\mid h)\Big)\,d\Gamma^f_g(\beta'\mid h),\tag{C.31}$$

where β^+ and p^f use the common aggregates.

Lemma 17. *Define the aggregate belief CDF* $\Gamma^f(\beta\mid h):=[\Lambda^f_I(h)\,\Gamma^f_I(\beta\mid h)+\Lambda^f_U(h)\,\Gamma^f_U(\beta\mid h)]/\Lambda^f(h)$ *and the* mixture entrant CDF

$$H_\mu(\beta\mid h)\ :=\ \mu\,H_I(\beta\mid h)+(1-\mu)\,H_U(\beta\mid h).\tag{C.32}$$

Then $\Gamma^f(\cdot \mid h)$ satisfies the same flow-balance equation as a homogeneous market with entrant distribution H_μ:

$$\Lambda^f(h)\,\Gamma^f(\beta \mid h) \;=\; \lambda^f(h)\,H_\mu(\beta \mid h) + (1-\delta)\int_{\{\beta':\beta^+(\beta')\le\beta\}}\Big(1-p^f(w^*(\beta') \mid h)\Big)\,\Lambda^f(h)\,d\Gamma^f(\beta' \mid h). \tag{C.33}$$

Consequently, the aggregate equilibrium of the heterogeneous market—i.e., the aggregate belief CDF Γ^f, the tightness ξ, the wage distribution W, and the common wage and value functions w^*, V^f—coincides with the equilibrium of a homogeneous market with signal distribution H_μ.

Proof. Sum the group-specific flow-balance equations (C.31) over $g \in \{I,U\}$, noting that w^*, β^+, and p^f are common:

$$\begin{aligned}\sum_g \Lambda^f_g(h)\,\Gamma^f_g(\beta \mid h) = &\sum_g \mu_g\,\lambda^f(h)\,H_g(\beta \mid h)\\ &+ (1-\delta)\int_{\{\beta':\beta^+\le\beta\}}\Big(1-p^f(w^*(\beta') \mid h)\Big)\sum_g \Lambda^f_g(h)\,d\Gamma^f_g(\beta' \mid h).\end{aligned}$$

The left side is $\Lambda^f(h)\,\Gamma^f(\beta \mid h)$. The first term on the right is $\lambda^f(h)[\mu H_I + (1-\mu)H_U] = \lambda^f(h)\,H_\mu(\beta \mid h)$. The integrand on the right uses $\sum_g \Lambda^f_g\,d\Gamma^f_g = \Lambda^f\,d\Gamma^f$ (by definition of Γ^f). This gives (C.33).

The aggregate stock satisfies $\Lambda^f(h) = \lambda^f(h)/[\delta + (1-\delta)\bar p^f(h)]$ where $\bar p^f(h) = \int p^f(w^*(\beta) \mid h)\,d\Gamma^f(\beta \mid h)$ is the aggregate hiring probability, and the aggregate wage CDF is $W(w \mid h) = \Gamma^f(w^{*-1}(w) \mid h)$. Since w^* and V^f solve the Bellman equation with these aggregates, the quadruple (Γ^f, ξ, W, w^*) constitutes an equilibrium for a homogeneous market with entrant distribution H_μ. In particular, the aggregate objects of the heterogeneous market satisfy the same fixed-point equations as a homogeneous market with signal H_μ. By uniqueness of the steady-state belief distributions (Lemma 4) and firm stocks (Lemma 5) for any given wage strategy, any equilibrium of the homogeneous market with the same wage function w^* must produce the same aggregates. □

The belief comparison for groups within the same market uses the same upper/lower bound technique as for separate markets, but is simpler because both groups face common aggregates.

Lemma 18. *Under the FOSD assumption on signal distributions, $H_I(\beta \mid h) \le H_U(\beta \mid h)$ for all $\beta \in [0,1]$, with strict inequality on an open subset.*

Proof. The initial belief $\beta_0(s) = \pi(h)\,h(s \mid h)/[\pi(h)\,h(s \mid h) + \pi(l)\,h(s \mid l)]$ is strictly increasing in s by the strict MLRP of the signal density. Since β_0 is increasing and $S_I \mid h \succ_{\text{FOSD}} S_U \mid h$, the composition theorem gives $\beta_0(S_I) \mid h \succ_{\text{FOSD}} \beta_0(S_U) \mid h$, i.e., $H_I(\beta \mid h) \le H_U(\beta \mid h)$ for all β, with strict inequality wherever the signal FOSD is strict. □

The following technical lemma establishes that, for sufficiently large $\lambda^f(h)$, informed firms within the shared market have stochastically higher steady-state beliefs on a compact sub-interval bounded away from the endpoints of $[0,1]$.

Lemma 19. *Under the hypotheses of Proposition 2, for any $\epsilon \in (0,1/2)$ and any $\underline{\beta} \in (0,\epsilon)$ such that the strict FOSD inequality in Lemma 18 holds on a subset of $[\underline{\beta}, 1-\epsilon]$ of positive measure, there exists $\lambda^f_* = \lambda^f_*(\epsilon,\underline{\beta}) > 0$ such that for all $\lambda^f(h) > \lambda^f_*$: $\Gamma^f_I(\beta \mid h) < \Gamma^f_U(\beta \mid h)$, $\forall \beta \in [\underline{\beta}, 1-\epsilon]$.*

Proof. The flow-balance (C.31) for each group g has entrant term $T_g(\beta) := [\mu_g \lambda^f(h)/\Lambda^f_g(h)]\, H_g(\beta \mid h)$ and identical survivor dynamics (common w^*, β^+, p^f).

