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The Paradox of Inequality

*Oded Galor, Luca Pensieroso, Daniel C. Wainstock**

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Abstract

How could inequality between social classes have ever persisted during the Malthusian epoch, as affluence led to greater reproductive success, diluting inherited wealth and narrowing disparities over time? This paper uncovers the Paradox of Inequality, establishes its robust theoretical foundations, and provides its resolution. The analysis suggests that elites' lavish, nonproductive displays of status inadvertently diverted resources from reproduction, limiting the dilution of inherited wealth and sustaining inequality across generations. Ironically, extravagance preserved the wealth it seemed to squander.

Keywords: Inequality, Malthusian Epoch, Conspicuous consumption, Fertility, Paradox

JEL codes: O10, Z10

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1 Introduction

For most of human history, the course of development was governed by the iron law of the Malthusian trap. The reinforcing interplay between population growth and technological progress expanded productive capacity and sustained a larger population. Yet, despite these advances, the material well-being of humanity remained nearly unchanged. Technological progress and land expansion fueled rising birth rates and declining mortality, and inevitably brought about proportional increases in both population and resources, leading to stagnation in living standards over the long run (Malthus, 1798; Galor, 2011, 2022).¹

This powerful, well-established account of the economic ice age that spanned most of human existence conceals the striking yet overlooked Paradox of Inequality. Although income inequality between social classes was widespread and persistent across preindustrial agrarian societies, Malthusian population dynamics would have eroded these disparities over generations, rendering them unsustainable. How could inequality have ever persisted during this epoch, when affluence increased reproductive success, diluting inherited resources among a growing number of heirs and diminishing economic disparities over time? What latent force operated to offset this seemingly inevitable trend?

This paper uncovers the Paradox of Inequality, establishes its robust theoretical foundations, and provides its resolution. It advances the hypothesis that, perhaps paradoxically, the longstanding practice of conspicuous consumption among elites sustained inequality in the Malthusian epoch. In stratified societies, landed elites signaled status through lavish, visible, nonproductive expenditures, such as extravagant banquets, ornamental dress, monumental architecture, and elaborate burial rituals (Veblen, 1899). These nonproductive displays inadvertently diverted resources from reproduction, limiting the dilution of inherited wealth and sustaining inequality across generations. Ironically, a practice that might have hindered asset accumulation and prosperity in the modern era preserved elite economic dominance under Malthusian conditions. Extravagance preserved the wealth it seemed to squander.²

The theory yields two fundamental insights. First, it uncovers the Paradox of Inequality, establishing that income inequality between social classes would vanish asymptotically in a Malthusian environment because the greater affluence of landowners would inevitably generate greater reproductive success, diluting their wealth until incomes are equalized across social classes in the long run. This pattern need not depend on competitive factor pricing. It can also arise when the division of output reflects the power of elites to extract resources from the populations they govern. Second, as long as the wealthy are compelled to signal status through conspicuous consumption, prestige-

¹The validity of the Malthusian hypothesis is supported empirically by cross-country evidence (Ashraf & Galor, 2011), quantitative analysis of preindustrial Europe (Lagerlöf, 2019; Cervellati *et al.*, 2023), and evidence from a range of countries across the globe (e.g., Chen & Kung, 2016; Klemp & Møller, 2016).

²Primogeniture could have preserved the concentration of landownership in societies where this social norm prevailed. Yet it cannot account for the persistence of inequality across time and space in the preindustrial world, where more partible inheritance was widespread. In particular, of the 597 societies with a recorded rule for distributing inherited land in the Ethnographic Atlas, 307 practice equal or relatively equal division of land among heirs, while primogeniture is recorded in 248 (Murdock *et al.*, 1999).

oriented spending would divert resources from reproduction, curbing the expansion of elite families and limiting the dilution of inherited wealth. Paradoxically, this well-documented nonproductive expenditure preserves the concentration of wealth and sustains elite dominance across generations.³

2 The Paradox of Inequality

Consider a perfectly competitive overlapping-generations economy that captures the two defining characteristics of the Malthusian epoch: (i) a positive relationship between income and reproductive success, and (ii) a fixed factor of production, land, which gives rise to diminishing returns to labor.

The society consists of two social classes: landowners and workers. Individuals within and across classes are otherwise identical, and income differences arise from their distinct ownership of productive resources. At any time t , there are R_t landowners and L_t workers. Each landowner owns an equal share of the economy's total land endowment, X , and inelastically supplies X/R_t units of land. Children inherit their parental class, and each landowner divides their land equally among their children.^{4,5} Each worker supplies inelastically one unit of labor and total labor supply is L_t . In every period t , the economy produces a single homogeneous good using land and labor. The supply of land, X , is constant over time, whereas the numbers of workers and landowners in period $t + 1$ are determined by their respective population sizes and fertility choices in period t .

2.1 Production

Production occurs according to a constant-returns-to-scale technology. Output in period t , Y_t , is given by:

$$Y_t = (AX)^\alpha L_t^{1-\alpha}; \quad \alpha \in (0, 1), \quad (1)$$

where L_t and X are, respectively, labor and land employed in production in period t , and A measures the level of technology. Thus, AX represents effective land employed in production.

Output per worker, $y_t \equiv Y_t/L_t$, is therefore a strictly concave function of effective land per worker:

$$y_t = (AX/L_t)^\alpha. \quad (2)$$

Under perfect competition, the wage, w_t , and the rental rate on land, r_t , are therefore:

$$w_t = (1 - \alpha)Y_t/L_t; \quad r_t = \alpha Y_t/X. \quad (3)$$

³Appendix E establishes that inequality emerges and persists even when individuals initially have identical land and labor endowments and their predisposition toward conspicuous consumption has no systematic ex ante association with landownership. Differences in fertility generate unequal land inheritances, endogenously creating a persistent association between conspicuous consumption, larger landholdings, and higher incomes.

⁴Appendix B relaxes this assumption, permitting some children of landowners to be disinherited and enter the working class, and establishes that incomes remain equal across classes in the steady state.

⁵Appendix C establishes that income inequality dissipates even in the presence of a land market and endogenous bequests.

Total output produced at any time t , Y_t , is therefore divided between the earnings of workers, $w_t L_t = (1 - \alpha)Y_t$, and the rents received by landowners, $r_t X = \alpha Y_t$.⁶

2.2 Individuals

Individuals of both classes, workers, L , and landowners, R , live for two periods. In the first period of life, childhood, they are supported by their parents, whereas in the second period, adulthood, they inelastically supply their factor endowment and allocate the resulting income between consumption and child rearing.