Upper bound for group I:

$$\Gamma^f_I(\beta \mid h) \;\le\; \frac{1}{\delta} T_I(\beta) \;=\; \frac{1}{\delta}\, \frac{\mu \lambda^f(h)}{\Lambda^f_I(h)}\, H_I(\beta \mid h). \tag{C.34}$$

Refined lower bound for group U: Fix $\beta \le 1-\epsilon$. Every predecessor belief β' with $\beta^+(\beta') \le \beta$ satisfies $\beta' \le 1-\epsilon$. Using $1 - p^f(w^*(\beta') \mid h) \ge 1 - \bar{p}^\epsilon_U(h)$ (the truncated hiring probability bound from Lemma 16(ii)):

$$\Gamma^f_U(\beta \mid h) \;\ge\; \frac{T_U(\beta)}{\delta + (1-\delta)\,\bar{p}^\epsilon_U(h)} \;=\; \frac{(1-\mu)\,\lambda^f(h)/\Lambda^f_U(h)}{\delta + (1-\delta)\,\bar{p}^\epsilon_U(h)}\, H_U(\beta \mid h). \tag{C.35}$$

From (C.30), $\mu_g \lambda^f(h)/\Lambda^f_g(h) = \delta + (1-\delta)\bar{p}^f_g(h)$. Define

$$\rho_\mu \;:=\; \max_{\beta \in [\underline{\beta}, 1-\epsilon]} \frac{T_I(\beta)}{T_U(\beta)} \;=\; \frac{\delta + (1-\delta)\bar{p}^f_I(h)}{\delta + (1-\delta)\bar{p}^f_U(h)} \cdot \max_{\beta \in [\underline{\beta}, 1-\epsilon]} \frac{H_I(\beta \mid h)}{H_U(\beta \mid h)}.$$

By Lemma 16 (applied with common aggregates), $\bar{p}^f_g(h) \to 0$ as $\lambda^f(h) \to \infty$. Since $\max_\beta H_I/H_U < 1$ on $[\underline{\beta}, 1-\epsilon]$ (by Lemma 18), we have $\rho_\mu \to \bar{r} < 1$. Choose $\lambda^f(h)$ large enough that $\rho_\mu < 1$ and $\frac{\delta}{\delta + (1-\delta)\bar{p}^\epsilon_U(h)} > \rho_\mu$. Then the chain of inequalities yields:

$$\begin{aligned}
\Gamma^f_I(\beta \mid h) \;&\le\; \frac{1}{\delta} T_I(\beta) && \text{(C.36)}\\
&\le\; \frac{\rho_\mu}{\delta} T_U(\beta) && \text{(C.37)}\\
&<\; \frac{1}{\delta + (1-\delta)\,\bar{p}^\epsilon_U(h)} T_U(\beta) && \text{(C.38)}\\
&\le\; \Gamma^f_U(\beta \mid h). && \text{(C.39)}
\end{aligned}$$

The first line is (C.34); the second uses $T_I \le \rho_\mu T_U$; the third uses the condition above; the fourth is (C.35). In particular, $\Gamma^f_I(\beta \mid h) < \Gamma^f_U(\beta \mid h)$ for all $\beta \in [\underline{\beta}, 1-\epsilon]$. □

C.4.1 Proof of Proposition 2

Proof. Fix $\epsilon \in (0,1/2)$ and $\underline{\beta} \in (0,\epsilon)$ such that the strict FOSD inequality in Lemma 18 holds on a subset of $[\underline{\beta}, 1-\epsilon]$ of positive measure. Let $\bar{\lambda}^f := \lambda^f_*(\epsilon, \underline{\beta})$ from Lemma 19, possibly enlarged so that the tail contributions below are controlled. Fix $\lambda^f(h) > \bar{\lambda}^f$. By Lemma 19, $\Gamma^f_I(\beta \mid h) < \Gamma^f_U(\beta \mid h)$

on $[\underline{\beta}, 1-\epsilon]$.

Part (a): Average wages. Since both groups use the *same* optimal wage function $w^*(\beta)$, which is increasing in β (Lemma 11):

$$E_I[w \mid h] = \int_0^1 w^*(\beta)\, d\Gamma_I^f(\beta \mid h), \qquad E_U[w \mid h] = \int_0^1 w^*(\beta)\, d\Gamma_U^f(\beta \mid h).$$

By Lemma 19, $\Gamma_I^f(\cdot \mid h)$ FOSD-dominates $\Gamma_U^f(\cdot \mid h)$ on $[\underline{\beta}, 1-\epsilon]$. Since w^* is increasing (and continuous), the standard FOSD comparison theorem gives $\int_{[\underline{\beta},1-\epsilon]} w^*\, d\Gamma_I^f \geq \int_{[\underline{\beta},1-\epsilon]} w^*\, d\Gamma_U^f$, with strict inequality because the FOSD is strict on a set of positive measure. By (C.34) the tail mass on $[0, \underline{\beta})$ of either group satisfies $\Gamma_g^f(\underline{\beta}|h) \leq (1/\delta)(\mu_g \lambda^f(h)/\Lambda_g^f(h)) H_g(\underline{\beta}|h) \leq K_1 \cdot H(\underline{\beta}|h)$ for a constant K_1 uniform over g; similarly the tail mass on $(1-\epsilon, 1]$ is bounded by $K_2 \cdot (1 - H(1-\epsilon|h))$. Both tail bounds vanish as $\underline{\beta} \to 0$ and $\epsilon \to 0$ by continuity of $H(\cdot|h)$ with $H(0|h) = 0, H(1|h) = 1$. The interior gap $\Delta_{\text{int}} := \int_{[\underline{\beta},1-\epsilon]} w^*\, d(\Gamma_U^f - \Gamma_I^f) > 0$ is bounded below by a constant determined by the strict FOSD set in Lemma 18 and the slope of w^* from Lemma 2; in particular, Δ_{int} does not vanish as $\underline{\beta}, \epsilon \to 0$. Choosing $\underline{\beta}, \epsilon$ small enough that $y \cdot [K_1 H(\underline{\beta}|h) + K_2(1 - H(1-\epsilon|h))] < \Delta_{\text{int}}/2$, and enlarging $\bar{\lambda}^f$ so the interior FOSD continues to hold, yields $E_I[w \mid h] > E_U[w \mid h]$.

Part (b): Average hiring probability. The actual hiring probability at the equilibrium wage is $p^f(w^*(\beta) \mid h) = G(w^*(\beta))\, e^{-\xi(h)(1-W(w^*(\beta)|h))}$. This is increasing in β: $w^*(\beta)$ is increasing, G is increasing, and $W(w^*(\beta) \mid h) = \Gamma^f(\beta \mid h)$ is increasing (so $1 - W$ is decreasing and the exponential is increasing). By the same FOSD argument as Part (a):

$$\bar{p}_I^f(h) = \int p^f(w^*(\beta) \mid h)\, d\Gamma_I^f(\beta \mid h) \;>\; \int p^f(w^*(\beta) \mid h)\, d\Gamma_U^f(\beta \mid h) = \bar{p}_U^f(h).$$

□

C.4.2 Proof of Proposition 4

Proof. By the aggregation lemma (Lemma 17), the aggregate equilibrium of market μ coincides with the equilibrium of a homogeneous market whose entrant CDF is $H_\mu(\beta \mid h) = \mu\, H_I(\beta \mid h) + (1-\mu)\, H_U(\beta \mid h)$.

Step 1: FOSD ordering of entrant distributions. For $\mu^{IM} > \mu^{UM}$:

$$\begin{aligned} H_{IM}(\beta \mid h) - H_{UM}(\beta \mid h) &= (\mu^{IM} - \mu^{UM})\big[H_I(\beta \mid h) - H_U(\beta \mid h)\big] \\ &\leq 0 \qquad \forall \beta \in [0,1], \end{aligned}$$

with strict inequality on the open set where $H_I < H_U$. Hence $H_{IM}(\cdot \mid h) \succ_{\text{FOSD}} H_{UM}(\cdot \mid h)$.

Step 2: Belief-CDF FOSD ordering. Fix $\epsilon \in (0, 1/2)$ and $\underline{\beta} \in (0, \epsilon)$ such that the strict entrant FOSD inequality $H_{IM}(\beta \mid h) < H_{UM}(\beta \mid h)$ holds on a subset of $[\underline{\beta}, 1-\epsilon]$ of positive measure. Each market is a homogeneous-market equilibrium with its own entrant CDF H_g and its own aggregates $(\xi_g(h), W_g(\cdot|h), w_g^*(\cdot), \Gamma_g^f(\cdot|h))$ by Step 1. Applying the upper and lower bounds (C.34) and (C.35) separately within each market—with $T_g(\beta) := [\lambda^f(h)/\Lambda_g^f(h)]H_g(\beta \mid h)$ and the truncated hiring probability bound from Lemma 16(ii)—the argument of Lemma 19 carries through: there exists $\bar{\lambda}^{f'} = \bar{\lambda}^{f'}(\epsilon, \underline{\beta}) > 0$ such that for all $\lambda^f(h) > \bar{\lambda}^{f'}$, $\Gamma_{IM}^f(\beta \mid h) < \Gamma_{UM}^f(\beta \mid h), \forall \beta \in [\underline{\beta}, 1-\epsilon]$.

Step 3: Wage-function ordering via Assumption 3. The two markets share identical primitives apart from their entrant CDFs, so the standing parameter restrictions of Appendix C.2 hold uniformly across them. By Step 2, the belief CDFs satisfy $\Gamma_{IM}^f(\cdot \mid h) \leq \Gamma_{UM}^f(\cdot \mid h)$ on $[\underline{\beta}, 1-\epsilon]$, with strict inequality on a set of positive measure. Assumption 3 then yields the wage-function ordering $w_{IM}^*(\beta) \geq w_{UM}^*(\beta)\ \forall \beta \in [\underline{\beta}, 1-\epsilon]$.

Step 4: Wage-CDF FOSD. The wage CDF in each market is $W_g(w \mid h) = \Gamma_g^f(w_g^{*-1}(w) \mid h)$. From Step 3, $w_{IM}^* \geq w_{UM}^*$ on the interior interval, hence $w_{IM}^{*-1}(w) \leq w_{UM}^{*-1}(w)$ for w in the image of $[\underline{\beta}, 1-\epsilon]$ under w_{UM}^*. Combining with the strict belief FOSD from Step 2: $W_{IM}(w \mid h) = \Gamma_{IM}^f(w_{IM}^{*-1}(w) \mid h) \leq \Gamma_{IM}^f(w_{UM}^{*-1}(w) \mid h) < \Gamma_{UM}^f(w_{UM}^{*-1}(w) \mid h) = W_{UM}(w \mid h)$, so $W_{IM}(\cdot \mid h) \prec_{\text{FOSD}} W_{UM}(\cdot \mid h)$ strictly on the interior wage interval $[w_{UM}^*(\underline{\beta}), w_{UM}^*(1-\epsilon)]$.

Step 5: Tightness comparison. Define $\mathcal{P}(\xi, W) := \int_c^y G(w)e^{-\xi(1-W(w|h))}\,dW(w|h)$, which is the aggregate hiring probability in a market with tightness ξ and wage CDF W. In equilibrium each market satisfies $\xi_g(h) = \lambda^f(h)/[\lambda^a(\delta + (1-\delta)\mathcal{P}(\xi_g(h), W_g))]$. By Step 4, $W_{IM}(\cdot \mid h) < W_{UM}(\cdot \mid h)$ on the interior with positive-measure strict inequality, so for any common ξ, $\mathcal{P}(\xi, W_{IM}) > \mathcal{P}(\xi, W_{UM})$. Evaluating the IM fixed-point map at $\xi = \xi_{UM}(h)$:

$$\frac{\lambda^f(h)/\lambda^a}{\delta + (1-\delta)\mathcal{P}(\xi_{UM}(h), W_{IM})} < \frac{\lambda^f(h)/\lambda^a}{\delta + (1-\delta)\mathcal{P}(\xi_{UM}(h), W_{UM})} = \xi_{UM}(h).$$

By the uniqueness and strict monotonicity of the stock fixed-point map from Lemma 5—which holds under the contraction condition (C.7) maintained throughout the appendix—the fixed point $\xi_{IM}(h)$ of the IM map satisfies $\xi_{IM}(h) < \xi_{UM}(h)$.

Step 6: Hiring probability and average wages. Combining Steps 4 and 5:

$$\bar{p}_{IM}^f(h) = \mathcal{P}(\xi_{IM}(h), W_{IM}) > \mathcal{P}(\xi_{UM}(h), W_{IM}) > \mathcal{P}(\xi_{UM}(h), W_{UM}) = \bar{p}_{UM}^f(h),$$

where the first strict inequality uses $\xi_{IM}(h) < \xi_{UM}(h)$ with $\mathcal{P}$ strictly decreasing in ξ, and the second uses Step 4. For average wages, integration by parts gives $E_g[w \mid h] = y - \int_c^y W_g(w \mid h)\,dw$. By Step 4 the integrand on $[w_{UM}^*(\underline{\beta}), w_{UM}^*(1-\epsilon)]$ satisfies $W_{IM} < W_{UM}$ strictly, producing an interior gap $\Delta_{\text{int}} > 0$. The tail contributions on $[c, w_{UM}^*(\underline{\beta})]$ and $[w_{UM}^*(1-\epsilon), y]$ are each bounded by $(y-c)$ times the corresponding entrant tail mass $H_{UM}(\underline{\beta}|h) + (1 - H_{UM}(1-\epsilon|h))$, which tends to 0 as $\underline{\beta} \to 0$ and $\epsilon \to 0$. Choosing $\underline{\beta}, \epsilon$ small (and enlarging $\bar{\lambda}^{f'}$ accordingly so the interior FOSD continues to hold) yields $E_{IM}[w \mid h] > E_{UM}[w \mid h]$. □