Adults of class i in period t , $i \in \{L, R\}$, derive utility from their consumption, c_t^i , and the number of their children, n_t^i .^{7,8} In particular,

$$u_t^i = (1 - \gamma) \ln(c_t^i) + \gamma \ln(n_t^i), \quad \gamma \in (0, 1). \quad (4)$$

They allocate their income I_t^i between their own consumption, c_t^i , and expenditure on children, ρn_t^i , where ρ is the cost of raising a child, so as to maximize their utility function. Their budget constraint is therefore:

$$\rho n_t^i + c_t^i \leq I_t^i = \begin{cases} w_t & \text{if } i = L \\ r_t(X/R_t) & \text{if } i = R. \end{cases} \quad (5)$$

Given the utility function, adults of class i in period t , $i = L, R$, allocate a fraction $(1 - \gamma)$ of their income to consumption and a fraction γ to childrearing. Hence,

$$c_t^i = (1 - \gamma)I_t^i; \quad n_t^i = (\gamma/\rho)I_t^i, \quad (6)$$

where, in accordance with the Malthusian paradigm, income has a positive effect on the number of children of workers, n_t^L , and landowners, n_t^R .

2.3 Population Dynamics

At time 0, there are L_0 adult workers and R_0 adult landowners in the economy. Their initial population sizes satisfy

$$L_0 > [(1 - \alpha)/\alpha]R_0. \quad (7)$$

⁶As established in Appendix A, the qualitative prediction that inequality dissipates in the long run holds in a non-competitive economy in which the division of output between workers and landowners follows alternative allocation rules that permit varying degrees of worker exploitation by the landed elite.

⁷Alternative formulations incorporating a subsistence consumption constraint or a common child-survival rate would not affect the qualitative results (Galor, 2011; Dalgaard & Strulik, 2015).

⁸As established in Appendix D, the result extends to an economy in which individuals invest in their children's productive skills. When workers and landowners share the same preferences for child quality and the same skill-acquisition technology, the quantity-quality trade-off preserves the proportional relationship between income and fertility, and the income gap between the two classes vanishes in the long run.

Under perfect competition, the incomes of each landowner and each worker at time 0 are, respectively, $I_0^R = r_0 X / R_0 = \alpha Y_0 / R_0$ and $I_0^L = w_0 = (1 - \alpha) Y_0 / L_0$. Hence, it follows from (7) that

$$I_0^R > I_0^L, \quad (8)$$

and inequality prevails in the Malthusian economy in period 0.

2.3.1 Population Dynamics: Workers

The evolution of the working population is determined by the initial size of the working population and the number of children per worker. Specifically, the size of the working population in period $t + 1$, L_{t+1} , is:

$$L_{t+1} = n_t^L L_t. \quad (9)$$

Hence, it follows from (1), (3), (5), and (6) that the time path of the working population is governed by the following first-order difference equation:

$$L_{t+1} = (1 - \alpha)(\gamma/\rho)(AX)^\alpha L_t^{1-\alpha} \equiv \phi(L_t), \quad (10)$$

where $\phi'(L_t) > 0$ and $\phi''(L_t) < 0$. Hence, $\phi(L_t)$ is strictly concave in L_t , where $\phi(0) = 0$, $\lim_{L_t \rightarrow 0} \phi'(L_t) = \infty$, and $\lim_{L_t \rightarrow \infty} \phi'(L_t) = 0$.

Hence, as depicted in Figure 1, for a given amount of land X and a fixed level of technology A , if $L_0 < \bar{L}$, the size of the working population increases over time and converges from its initial level, L_0 , to a unique positive steady-state level $\bar{L}$:

$$\bar{L} = [(1 - \alpha)(\gamma/\rho)]^{1/\alpha} AX > 0. \quad (11)$$

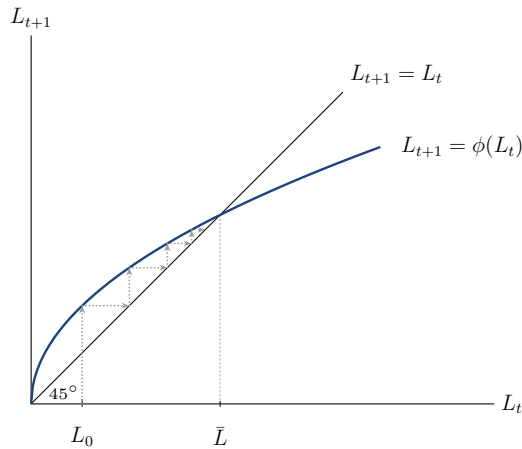


Figure 1: The Evolution of the Population of Workers

2.3.2 Population Dynamics: Landowners

The evolution of the landowning population is determined by its initial size and the number of children per landowner. Specifically, the size of the landowning population in period $t + 1$, R_{t+1} , is:

$$R_{t+1} = n_t^R R_t. \quad (12)$$

Hence, it follows from (1), (3), (5), and (6) that the time path of the landowning population is:

$$R_{t+1} = (\gamma/\rho)\alpha(AX)^\alpha L_t^{1-\alpha} \equiv R(L_t), \quad (13)$$

which is independent of R_t .^{9,10}

As the working population L_t increases monotonically toward its steady-state level $\bar{L}$, the landowning population increases monotonically toward its steady-state level $\bar{R}$:

$$\bar{R} = \alpha(1 - \alpha)^{\frac{1-\alpha}{\alpha}} (\gamma/\rho)^{\frac{1}{\alpha}} AX. \quad (14)$$

2.4 Earnings Dynamics

2.4.1 The Time Path of Output Per Worker

The evolution of output per worker is determined by its initial level and the number of children per worker. Noting (2) and (9), since $n_t^L = (\gamma/\rho)(1 - \alpha)y_t$, the time path of output per worker is governed by the first-order difference equation:

$$y_{t+1} = \left[\frac{AX}{L_{t+1}} \right]^\alpha = \left[\frac{AX}{n_t^L L_t} \right]^\alpha = \left[\frac{\rho}{(1 - \alpha)\gamma} \right]^\alpha y_t^{1-\alpha} \equiv \psi(y_t), \quad (15)$$

where $\psi(y_t)$ is strictly concave, $\psi(0) = 0$, $\lim_{y_t \rightarrow 0} \psi'(y_t) = \infty$, and $\lim_{y_t \rightarrow \infty} \psi'(y_t) = 0$.

Consequently, as depicted in Figure 2, for a given amount of land X and a fixed level of technology A , output per worker converges to a unique, stable steady-state level, $\bar{y}$:

$$\bar{y} = \rho/[(1 - \alpha)\gamma]. \quad (16)$$

⁹The size of the landowning population does not affect the total amount of land employed in production, the landowners' share of aggregate output, or landowners' aggregate income. Since each landowner devotes the same fraction of income to childrearing, total expenditure on children, and hence the total number of children born to landowners, is independent of the number of landowners. In contrast, a larger working population, L_t , raises the rental rate on land and therefore the income and fertility of each landowner.