C.5 Proof of Proposition 3

The state transition probability p_{hh} affects four objects: (1) The next-period belief after hiring failure is $\beta^+(\beta;p_{hh}) = p_{hh}\tilde{\beta}(\beta) + (1-p_{ll})(1-\tilde{\beta}(\beta))$, which is increasing in p_{hh}. (2) In steady state, $\pi(\omega^- = h \mid \omega = h) = p_{hh}$ and $\pi(\omega^- = l \mid \omega = h) = 1-p_{hh}$. Higher p_{hh} places more weight on state-h predecessors in the flow-balance equation. (3) $\pi_k(h) = (1-p_{ll})/(2-p^k_{hh}-p_{ll})$ is increasing in p^k_{hh}. (4) The stock equations (B.3) couple the two states:

$$\Lambda^f_k(h) = \lambda^f(h) + (1-\delta)\Big[p^k_{hh}\, S_k(h) + (1-p^k_{hh})\, S_k(l)\Big], \tag{C.40}$$

$$\Lambda^f_k(l) = \lambda^f(l) + (1-\delta)\Big[p_{ll}\, S_k(l) + (1-p_{ll})\, S_k(h)\Big], \tag{C.41}$$

where $S_k(\omega) := \Lambda^f_k(\omega)(1-\bar{p}^f_k(\omega))$ is the survivor mass.

Intuitively, higher state persistence raises the stakes of failure by making future periods more likely to mirror the current tightness. A failed hiring attempt signals high competition ($\omega = h$), updating the belief toward h. With higher p_{hh}, the next period is more likely to remain tight, lowering the continuation value after failure (as hiring tomorrow will be harder with a low wage). This incentivizes firms to offer higher wages today to secure a match and avoid prolonged search costs. Over time, this leads to a steady-state distribution of beliefs that, combined with aggressive bidding, results in higher average wages in market A.

The proof of Proposition 3 proceeds through five lemmas.

Lemma 20. *Fix any $p_{ll} \in (0,1)$, any wage strategy $w(\cdot) \in \mathcal{W}$, and any aggregates $(\xi(\omega), W(\cdot|\omega))$ satisfying the* hiring difficulty ordering*:*

$$\xi(h)\Big(1-W(w \mid h)\Big) \;\geq\; \xi(l)\Big(1-W(w \mid l)\Big) \qquad \forall\, w \in [c,y], \tag{C.42}$$

with strict inequality for w in the interior of the wage support. Let $\tilde{\beta}(\beta)$ denote the within-period posterior about $\omega = h$ after hiring failure:

$$\tilde{\beta}(\beta) \;=\; \frac{\beta\,[1-p^f(w(\beta) \mid h)]}{\beta\,[1-p^f(w(\beta) \mid h)] + (1-\beta)\,[1-p^f(w(\beta) \mid l)]}.$$

Then: (i) $\tilde{\beta}(\beta) > \beta$ for all $\beta \in (0,1)$; (ii) The next-period belief $\beta^+(\beta;p_{hh}) := p_{hh}\tilde{\beta}(\beta) + (1-p_{ll})(1-\tilde{\beta}(\beta))$ is strictly increasing in p_{hh}:

$$\frac{\partial \beta^+}{\partial p_{hh}} \;=\; \tilde{\beta}(\beta) \;>\; 0 \qquad \forall\, \beta \in (0,1). \tag{C.43}$$

(iii) For $p^A_{hh} > p^B_{hh}$: $\beta^+_A(\beta) > \beta^+_B(\beta)$ and $(\beta^+_A)^{-1}(\beta) < (\beta^+_B)^{-1}(\beta)$ for all β in the interior of the range of β^+.

Proof. Part (i). Define $\eta(\beta) := [1-p^f(w(\beta)|h)]/[1-p^f(w(\beta)|l)]$. By (C.42),

$p^f(w|h) = G(w)\,e^{-\xi(h)(1-W(w|h))} \le G(w)\,e^{-\xi(l)(1-W(w|l))} = p^f(w|l)$ for every w, so $\eta(\beta) \ge 1$, with $\eta > 1$ whenever the inequality in (C.42) is strict (which holds for w in the interior of the wage support). Then $\tilde\beta = \frac{\beta\eta}{\beta\eta+(1-\beta)}$ and $\tilde\beta - \beta = \frac{\beta(1-\beta)(\eta-1)}{\beta\eta+(1-\beta)} > 0$. Moreover, $\tilde\beta$ is strictly increasing in β: since $\tilde\beta = \beta\eta/(\beta\eta+1-\beta)$ with $\eta > 1$ fixed, differentiating gives $\partial\tilde\beta/\partial\beta = \eta/[\beta\eta+(1-\beta)]^2 > 0$.

Part (ii). By direct differentiation: $\beta^+ = p_{hh}\tilde\beta + (1-p_{ll})(1-\tilde\beta) = (p_{hh}+p_{ll}-1)\tilde\beta + (1-p_{ll})$. Hence $\partial\beta^+/\partial p_{hh} = \tilde\beta > 0$.

Part (iii). The pointwise inequality $\beta^+_A > \beta^+_B$ is immediate from Part (ii). Since β^+ is strictly increasing in β along the equilibrium wage path by the maintained composite-update monotonicity condition, the generalized inverse $(\beta^+)^{-1}$ is well-defined and decreasing in p_{hh}. □

Lemma 21. *Under the setting of Proposition 3 (i.e., $p^k_{hh} \in (1-p_{ll}, 1)$, $p_{ll} \in (0,1)$ fixed), if $\lambda^f(h) \to \infty$ with $\lambda^f(l)$ fixed: (i) $\Lambda^f_k(h) \sim C^k_h\,\lambda^f(h)$ and $\Lambda^f_k(l) \sim C^k_l\,\lambda^f(h)$, where, with $\rho_\infty := (1-\delta)(1-p_{ll})/(1-(1-\delta)p_{ll})$ and $D_k := 1-(1-\delta)[\rho_\infty + p^k_{hh}(1-\rho_\infty)]$,*

$$C^k_h := \frac{1}{D_k} + o(1), \qquad C^k_l := \rho_\infty\,C^k_h + o(1).$$

(ii) When $\xi_k(h)$ and $\xi_k(l)$ tend to infinity, and the ratio $\xi_k(h)/\xi_k(l) \to C^k_h/C^k_l = 1/\rho_\infty > 1$. (iii) Consequently both $\bar p^f_k(h) \to 0$ and $\bar p^f_k(l) \to 0$, and the state-l survivor contribution to the state-h flow-balance, $(1-p^k_{hh})\Lambda^f_k(l)/\Lambda^f_k(h)$, converges to the finite constant $(1-p^k_{hh})\,C^k_l/C^k_h$, which is bounded away from 1.

Proof. From the stock equations (C.40)–(C.41) with $S_k(\omega) = \Lambda^f_k(\omega)(1-\bar p^f_k(\omega))$,

$$\Lambda^f_k(h)\Big[1-(1-\delta)p^k_{hh}(1-\bar p^f_k(h))\Big] = \lambda^f(h) + (1-\delta)(1-p^k_{hh})\,\Lambda^f_k(l)(1-\bar p^f_k(l)),$$
$$\Lambda^f_k(l)\Big[1-(1-\delta)p_{ll}(1-\bar p^f_k(l))\Big] = \lambda^f(l) + (1-\delta)(1-p_{ll})\,\Lambda^f_k(h)(1-\bar p^f_k(h)).$$

Lemma 16(i) applied to state h gives $\bar p^f_k(h) \le \phi(\xi_k(h))$, and since $\xi_k(h) \ge \Lambda^f_k(h)/\lambda^a \ge \lambda^f(h)/\lambda^a \to \infty$, we have $\bar p^f_k(h) \to 0$. Substituting and dividing both equations by $\lambda^f(h)$:

$$\frac{\Lambda^f_k(h)}{\lambda^f(h)}[1-(1-\delta)p^k_{hh}+o(1)] = 1 + (1-\delta)(1-p^k_{hh})\,\frac{\Lambda^f_k(l)}{\lambda^f(h)}\,(1-\bar p^f_k(l)),$$
$$\frac{\Lambda^f_k(l)}{\lambda^f(h)}[1-(1-\delta)p_{ll}(1-\bar p^f_k(l))] = o(1) + (1-\delta)(1-p_{ll})\,\frac{\Lambda^f_k(h)}{\lambda^f(h)}\,(1-\bar p^f_k(h)).$$

The second equation gives $\Lambda^f_k(l)/\lambda^f(h) \le [(1-\delta)(1-p_{ll})\cdot\Lambda^f_k(h)/\lambda^f(h) + o(1)]/[1-(1-\delta)p_{ll} + o(1)]$; substituting into the first yields a 2×2 linear system with positive solution. Letting $a := \Lambda^f_k(h)/\lambda^f(h)$ and $b := \Lambda^f_k(l)/\lambda^f(h)$, the limit system is

$$a\Big(1-(1-\delta)p^k_{hh}\Big) = 1 + (1-\delta)(1-p^k_{hh})\,b,$$
$$b\Big(1-(1-\delta)p_{ll}\Big) = (1-\delta)(1-p_{ll})\,a.$$

The second gives $b = [(1-\delta)(1-p_{ll})/(1-(1-\delta)p_{ll})]\,a$. Substituting:

$$a\Big(1-(1-\delta)p^k_{hh}\Big) - (1-\delta)(1-p^k_{hh})\,\frac{(1-\delta)(1-p_{ll})}{1-(1-\delta)p_{ll}}\,a = 1.$$

The bracketed coefficient simplifies (using $p^k_{hh} + p_{ll} > 1$) to a strictly positive constant; explicit computation gives the stated C^k_h and C^k_l to leading order. Then $\bar{p}^f_k(l) \le \phi(\xi_k(l)) \to 0$ since $\xi_k(l) = \Lambda^f_k(l)/\lambda^a \to \infty$, and the $o(1)$ corrections in the system go to zero, establishing parts (i)–(iii). The strict ordering $C^k_h > C^k_l$ follows from $C^k_l = \rho_\infty C^k_h$ with $\rho_\infty < 1$ for $\delta > 0$. Each quantity above converges to its displayed limit as $\lambda^f(h)$ grows, and the limiting orderings are strict. Hence there is a finite threshold $\bar{\lambda}^{f''}$ such that these orderings already hold at finite $\lambda^f(h)$ whenever $\lambda^f(h) > \bar{\lambda}^{f''}$. □

Lemma 22. *For any $\epsilon > 0$ and $\underline{\beta} > 0$, there exists $\bar{\lambda}^{f''} = \bar{\lambda}^{f''}(\epsilon, \underline{\beta}) > 0$ such that for all $\lambda^f(h) > \bar{\lambda}^{f''}$: $\Gamma^f_A(\beta \mid h) \;<\; \Gamma^f_B(\beta \mid h), \forall \beta \in [\underline{\beta}, 1-\epsilon]$.*

Proof. The flow-balance equation for market k in state h reads, after dividing by $\Lambda^f_k(h)$:

$$\Gamma^f_k(\beta|h) = \underbrace{\frac{\lambda^f(h)}{\Lambda^f_k(h)}\,H(\beta|h)}_{=:T_k(\beta)} + (1-\delta)\Big[p^k_{hh}\,J^h_k(\beta) + (1-p^k_{hh})\,\frac{\Lambda^f_k(l)}{\Lambda^f_k(h)}\,J^l_k(\beta)\Big], \tag{C.44}$$

where, for $\omega \in \{h, l\}$, $J^\omega_k(\beta) := \int_{\{\beta' : \beta^+_k(\beta') \le \beta\}} \Big(1 - p^f(w^*_k(\beta')|\omega)\Big)\, d\Gamma^f_k(\beta'|\omega)$.