¹⁰In the extension with downward social mobility in Appendix B, the landowning population affects fertility and hence the fraction of children who inherit, making R_{t+1} dependent on R_t without altering steady-state income equality.

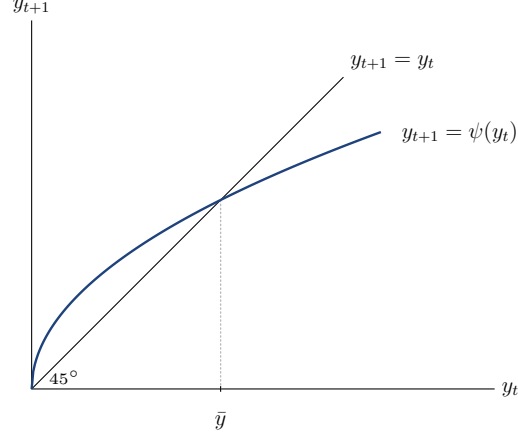


Figure 2: The Evolution of Output Per Worker

2.4.2 Vanishing Earnings Inequality between Social Classes

This section establishes that, regardless of the initial income gap between landowners and workers, this gap vanishes in the long run, as affluence generates greater reproductive success, diluting inherited wealth and narrowing disparities over time.

As the economy approaches the steady state, noting (11), (14), and (16), the steady-state income of each landowner, $\bar{I}^R$, is

$$\bar{I}^R = \bar{r}X/\bar{R} = \alpha(AX)^\alpha \bar{L}^{1-\alpha}/\bar{R} = \rho/\gamma, \quad (17)$$

whereas the steady-state income of each worker, $\bar{I}^L$, is:

$$\bar{I}^L = \bar{w} = (1 - \alpha)\bar{y} = \rho/\gamma. \quad (18)$$

Thus Proposition 1 follows:

Proposition 1 (The Paradox of Inequality). *For any initial earnings inequality between landowners and workers such that $I_0^R > I_0^L$, the income gap between the two classes vanishes in the long run:*

$$\lim_{t \rightarrow \infty} (I_t^R - I_t^L) = 0. \quad (19)$$

Proof: The proposition follows from (17) and (18). $\square$

Given the proportional relationship between income and fertility, the income gap between landowners and workers vanishes after one generation. In particular, as follows from (10) and (13), $R_{t+1}/L_{t+1} = \alpha/(1 - \alpha)$, and hence $I_{t+1}^R/I_{t+1}^L = [\alpha/(1 - \alpha)](L_{t+1}/R_{t+1}) = 1$. The earnings of landowners and workers remain equal and, if $L_0 < \bar{L}$, they jointly decline toward their common steady-state level.

These findings constitute the Paradox of Inequality. Landowners' initial affluence generates greater reproductive success, diluting their landholdings among their offspring and eliminating

their earnings advantage. What, then, explains the persistence of income disparities between social classes during this economic ice age?

3 Conspicuous Consumption and Persistent Inequality between Social Classes

This section establishes that conspicuous consumption can resolve the Paradox of Inequality. Elites' lavish, nonproductive displays of status inadvertently diverted resources from reproduction, limiting the dilution of inherited wealth and sustaining inequality across generations.¹¹

Consider the two-class overlapping-generations economy developed in Section 2, in which landowners derive utility from displaying their affluence, in addition to consumption and the number of their children. The production technology, factor endowments, and competitive allocation of output remain unchanged as specified in Section 2.

3.1 Individuals

Individuals, workers, L , and landowners, R , live for two periods. In the first period of life, childhood, they are supported by their parents, whereas in the second period, adulthood, they inelastically supply their factor endowment and allocate the resulting income among consumption, displays of affluence, and child rearing.

Adults of class i in period t , $i \in \{L, R\}$, derive utility from their consumption, c_t^i , the number of their children, n_t^i , and, depending on their predisposition, displays of affluence, x_t^i . Consumption, c_t^i , provides direct material benefits, whereas conspicuous expenditure, x_t^i , provides social recognition through visible displays of affluence. These expenditures therefore yield distinct benefits rather than entering utility as interchangeable forms of consumption. The utility derived from conspicuous expenditure captures, in reduced form, the value individuals attach to the recognition conferred by such displays. In particular,

$$u_t^i = (1 - \gamma) \ln c_t^i + (1 - \gamma)\theta^i \ln x_t^i + \gamma \ln n_t^i, \quad \gamma \in (0, 1), \quad (20)$$

where $\theta^i \in \{0, 1\}$ is the predisposition of individuals of class i towards the displays of affluence; $\theta^R = 1$ and $\theta^L = 0$.¹²

Adults of class i in period t allocate their income I_t^i among consumption, c_t^i , conspicuous consumption, x_t^i , and expenditure on children, ρn_t^i , where ρ is the cost of raising a child, so as to

¹¹Conspicuous consumption could also encompass lavish investment in children's nonproductive attributes that serve to display family wealth and status. If such expenditure is independent of the number of children, the formulation and results remain unchanged. If, instead, x_t^i represents expenditure per child, the budget constraint becomes $c_t^i + n_t^i(\rho + x_t^i) \leq I_t^i$. Under the parameter restriction ensuring an interior solution, the qualitative results remain unchanged.

¹²Appendix E establishes that inequality can emerge and persist even when individuals initially have identical land and labor endowments. An inherited predisposition toward conspicuous consumption lowers fertility, resulting in larger land inheritances per child and a persistent income advantage for their descendants.

maximize their utility function. Their budget constraint is therefore:

$$c_t^i + x_t^i + \rho n_t^i \leq I_t^i = \begin{cases} w_t & \text{if } i = L \\ r_t(X/R_t) & \text{if } i = R. \end{cases} \quad (21)$$

Given the homotheticity of the utility function, workers allocate a fraction $(1 - \gamma)$ of their income to consumption and a fraction γ to childrearing, as in Section 2. They do not value displays of affluence and choose $x_t^L = 0$. Hence,

$$c_t^L = (1 - \gamma)I_t^L; \quad n_t^L = (\gamma/\rho)I_t^L; \quad x_t^L = 0. \quad (22)$$

Landowners allocate a fraction $(1 - \gamma)/(2 - \gamma)$ of their income to consumption, the same fraction to displays of affluence, and a fraction $\gamma/(2 - \gamma)$ to childrearing. Hence,

$$c_t^R = \frac{1 - \gamma}{2 - \gamma}I_t^R; \quad x_t^R = \frac{1 - \gamma}{2 - \gamma}I_t^R; \quad n_t^R = \frac{\gamma}{\rho(2 - \gamma)}I_t^R. \quad (23)$$

Thus, conspicuous consumption rises with landowners' income. Income continues to have a positive effect on the number of children in both groups. In particular,

$$n_t^L = (\gamma/\rho)w_t; \quad n_t^R = \frac{\gamma}{\rho(2 - \gamma)}r_t(X/R_t). \quad (24)$$

3.2 Population Dynamics

At time 0, there are L_0 adult workers and R_0 adult landowners in the economy. Their initial population sizes satisfy

$$L_0 > [(1 - \alpha)/\alpha]R_0. \quad (25)$$

Under perfect competition, the incomes of each landowner and each worker at time 0 are, respectively, $I_0^R = r_0X/R_0 = \alpha Y_0/R_0$ and $I_0^L = w_0 = (1 - \alpha)Y_0/L_0$. Hence, it follows from (25) that

$$I_0^R > I_0^L, \quad (26)$$

and inequality prevails in the Malthusian economy in period 0.