Upper bound for market A. Bounding $1 - p^f \le 1$ and the integral by $\Gamma^f_k(\beta|h)$:

$$\Gamma^f_A(\beta|h) \;\le\; \frac{1}{\delta}\,T_A(\beta) \;=\; \frac{1}{\delta}\,\frac{\lambda^f(h)}{\Lambda^f_A(h)}\,H(\beta|h). \tag{C.45}$$

Refined lower bound for market B. We isolate the h-state survivor term, which dominates for large $\lambda^f(h)$. For $\beta \le 1-\epsilon$, every predecessor β' with $\beta^+_B(\beta') \le \beta$ satisfies $\beta' \le 1-\epsilon$ (since $\beta^+_B \ge \beta'$ on $(0,1)$). Using $1 - p^f(w^*_B(\beta')|h) \ge 1 - \bar{p}^\epsilon_B(h)$: $J^h_B(\beta) \;\ge\; \Big(1 - \bar{p}^\epsilon_B(h)\Big)\,\Gamma^f_B\Big((\beta^+_B)^{-1}(\beta) \mid h\Big)$. Using the cruder bound and iterating:

$$\Gamma^f_B(\beta|h) \;\ge\; \frac{T_B(\beta)}{\delta + (1-\delta)\bar{p}^\epsilon_B(h)}. \tag{C.46}$$

Entrant term ratio. Since both markets share the same H: $\frac{T_A(\beta)}{T_B(\beta)} = \frac{\Lambda^f_B(h)}{\Lambda^f_A(h)} = \frac{\delta + (1-\delta)\bar{p}^f_A(h)}{\delta + (1-\delta)\bar{p}^f_B(h)} =: \Theta_3$. By Lemma 16, $\bar{p}^f_k(h) \to 0$, so $\Theta_3 \to 1$.

We need

$$\frac{1}{\delta}\,T_A(\beta) \;<\; \frac{T_B(\beta)}{\delta + (1-\delta)\bar{p}^\epsilon_B(h)}. \tag{C.47}$$

Substituting $T_A = \Theta_3 T_B$:

$$\Theta_3 \; < \; \frac{\delta}{\delta + (1-\delta)\bar{p}^{\epsilon}_B(h)}. \tag{C.48}$$

Since $\Theta_3 \to 1$ and $\bar{p}^{\epsilon}_B(h) \to 0$, the right side tends to 1 as well.

Two-state asymptotic reduction. By Lemma 21, the cross-state survivor contribution does not vanish: $\Lambda^f_k(l)/\Lambda^f_k(h) \to C^k_l/C^k_h = \rho_\infty := (1-\delta)(1-p_{ll})/(1-(1-\delta)p_{ll})$, a finite positive constant that does *not* depend on p^k_{hh}. Hence both markets share the same asymptotic stock ratio, and the cross-state survivor flow contributes to leading order. When $\bar{p}^f_k(\omega) \to 0$ in both states and using $\lambda^f(h)/\Lambda^f_k(h) \to 1/C^k_h$, $\lambda^f(l)/\Lambda^f_k(l) \to 0$, the asymptotic joint belief equations become

$$F^k_h(\beta) \; = \; \frac{1}{C^k_h} H(\beta|h) \; + \; a^k F^k_h\left(\bar{\beta}^k\right) \; + \; b^k F^k_l\left(\bar{\beta}^k\right), \tag{C.49}$$

$$F^k_l(\beta) \; = \; c\, F^k_h\left(\bar{\beta}^k\right) \; + \; d\, F^k_l\left(\bar{\beta}^k\right), \tag{C.50}$$

where $a^k = (1-\delta)p^k_{hh}$, $b^k = (1-\delta)(1-p^k_{hh})\rho_\infty$, $c = 1-(1-\delta)p_{ll}$, $d = (1-\delta)p_{ll}$, and $\bar{\beta}^k := (\beta^+_k)^{-1}(\beta)$. Direct computation gives $1/C^k_h = 1-(1-\delta)[p^k_{hh}(1-\rho_\infty)+\rho_\infty]$, which is *decreasing* in p^k_{hh}. The joint operator $\mathcal{G}^k = (\mathcal{G}^k_h, \mathcal{G}^k_l)$ mapping (F_h, F_l) to the right-hand sides of (C.49)–(C.50) is non-expansive in the sup norm: its row sums are $a^k + b^k = (1-\delta)[p^k_{hh}(1-\rho_\infty)+\rho_\infty] < 1$ in the h-equation and $c + d = 1$ in the l-equation, the latter reflecting that state l has no entry term in the asymptotic limit. The borderline l-row is eliminated next. Substituting (C.50) recursively into (C.49) yields a closed equation for F^k_h alone with contraction modulus $a^k + b^k c/(1-d)$, which is bounded above by a constant strictly less than 1 under the standing parameter restrictions.

Joint-operator sign analysis. For the comparative-statics argument that follows, we analyze the joint operator $\mathcal{G}^k = (\mathcal{G}^k_h, \mathcal{G}^k_l)$ from (C.49)–(C.50). At any common input (F_h, F_l), the cross-market operator difference admits the decomposition

$$\begin{aligned}
\mathcal{G}^B_h(F_h, F_l)(\beta) - \mathcal{G}^A_h(F_h, F_l)(\beta) = \; & \underbrace{(1/C^B_h - 1/C^A_h) H(\beta|h)}_{>0,\text{ entry channel}} \\
& + \underbrace{(1-\delta)(p^B_{hh} - p^A_{hh})[F_h(\bar{\beta}^B) - \rho_\infty F_l(\bar{\beta}^B)]}_{<0 \text{ when } F_h > \rho_\infty F_l,\text{ weight-shift channel}} \\
& + \underbrace{a^A[F_h(\bar{\beta}^B) - F_h(\bar{\beta}^A)] + b^A[F_l(\bar{\beta}^B) - F_l(\bar{\beta}^A)]}_{>0,\text{ pre-image channel}}.
\end{aligned}$$

The entry channel contributes positively because $1/C^B_h > 1/C^A_h$ (entry weight decreases with persistence, derived above). The pre-image channel contributes positively because $\bar{\beta}^B > \bar{\beta}^A$ (Lemma 20). The weight-shift channel has ambiguous sign: in steady state we expect $F_l > F_h$ pointwise (state l has more mass at low beliefs), so when $\rho_\infty < F_h/F_l$—which holds when ρ_∞ is sufficiently small—the bracket $F_h - \rho_\infty F_l > 0$ and the weight-shift channel contributes *negatively*. For $\mathcal{G}^B_h - \mathcal{G}^A_h > 0$ to hold uniformly on the comparison interval—which is what monotone iteration requires to deliver $F_h^A \le F_h^B$—the entry and pre-image channels must dominate the weight-shift channel. For $\mathcal{G}_h^B - \mathcal{G}_h^A > 0$ we therefore require the entrant inflow to be concentrated enough on the interior that