3.2.1 Population Dynamics: Workers

The evolution of the working population is determined by the initial size of the working population and the number of children per worker. Specifically, the size of the working population in period $t + 1$, L_{t+1} , is:

$$L_{t+1} = n_t^L L_t. \quad (27)$$

Hence, the time path of the working population is governed by the following first-order difference equation:

$$L_{t+1} = (1 - \alpha)(\gamma/\rho)(AX)^\alpha L_t^{1-\alpha} \equiv \phi(L_t), \quad (28)$$

where $\phi'(L_t) > 0$ and $\phi''(L_t) < 0$. Hence, $\phi(L_t)$ is strictly concave in L_t , where $\phi(0) = 0$, $\lim_{L_t \rightarrow 0} \phi'(L_t) = \infty$, and $\lim_{L_t \rightarrow \infty} \phi'(L_t) = 0$.

For a given amount of land X and a fixed level of technology A , if $L_0 < \bar{L}$, the size of the working population increases over time and converges from its initial level, L_0 , to a unique positive steady-state level $\bar{L}$:

$$\bar{L} = [(1 - \alpha)(\gamma/\rho)]^{1/\alpha} AX > 0. \quad (29)$$

Thus, the evolution of the working population and its steady-state level are unaffected by landowners' conspicuous consumption.

3.2.2 Population Dynamics: Landowners

The evolution of the landowning population is determined by its initial size and the number of children per landowner. Specifically, the size of the landowning population in period $t + 1$, R_{t+1} , is:

$$R_{t+1} = n_t^R R_t. \quad (30)$$

Hence, it follows from (1), (3), (21), and (23) that the time path of the landowning population is:

$$R_{t+1} = \frac{\alpha\gamma}{\rho(2 - \gamma)} (AX)^\alpha L_t^{1-\alpha} \equiv R(L_t), \quad (31)$$

which is independent of R_t . As in Section 2, the number of landowners does not affect their aggregate income and aggregate fertility. Since each landowner devotes the same fraction of income to childrearing, the total number of children born to landowners is independent of the number of landowners. Conspicuous consumption, however, reduces this fraction of income devoted to fertility from γ to $\gamma/(2 - \gamma)$.

As the working population L_t converges to its steady-state level $\bar{L}$, the landowning population converges to its steady-state level $\bar{R}$:

$$\bar{R} = \frac{\alpha(1 - \alpha)^{\frac{1-\alpha}{\alpha}} (\gamma/\rho)^{\frac{1}{\alpha}} AX}{2 - \gamma}. \quad (32)$$

Thus, the steady-state landowning population is smaller than in Section 2, preserving larger landholdings per landowner.¹³

3.3 Earnings Dynamics

3.3.1 The Time Path of Output Per Worker

The evolution of output per worker is determined by its initial level and the number of children per worker. Since $n_t^L = (\gamma/\rho)(1 - \alpha)y_t$, the time path of output per worker, as was the case in

¹³Unlike in Section 2, initial earnings inequality in favor of landowners does not by itself guarantee that $R_1 > R_0$, since landowners devote a smaller share of income to childrearing. Their population increases initially if $I_0^R > \rho(2 - \gamma)/\gamma$.

Section 2, is governed by the first-order difference equation:

$$y_{t+1} = \left[\frac{\rho}{(1-\alpha)\gamma} \right]^\alpha y_t^{1-\alpha} \equiv \psi(y_t), \quad (33)$$

where $\psi(y_t)$ is strictly concave, $\psi(0) = 0$, $\lim_{y_t \rightarrow 0} \psi'(y_t) = \infty$, and $\lim_{y_t \rightarrow \infty} \psi'(y_t) = 0$. Consequently, for a given amount of land X and a fixed level of technology A , output per worker converges to a unique, stable steady-state level, $\bar{y} = \rho/[(1-\alpha)\gamma]$. Thus, the evolution of output per worker reflects Malthusian dynamics and its steady-state level is unaffected by landowners' conspicuous consumption.

3.3.2 Persistent Earnings Inequality between Social Classes

This section establishes that, regardless of the initial income gap between landowners and workers, conspicuous consumption sustains the income advantage of landowners in the long run by diverting resources from reproduction and limiting the dilution of inherited wealth.

The steady-state income of each landowner, $\bar{I}^R$, is

$$\bar{I}^R = \bar{r}X/\bar{R} = \alpha(AX)^\alpha \bar{L}^{1-\alpha}/\bar{R} = \rho(2-\gamma)/\gamma, \quad (34)$$

whereas the steady-state income of each worker, $\bar{I}^L$, is:

$$\bar{I}^L = \bar{w} = (1-\alpha)\bar{y} = \rho/\gamma, \quad (35)$$

Hence, the income gap between the two classes persists:

$$\bar{I}^R/\bar{I}^L = 2-\gamma > 1. \quad (36)$$

Proposition 2 (Persistent Inequality). *For any initial earnings inequality such that $I_0^R > I_0^L$, landowners' predisposition toward conspicuous consumption sustains their income advantage over workers in the long run:*

$$\lim_{t \rightarrow \infty} (I_t^R - I_t^L) > 0. \quad (37)$$

Proof: The proposition follows from (34) and (35). □

Given the proportional relationship between income and fertility within each group, the earnings ratio reaches its long-run level from period 1 onward. In particular, as follows from (28) and (31), $R_{t+1}/L_{t+1} = \alpha/[(1-\alpha)(2-\gamma)]$, and hence $I_{t+1}^R/I_{t+1}^L = [\alpha/(1-\alpha)](L_{t+1}/R_{t+1}) = 2-\gamma > 1$. If $L_0 < \bar{L}$, the earnings of landowners and workers decline from period 1 onward toward their respective steady-state levels, while their ratio remains unchanged. Long-run relative inequality is higher or lower than initial relative inequality according to whether I_0^R/I_0^L is below or above $2-\gamma$.

These findings provide a resolution to the Paradox of Inequality. Landowners' displays of affluence divert resources from reproduction, reducing the number of children they have at a given income and limiting the dilution of inherited wealth. In the steady state, both workers and landowners have replacement fertility, but landowners require a higher income to attain it because they de-

vote a smaller share of income to childrearing. Their conspicuous consumption rises with income, sustaining their earnings advantage across generations.¹⁴

4 Historical Foundations of the Paradox of Inequality and Its Resolution

The historical record reveals two enduring features of preindustrial societies: persistent inequality and lavish displays of affluence among elites. Their coexistence across civilizations and millennia provides a historical foundation for the paradox and lends credence to its proposed resolution.

4.1 Persistent Inequality and Class Stratification in Human History

The diffusion of agricultural practices in the wake of the Neolithic Revolution, nearly 12,000 years ago, transformed the scope for wealth accumulation and set the stage for enduring economic disparities. Cultivation and animal husbandry elevated the importance of land and livestock as sources of livelihood and stores of wealth that could be transmitted across generations, transforming unequal ownership of productive resources into inherited economic advantage. The emergence of agriculture therefore provided the material foundations upon which inequality could take root and persist across generations.

In the early civilizations of Mesopotamia, Egypt, the Indus Valley, and China, economic disparities were reinforced by institutions governing landownership, labor, and taxation. Control over productive resources and the power to exact tribute enabled ruling elites to extract wealth from the populations they governed, creating and sustaining inequality across generations. Legal distinctions and unequal access to education and positions of authority further entrenched these disparities, restricting mobility and perpetuating inherited privilege (Diamond, 1997; Galor, 2022).

In ancient Greece and Rome, landownership and control over coerced labor supported pronounced wealth disparities. Military conquest enlarged elite fortunes through tribute and the acquisition of slaves and land, while political privileges reinforced the economic advantages of wealthy families. Across these civilizations, hereditary privileges and legal protections secured elite control over productive resources, restricting mobility and perpetuating wealth inequality (Finley, 1973).

In medieval Europe, the concentration of landed wealth placed economic resources and political authority in the hands of a narrow elite. Aristocratic families and the Church drew their affluence from the rents and labor of peasant populations, whose obligations and restricted mobility limited their prospects of escaping poverty. The inheritance of estates and the persistence of these relations of dependence entrenched economic disparities across generations (Lagerlöf, 2009; Ashraf *et al.*, 2025).

¹⁴The conspicuous consumption mechanism also extends to status-oriented investment in children’s skills. As established in Appendix D, a common preference for child quality does not prevent income equalization. When landowners additionally value children’s accomplishments as displays of family wealth and status, this greater valuation raises investment per child and reduces fertility at a given income, sustaining inequality.

In China, concentrated landownership and commercial wealth sustained the economic advantages of the landed gentry and merchant elites, despite an imperial examination system that offered a path to upward mobility through entry into government service. In India, the caste system tied occupation and social standing to birth, while restrictions on marriage across caste boundaries limited the opportunities for families to escape their inherited position, reinforcing economic divisions across generations. In pre-Columbian civilizations, elites extracted tribute and labor from subordinate populations, concentrating wealth and sustaining pronounced economic hierarchies (Morris, 2010).

By the eve of the Industrial Revolution, economic inequality had persisted for millennia across agricultural societies. Despite differences in political institutions and social organization, concentrated ownership of productive resources and inherited privilege remained defining features of these societies. This enduring inequality presents the paradox of why the greater reproductive success associated with affluence failed to dissipate the income advantage of landowning elites.

4.2 The Historical Prevalence of Conspicuous Consumption among Elites

Alongside the persistence of inequality, agrarian societies exhibited another enduring feature of elite life—the lavish display of affluence. Across agrarian civilizations, ruling elites proclaimed their status through imposing palaces while seeking to carry their earthly splendor into eternity through monumental tombs, gold ornaments, and elaborate burial goods.¹⁵

Beyond monumental architecture and burial, elite affluence found expression in courtly ceremony, communal celebration, and domestic life. In ancient Persia, royal residences, sumptuous feasts, lavish gifts, and luxurious attire made visible expenditure an integral part of elite standing. In ancient Greece, aristocratic banquets brought wealth before an audience through costly vessels, entertainment, and elaborate hospitality. The splendor of the palace and the extravagance of the gathering served to display privilege and reaffirm social standing.

In imperial China and Rome, this association between affluence and display permeated both public and private life. Elaborate burials and ceremonial objects affirmed elite distinction in China, while palatial residences, extravagant banquets, and ostentatious displays of gold and jewels marked the lives of wealthy Romans.

The civilizations of the pre-Columbian Americas gave equally striking expression to the association between privilege and display. Maya rulers adorned themselves with precious jade and carried elaborate ornaments into their royal tombs. Among the Aztec and Inca elites, gold ornaments, exquisite featherwork, and ceremonial objects transformed rare materials and painstaking craftsmanship into visible expressions of power. Monumental temples and pyramids joined earthly authority with divine grandeur, while the splendor surrounding rulers distinguished them from the populations they governed.

¹⁵For historical accounts of elite consumption in different societies, see, e.g., Briant (1996); Loewe & Shaughnessy (1999); Girouard (1978).

These traditions of display endured into the early modern era, when aristocratic and courtly elites across Europe and Asia—from France and Russia to China, Japan, and Korea—devoted fortunes to gilded interiors, ornamental gardens, elaborate processions, and extravagant ceremonies displaying their wealth and status. Palaces and grand residences gave enduring physical form to elite privilege, while their upkeep and the rituals of courtly life placed recurrent demands on the fortunes that sustained them.

While the forms of extravagance changed, the cultural demands of distinction endured. This persistent association between privilege and costly display provides the historical motivation for the mechanism examined in the theory. Under the demographic conditions of the Malthusian epoch, resources absorbed by lavish, nonproductive expenditure could otherwise have supported larger families, multiplying heirs and fragmenting inherited fortunes. The imperative to display affluence may thus have restrained the very demographic force that threatened the persistence of elite wealth. Ironically, extravagance could preserve the wealth it appeared to dissipate.

5 Concluding Remarks

This paper uncovers the Paradox of Inequality, establishes its robust theoretical foundations, and provides a resolution to this conundrum. During the Malthusian epoch, greater affluence would have sustained larger families, dividing inherited resources among a growing number of heirs and progressively eroding the economic advantage of the wealthy. Yet income disparities between social classes persisted throughout this epoch, contradicting the equalizing demographic consequences of affluence.

The proposed resolution, which may itself appear paradoxical, lies in the demographic consequences of extravagance. Elites' lavish, nonproductive displays of status absorbed resources that would otherwise have supported reproductive expansion, limiting the fragmentation of inherited wealth and preserving their economic advantage across generations. By choosing cake over bread, elites inadvertently preserved the fortunes their extravagance seemed destined to erode.