$$(1-\rho_\infty)(C_h^B)^2\underline{H} > \sup_{\beta\in[\underline{\beta},1-\epsilon]}\Big[F_h^B(\bar{\beta}^B) - \rho_\infty F_l^B(\bar{\beta}^B)\Big], \tag{C.51}$$

where $\underline{H} > 0$ lower-bounds $H(\beta|h)$ for $\beta \ge \underline{\beta}$. This requirement reduces to a single-state interior-concentration condition on the entrant signal when $\rho_\infty \to 0$ (the absorbing-state limit $p_{ll} \to 1$). Under it, monotone iteration of $\mathcal{G}^k$ starting from a common seed preserves the FOSD direction $F_h^A \le F_h^B$ on $[\underline{\beta}, 1-\epsilon]$; the state-$l$ comparison $F_l^A \le F_l^B$ follows from (C.50) via the pre-image channel alone, without further conditions. Identifying $F_h^k(\beta)$ with the high-state belief CDF $\Gamma_k^f(\beta|h)$, this establishes the dominance $\Gamma_A^f(\beta|h) \le \Gamma_B^f(\beta|h)$ on $[\underline{\beta}, 1-\epsilon]$ asserted in the lemma, with strict inequality on a positive-measure subset where the entry channel is strict, for $\lambda^f(h)$ sufficiently large. □

Lemma 23. *Under Assumption (3), for $\lambda^f(h) > \bar{\lambda}^{f''}$ (Lemma 22), the equilibrium wage functions satisfy $w_A^*(\beta) \ge w_B^*(\beta)$ for all $\beta \in [\underline{\beta}, 1-\epsilon]$. Moreover, the equilibrium wage CDFs satisfy the* strict *FOSD inequality*

$$W_A(w \mid h) < W_B(w \mid h) \qquad \forall\, w \in \Big[w_B^*(\underline{\beta}),\, w_B^*(1-\epsilon)\Big]. \tag{C.52}$$

Proof. Weak inequality $w_A^* \ge w_B^*$. By the cross-market wage-monotonicity selection, since the equilibrium in market A has entrant CDF $H_A = H$ and state persistence p_{hh}^A, and market B has the same H but lower persistence $p_{hh}^B < p_{hh}^A$, the monotone comparative statics condition yields $w_A^*(\beta) \ge w_B^*(\beta)$ for $\beta \in [\underline{\beta}, 1-\epsilon]$ with Assumption 3.

Strictness. Even if $w_A^* = w_B^*$ on $[\underline{\beta}, 1-\epsilon]$, the wage *distribution* FOSD still holds strictly, because the strict belief FOSD from Lemma 22 is sufficient. To see this: with $w_A^* = w_B^* =: w^*$ on $[\underline{\beta}, 1-\epsilon]$, the wage CDF satisfies $W_k(w|h) = \Gamma_k^f(w^{*-1}(w)|h)$ for w in the image of $[\underline{\beta}, 1-\epsilon]$. By Lemma 22, $\Gamma_A^f(\beta|h) < \Gamma_B^f(\beta|h)$ strictly on $[\underline{\beta}, 1-\epsilon]$, so $W_A(w|h) = \Gamma_A^f(w^{*-1}(w)|h) < \Gamma_B^f(w^{*-1}(w)|h) = W_B(w|h)$ for w in the interior of the wage image. Note that this argument uses the belief FOSD directly and does *not* require the wage functions to differ pointwise—the strict FOSD of wages follows from the strict FOSD of beliefs through the common increasing wage function w^*.

More generally, when $w_A^* \ge w_B^*$ with at least weak inequality, the chain $W_A(w|h) = \Gamma_A^f(w_A^{*-1}(w)|h) \le \Gamma_A^f(w_B^{*-1}(w)|h) < \Gamma_B^f(w_B^{*-1}(w)|h) = W_B(w|h)$ holds for $w \in [w_B^*(\underline{\beta}), w_B^*(1-\epsilon)]$, using $w_A^{*-1} \le w_B^{*-1}$ (from $w_A^* \ge w_B^*$) and the strict FOSD of Γ. □

Lemma 24. *Under the hypotheses of Lemma 23, there exists $\bar{\lambda}^{f''} > 0$ such that for all $\lambda^f(h) > \bar{\lambda}^{f''}$ the tightness ordering $\xi_A(h) > \xi_B(h)$ holds; moreover, provided entrant signals are imperfect and the equilibrium wage schedule is sufficiently responsive to beliefs (the two requirements stated in the proof), $\bar{p}_A^f(h) > \bar{p}_B^f(h)$.*

Proof. Because $p_{hh}^k < 1$, the stocks are governed by the two-state coupled system, so we compare $\xi_A(h)$ and $\xi_B(h)$ using the asymptotic stock analysis of Lemma 21.

By Lemma 21, $\Lambda_k^f(h) \sim \lambda^f(h)/D_k$, so

$$\xi_k(h) \;=\; \frac{\lambda^f(h)/\lambda^a}{D_k}\,(1+o(1)), \qquad D_k := 1-(1-\delta)\Big[\rho_\infty + p_{hh}^k(1-\rho_\infty)\Big] \in (0,1),$$

with $\rho_\infty := (1-\delta)(1-p_{ll})/(1-(1-\delta)p_{ll})$. Since $1-\rho_\infty > 0$, D_k is decreasing in p_{hh}^k, so $\xi_A(h) > \xi_B(h)$ in the limit; because this ordering is strict, it already holds at every finite $\lambda^f(h) > \bar{\lambda}^{f''}$.

Hiring probability via the matching identity. We use the matching identity $\bar{p}_k^f(h) = \lambda^a(1-\Phi_k(h))/\Lambda_k^f(h)$. Here $\Phi_k(h) = \int_c^y e^{-\xi_k(h)(1-W_k(v|h))}\,dG(v)$ is the probability a random applicant receives no acceptable offer. As $\xi_k(h) \to \infty$, the integrand tends to 0 for every v with $W_k(v|h) < 1$ (i.e., $v < \bar{w}_k$) and to 1 for $v \geq \bar{w}_k$; by dominated convergence, $1-\Phi_k(h) \longrightarrow G(\bar{w}_k)$ where $\bar{w}_k = \sup\operatorname{supp} W_k(\cdot|h)$. Thus only applicants whose outside option lies below the highest posted wage can ever match; the wage channel does *not* disappear in the limit.

Combining with the stock limit $\Lambda_k^f(h) = \lambda^f(h)/D_k\cdot(1+o(1))$, $\bar{p}_k^f(h) \;=\; \frac{\lambda^a}{\lambda^f(h)}\,G(\bar{w}_k)\,D_k\,(1+o(1))$ hence $\frac{\bar{p}_A^f(h)}{\bar{p}_B^f(h)} \;\longrightarrow\; \frac{G(\bar{w}_A)\,D_A}{G(\bar{w}_B)\,D_B}$. The claim $\bar{p}_A^f(h) > \bar{p}_B^f(h)$ for large $\lambda^f(h)$ is therefore equivalent to $G(\bar{w}_A)D_A > G(\bar{w}_B)D_B$. The factor $D_A < D_B$ is the congestion penalty; $G(\bar{w}_A) \geq G(\bar{w}_B)$ is the wage gain.

Wage-support gap. Write $\bar{w}_k = \sup\operatorname{supp} W_k(\cdot|h) = w_k^*(\beta_k^{\sup})$ for the highest posted wage, where $\beta_k^{\sup}$ is the top of the steady-state belief support. Note that $\beta_k^{\sup}$ is determined by two things: first, entrants have beliefs up to the belief ceiling $B_0 := \sup_s \beta_0(s)$, which depends only on H and $(\lambda^f(h), \lambda^f(l))$ and is therefore common to both markets; second, beliefs of surviving firms evolve under the increasing map $\varphi_k(\beta) := \beta_k^+(w_k^*(\beta), \beta)$, whose iterates move monotonically toward its fixed point β_k^*. Since φ_k is increasing, $\sup_s \varphi_k^n(\beta_0(s)) = \varphi_k^n(B_0)$ for every n, so the top of the support is $\beta_k^{\sup} = \max\{B_0, \beta_k^*\}$. We therefore impose the entrant-ceiling condition $B_0 \leq \beta_B^*$, which holds whenever a single private signal is less informative than the structural persistence; under it the common ceiling does not bind and the belief gap is not truncated. The top of the support is the belief fixed point $\beta_k^{\sup} = \beta_k^*$ of β_k^+, which lies strictly below the persistence ceiling because mean reversion gives $\beta_k^+(p_{hh}^k) < p_{hh}^k$ and, in the congestion limit, equals the stationary prior $\beta_k^* \to \pi_k(h) = (1-p_{ll})/(2-p_{hh}^k-p_{ll})$, increasing in p_{hh}^k, so $\bar{w}_k = w_k^*(\beta_k^*)$. By Lemma 23, $w_A^* \geq w_B^*$ pointwise; by the lower Lipschitz bound ℓ on $w_B^* \in \mathcal{W}$ (Lemma 2),

$$\bar{w}_A - \bar{w}_B \;=\; w_A^*(\beta_A^*) - w_B^*(\beta_B^*) \;\geq\; w_B^*(\beta_A^*) - w_B^*(\beta_B^*) \;\geq\; \ell\,(\beta_A^* - \beta_B^*) \;>\; 0.$$

Hence, with $g_{\min} := \min_{[c,y]} g > 0$ and writing $\beta_A^* - \beta_B^* = \kappa_\pi\,(p_{hh}^A - p_{hh}^B)$ with $\kappa_\pi := (1-p_{ll})/[(2-p_{hh}^A-p_{ll})(2-p_{hh}^B-p_{ll})] > 0$, $G(\bar{w}_A) - G(\bar{w}_B) \geq g_{\min}\,\ell\,(\beta_A^*-\beta_B^*) = g_{\min}\,\ell\,\kappa_\pi\,(p_{hh}^A - p_{hh}^B)$.

Using $D_B - D_A = (1-\delta)(1-\rho_\infty)(p_{hh}^A - p_{hh}^B)$ and $G(\bar{w}_B) \leq 1$,

$$\begin{aligned} G(\bar{w}_A)D_A - G(\bar{w}_B)D_B &= [G(\bar{w}_A) - G(\bar{w}_B)]\,D_A - G(\bar{w}_B)\,(D_B - D_A) \\ &\geq (p_{hh}^A - p_{hh}^B)\Big[g_{\min}\,\ell\,\kappa_\pi\,D_A - (1-\delta)(1-\rho_\infty)\Big]. \end{aligned}$$

The persistence gap $p^A_{hh} - p^B_{hh} > 0$ factors out, so the sign turns on whether the wage schedule responds to beliefs steeply enough, with a dense enough applicant pool, to outweigh the congestion build-up: $g_{\min}\,\ell\,\kappa_\pi\,D_A \;>\; (1-\delta)(1-\rho_\infty)$. The left side measures the responsiveness of the highest posted wage to the persistence-induced rise in the top belief (steep schedule ℓ, dense pool $g_{\min}$); the right side is the rate at which persistence accumulates the firm stock. The inequality holds for a sufficiently responsive wage schedule and is independent of the persistence gap; under it the bracket is positive, so $G(\bar{w}_A)D_A > G(\bar{w}_B)D_B$ and $\bar{p}^f_A(h) > \bar{p}^f_B(h)$ for all $\lambda^f(h)$ large enough. □

C.5.1 Proof of Proposition 3

Proof. Fix $\epsilon \in (0,1/2)$ and $\underline{\beta} \in (0,\epsilon)$ with the FOSD strict inequality holding on a subset of $[\underline{\beta}, 1-\epsilon]$. Set $\bar{\lambda}^{f''} := \bar{\lambda}^{f''}(\epsilon, \underline{\beta})$. Fix $\lambda^f(h) > \bar{\lambda}^{f''}$.

Part (a): Average hiring probability. By Lemma 24, the asymptotic hiring-probability ratio satisfies $\bar{p}^f_A(h)/\bar{p}^f_B(h) \to G(\bar{w}_A)D_A/[G(\bar{w}_B)D_B] > 1$, so $\bar{p}^f_A(h) > \bar{p}^f_B(h)$ for all $\lambda^f(h) > \bar{\lambda}^{f''}$.

Part (b): Average wages. By integration by parts, $E_k[w|h] = y - \int_c^y W_k(w|h)\,dw$. By Lemma 23, $W_A(w|h) < W_B(w|h)$ on the interior interval $[w^*_B(\underline{\beta}), w^*_B(1-\epsilon)]$. The tail-control bookkeeping is the same as in Step 6 of the proof of Proposition 4. Hence $E_A[w|h] > E_B[w|h]$. □