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Online Appendix

A The Paradox of Inequality in a Non-Competitive Economy

This section establishes that, in a Malthusian economy, the Paradox of Inequality holds under any fixed division of output between landowners and workers, whether reflecting bargaining power or the degree of exploitation associated with extractive institutions.

Consider the Malthusian model presented in Section 2, and suppose that landowners are able to expropriate a portion $(1 - \eta)$ of the marginal productivity of labor; $\eta \in (0, 1)$.

The wage paid to each worker in period t , w_t , is therefore

$$w_t = \eta(1 - \alpha)Y_t/L_t, \quad (\text{A1})$$

whereas the R_t landowners in period t divide the remaining output, after the labor payment, among themselves. The income of each landowner, ζ_t , is therefore

$$\zeta_t \equiv (Y_t - w_t L_t) / R_t = [1 - \eta(1 - \alpha)](Y_t / R_t). \quad (\text{A2})$$

Given their preferences, individuals allocate their income between consumption and fertility. In particular, the optimal fertility is:

$$n_t^i = (\gamma/\rho)I_t^i, \quad (\text{A3})$$

where

$$I_t^i = \begin{cases} w_t & \text{if } i = L \\ \zeta_t & \text{if } i = R. \end{cases} \quad (\text{A4})$$

The evolution of the working population is determined by the initial size of the working population and the number of surviving children per worker. Specifically, the size of the working population in period $t + 1$, L_{t+1} , is:

$$L_{t+1} = n_t^L L_t. \quad (\text{A5})$$

Using (1), (A1) and (A3), the time path of the working population is governed by the following first-order difference equation:

$$L_{t+1} = \eta(1 - \alpha)(\gamma/\rho)Y_t = \eta(1 - \alpha)(\gamma/\rho)(AX)^\alpha L_t^{1-\alpha} = \phi(L_t), \quad (\text{A6})$$

where $\phi(L_t)$ is strictly concave in L_t , $\phi(0) = 0$, $\lim_{L_t \rightarrow 0} \phi'(L_t) = \infty$, and $\lim_{L_t \rightarrow \infty} \phi'(L_t) = 0$.

Thus, $\forall L_0 < \bar{L}$, the size of the working population increases over time and converges from its initial level L_0 to a unique and stable steady-state level $\bar{L}$.

$$\bar{L} = [\eta(1 - \alpha)(\gamma/\rho)]^{1/\alpha} AX. \quad (\text{A7})$$

The evolution of the landowning population is determined by its initial size and the number of surviving children per landowner. Specifically, the size of the landowning population in period $t + 1$, R_{t+1} , is given by:

$$R_{t+1} = n_t^R R_t. \quad (\text{A8})$$

Using (1), (A2) and (A3), the landowning population in period $t + 1$ is independent of its size in period t .

$$R_{t+1} = [1 - \eta(1 - \alpha)](\gamma/\rho)Y_t = [1 - \eta(1 - \alpha)](\gamma/\rho)(AX)^\alpha L_t^{1-\alpha} = R(L_t). \quad (\text{A9})$$

Hence, as the working population L_t converges to its steady-state level $\bar{L}$, the landowning population converges to its steady-state level $\bar{R}$:

$$\bar{R} = [1 - \eta(1 - \alpha)][\eta(1 - \alpha)]^{\frac{1-\alpha}{\alpha}} (\gamma/\rho)^{\frac{1}{\alpha}} AX. \quad (\text{A10})$$

Output per worker evolves according to

$$y_{t+1} = [(AX)/L_{t+1}]^\alpha = [(AX)/(n_t^L L_t)]^\alpha = y_t/(n_t^L)^\alpha. \quad (\text{A11})$$

Noting that $n_t^L = (\gamma/\rho) w_t$ and $w_t = \eta(1 - \alpha)y_t$, the time path of output per worker is governed by the first-order difference equation:

$$y_{t+1} = [\rho/(\eta(1 - \alpha)\gamma)]^\alpha y_t^{(1-\alpha)} \equiv \psi(y_t). \quad (\text{A12})$$

Thus the function $\psi(y_t)$ is strictly concave, where $\psi(0) = 0$, $\lim_{y_t \rightarrow 0} \psi'(y_t) = \infty$, and $\lim_{y_t \rightarrow \infty} \psi'(y_t) = 0$. Consequently, regardless of the quantity of land X or the level of technology A , output per worker converges to a unique and stable steady-state level, denoted $\bar{y}$:

$$\bar{y} = \rho/[\eta(1 - \alpha)\gamma]. \quad (\text{A13})$$

Since the number of workers and landowners as well as output per worker converge to their respective steady states, it follows that the steady-state income of workers, $\bar{I}^L$, equals $\bar{w}$, which in turn equals $\eta(1 - \alpha)\bar{y}$. As established in (A13), $\bar{y} = \rho/[\eta(1 - \alpha)\gamma]$, and therefore

$$\bar{I}^L = \eta(1 - \alpha)\bar{y} = \frac{\rho}{\gamma}. \quad (\text{A14})$$

The steady-state earnings of landowners, $\bar{I}^R$, are given by $\bar{\zeta} = [1 - \eta(1 - \alpha)]\bar{y}\bar{L}/\bar{R}$. Using the expressions for $\bar{L}$, $\bar{R}$, and $\bar{y}$ from (A7), (A10), and (A13), respectively, and simplifying terms one gets:

$$\bar{I}^R = \rho/\gamma. \quad (\text{A15})$$

The initial income gap between landowners and workers dissipates and the steady-state income of workers and landowners is identical (i.e., $\bar{I}^R = \bar{I}^L$).

Thus, a fixed degree of worker exploitation changes the steady-state population sizes but does not prevent the equalization of incomes between the two classes. The Paradox of Inequality therefore extends beyond competitive factor pricing.

B The Paradox of Inequality with Downward Social Mobility

This section establishes that the Paradox of Inequality persists even when the number of children who are disinherited and enter the working class increases with landowners' family size.

Consider the perfectly competitive overlapping-generations economy developed in Section 2. The society consists of R_t landowners and L_t workers. Each landowner owns an equal share of the economy's total land endowment, X , while each worker supplies inelastically one unit of labor. Unlike the baseline model, children of landowners need not inherit their parental class.

Let $p(n) \in [0, 1)$ denote the fraction of children who are disinherited by a landowner with n children. Disinherited children become landless workers, while the parental estate is divided equally among the remaining children. Children of workers remain workers. Suppose that $p(1) = 0$, i.e., an only child is not disinherited. Moreover, suppose that the number of disinherited children, $np(n)$, increases with n for $n > 1$, and that the number of inheriting children,

$$h(n_t^R) \equiv n_t^R[1 - p(n_t^R)], \quad (\text{A16})$$

is strictly increasing in n_t^R .

B.1 Production

Production and factor pricing are unchanged. Output in period t , Y_t , is given by:

$$Y_t = (AX)^\alpha L_t^{1-\alpha}; \quad \alpha \in (0, 1). \quad (\text{A17})$$

Output per worker, $y_t \equiv Y_t/L_t$, is therefore:

$$y_t = (AX/L_t)^\alpha. \quad (\text{A18})$$

Under perfect competition, the wage, w_t , and the rental rate on land, r_t , are:

$$w_t = (1 - \alpha)Y_t/L_t; \quad r_t = \alpha Y_t/X. \quad (\text{A19})$$

B.2 Individuals

Individuals share the preferences and childrearing costs specified in Section 2. They derive utility from their own consumption and the number of their children, irrespective of their children's subsequent class:

$$u_t^i = (1 - \gamma) \ln(c_t^i) + \gamma \ln(n_t^i), \quad \gamma \in (0, 1). \quad (\text{A20})$$

Their budget constraint is:

$$\rho n_t^i + c_t^i \leq I_t^i = \begin{cases} w_t & \text{if } i = L, \\ r_t(X/R_t) & \text{if } i = R. \end{cases} \quad (\text{A21})$$

Since preferences and budget constraints are unchanged, the optimal allocation remains:

$$c_t^i = (1 - \gamma)I_t^i; \quad n_t^i = (\gamma/\rho)I_t^i. \quad (\text{A22})$$

Hence, income has a positive effect on reproductive success in both classes.

B.3 Population Dynamics

At time 0, there are L_0 adult workers and R_0 adult landowners, where

$$L_0 > [(1 - \alpha)/\alpha]R_0. \quad (\text{A23})$$

As in the baseline model, this condition implies $I_0^R > I_0^L$, and income inequality initially prevails.

B.3.1 Population Dynamics: Workers

The working population in period $t + 1$ consists of workers' children and disinherited children of landowners:

$$L_{t+1} = n_t^L L_t + p(n_t^R) n_t^R R_t. \quad (\text{A24})$$

B.3.2 Population Dynamics: Landowners

The landowning population in period $t + 1$ consists of landowners' inheriting children:

$$R_{t+1} = [1 - p(n_t^R)] n_t^R R_t. \quad (\text{A25})$$

B.3.3 Steady-State Populations

In a positive steady state, $R_{t+1} = R_t = \bar{R} > 0$. Hence, (A25) requires

$$[1 - p(\bar{n}^R)] \bar{n}^R = 1. \quad (\text{A26})$$

Since $p(1) = 0$, this condition holds at $\bar{n}^R = 1$. Since the number of inheriting children, $n[1 - p(n)]$, is strictly increasing in n , this is the unique solution. Consequently, $p(\bar{n}^R) = 0$.

There is therefore no flow from the landowning class to the working class in the steady state. Setting $L_{t+1} = L_t = \bar{L} > 0$ in (A24) then gives $\bar{L} = \bar{n}^L \bar{L}$, and hence

$$\bar{n}^L = 1. \quad (\text{A27})$$

To determine the steady-state working population, note from (A19) and (A22) that

$$\bar{n}^L = (\gamma/\rho)(1 - \alpha)(AX/\bar{L})^\alpha = 1. \quad (\text{A28})$$

Solving for $\bar{L}$ yields

$$\bar{L} = [(1 - \alpha)(\gamma/\rho)]^{1/\alpha} AX > 0. \quad (\text{A29})$$

Similarly, replacement fertility among landowners requires

$$\bar{n}^R = (\gamma/\rho)\alpha\bar{Y}/\bar{R} = 1. \quad (\text{A30})$$

Since replacement fertility among workers implies $\bar{L} = (\gamma/\rho)(1 - \alpha)\bar{Y}$, it follows that

$$\bar{R} = \frac{\alpha}{1 - \alpha} \bar{L}. \quad (\text{A31})$$

These population sizes satisfy both population equations and constitute the unique positive steady state.

B.4 Vanishing Earnings Inequality between Social Classes

This section establishes that the economy converges to its unique positive steady state, where incomes are equal across social classes. Suppose that $p(n) = 0$ for $n \leq 1$, so that disinheritance occurs only when fertility exceeds replacement.

Since both classes have replacement fertility in the steady state, the common fertility–income relationship in (A22) implies that the income of each landowner is:

$$\bar{I}^R = \bar{r}X/\bar{R} = \rho/\gamma, \quad (\text{A32})$$

whereas the income of each worker is:

$$\bar{I}^L = \bar{w} = \rho/\gamma. \quad (\text{A33})$$

Thus, the following proposition holds:

Proposition 3 (The Paradox of Inequality with Downward Social Mobility). *Suppose that $p(n) = 0$ for $n \leq 1$ and that the number of inheriting children, $h(n) = n[1 - p(n)]$, is strictly increasing in fertility. Then, the economy converges to a unique positive steady state, and the incomes of landowners and workers converge to the same level:*

$$\lim_{t \rightarrow \infty} I_t^R = \lim_{t \rightarrow \infty} I_t^L = \rho/\gamma. \quad (\text{A34})$$

Proof: The uniqueness of the steady-state and income equality follow from (A29), (A31), (A32), and (A33).

Let $\ell_t = L_t/\bar{L}$ and $v_t = R_t/\bar{R}$. It follows that

$$\ell_{t+1} = \ell_t^{1-\alpha} \left[1 + \frac{\alpha}{1-\alpha} p(n_t^R) \right]; \quad v_{t+1} = \ell_t^{1-\alpha} [1 - p(n_t^R)], \quad (\text{A35})$$

where $n_t^R = \ell_t^{1-\alpha}/v_t$. Since $0 \leq p(n) < 1$, both populations are bounded from above. Moreover, $\ell_{t+1} \geq \ell_t^{1-\alpha}$, implying $\liminf_{t \rightarrow \infty} \ell_t \geq 1$. When $n_t^R \leq 1$, $v_{t+1} = \ell_t^{1-\alpha}$. When $n_t^R > 1$, strict monotonicity of h implies $v_t < v_{t+1} \leq \ell_t^{1-\alpha}$. Hence, $\liminf_{t \rightarrow \infty} v_t \geq 1$, and

$$(1 - \alpha)\ell_{t+1} + \alpha v_{t+1} = \ell_t^{1-\alpha}. \quad (\text{A36})$$

Let $M = \limsup_{t \rightarrow \infty} \ell_t \geq 1$. Since $\liminf v_t \geq 1$, this equation implies $(1 - \alpha)M + \alpha \leq M^{1-\alpha}$. Strict concavity implies that this inequality holds only at $M = 1$. Hence, $\ell_t \rightarrow 1$, and (A36) then implies $v_t \rightarrow 1$. Both populations therefore converge to their unique positive steady-state levels, and their incomes converge to ρ/γ . $\square$

Downward social mobility therefore does not prevent the equalization of incomes in the long run. Whenever landowners have above-replacement fertility, they have more than one inheriting child, despite the movement of some descendants into the working class. Greater reproductive success continues to dilute inherited landholdings. The disappearance of income inequality thus does not require permanent separation of the two classes.

C The Paradox of Inequality with a Land Market

This section establishes that the Paradox of Inequality persists in the presence of a land market. All individuals have identical preferences but differ initially in their inherited land endowments and, consequently, in their fertility and total land bequests. Greater wealth generates larger bequests but also more children, equalizing the land inherited per child and eliminating income inequality in the following generation.

Consider a perfectly competitive overlapping-generations economy with the production technology and childrearing costs specified in Section 2. Unlike the baseline model, all adults supply one unit of labor and can own land. Individuals differ initially in their inherited landholdings, while their preferences and labor endowments are identical. Landownership is therefore a source of income rather than a permanently inherited occupational status.

At any time t , there are N_t adults. Individual i inherits $a_t^i \geq 0$ units of land, and aggregate inherited landholdings equal the fixed land endowment, X . During adulthood, individuals earn wages and rental income, trade land, and divide their final holdings equally among their children.

C.1 Production

Production occurs according to a constant-returns-to-scale technology. Output in period t , Y_t , is given by:

$$Y_t = (AX)^\alpha N_t^{1-\alpha}; \quad \alpha \in (0, 1), \quad (\text{A37})$$

where N_t and X are, respectively, labor and land employed in production. Output per worker, $y_t \equiv Y_t/N_t$, is therefore:

$$y_t = (AX/N_t)^\alpha. \quad (\text{A38})$$

Under perfect competition, the wage, w_t , and the rental rate on land, r_t , are:

$$w_t = (1 - \alpha)Y_t/N_t; \quad r_t = \alpha Y_t/X. \quad (\text{A39})$$

The earnings of individual i are consequently:

$$I_t^i = w_t + r_t a_t^i. \quad (\text{A40})$$

Income differences arise entirely from differences in inherited landholdings.

C.2 Individuals

Individuals live for two periods. During childhood, they are supported by their parents. During adulthood, they supply their labor and inherited land to production and allocate their resources among consumption, childrearing, and land bequests.

Let $q_t > 0$ denote the price of land traded after production and the payment of current rents. Let $b_t^i > 0$ denote the amount of land bequeathed to each child of individual i . Preferences are:

$$u_t^i = (1 - \gamma) \ln(c_t^i) + \gamma \ln(n_t^i) + \beta \ln(b_t^i), \quad 0 < \beta < \gamma < 1. \quad (\text{A41})$$

The parameter β measures the common preference for land inherited per child. The restriction $\beta < \gamma$ ensures an interior solution.

An individual with n_t^i children leaves a total estate of $n_t^i b_t^i$ units of land. Net purchases are therefore $n_t^i b_t^i - a_t^i$. The budget constraint is:

$$c_t^i + \rho n_t^i + q_t(n_t^i b_t^i - a_t^i) \leq I_t^i. \quad (\text{A42})$$

Individuals who expand their estates pay for additional land, while those who reduce their estates receive the proceeds of land sales. Equivalently,

$$c_t^i + \rho n_t^i + q_t n_t^i b_t^i \leq W_t^i \equiv w_t + (r_t + q_t) a_t^i, \quad (\text{A43})$$

where W_t^i includes earnings and the market value of inherited land.

To derive the optimal allocation, let $s_t^i \equiv n_t^i b_t^i$ denote the total estate left to children. Utility can then be written as

$$u_t^i = (1 - \gamma) \ln(c_t^i) + (\gamma - \beta) \ln(n_t^i) + \beta \ln(s_t^i), \quad (\text{A44})$$

subject to $c_t^i + \rho n_t^i + q_t s_t^i \leq W_t^i$. Aggregate parental resources equal output plus the market value of the fixed land endowment:

$$\sum_{i=1}^{N_t} W_t^i = N_t w_t + (r_t + q_t) \sum_{i=1}^{N_t} a_t^i = Y_t + q_t X. \quad (\text{A45})$$

Hence,

$$c_t^i = (1 - \gamma) W_t^i; \quad \rho n_t^i = (\gamma - \beta) W_t^i; \quad q_t n_t^i b_t^i = \beta W_t^i. \quad (\text{A46})$$

In accordance with the Malthusian paradigm, greater inherited wealth raises reproductive success. It also raises expenditure on land bequests. Dividing total land bequests by the number of children gives:

$$b_t^i = \frac{\beta W_t^i}{q_t n_t^i} = \frac{\beta \rho}{(\gamma - \beta) q_t} \equiv b_t. \quad (\text{A47})$$

Thus, land inherited per child is independent of parental wealth. Wealthier parents leave larger total estates, but divide them among proportionally more children.

C.3 Land-Market Equilibrium

The land market clears when the total estates left to children equal the fixed land endowment, X . From (A43), aggregate parental resources are $Y_t + q_t X$. Since individuals devote a fraction β of their resources to land bequests, market clearing requires:

$$q_t X = \beta(Y_t + q_t X). \quad (\text{A48})$$

The equilibrium land price is therefore:

$$q_t = \frac{\beta}{1 - \beta} \frac{Y_t}{X} > 0. \quad (\text{A49})$$

This price supports land purchases by initially landless individuals and compensates sellers for the land they transfer. Substituting (A49) into aggregate parental resources gives:

$$Y_t + q_t X = \frac{Y_t}{1 - \beta}. \quad (\text{A50})$$

Although the market value of land enters individual budgets, land transactions do not create additional goods. Aggregate consumption and childrearing expenditure exhaust output:

$$C_t + \rho N_{t+1} = \frac{1-\gamma}{1-\beta} Y_t + \frac{\gamma-\beta}{1-\beta} Y_t = Y_t, \quad (\text{A51})$$

where C_t denotes aggregate consumption.

C.4 Population Dynamics

At time 0, there are $N_0 > 0$ adults with potentially unequal inherited landholdings. By (A40), individuals with more land have higher earnings. By (A46), they also have greater reproductive success.

The adult population in period $t+1$ equals the total number of children born in period t . Using (A46) and (A50),

$$N_{t+1} = \frac{\gamma-\beta}{\rho(1-\beta)} Y_t = \frac{\gamma-\beta}{\rho(1-\beta)} (AX)^\alpha N_t^{1-\alpha} \equiv \phi(N_t). \quad (\text{A52})$$

The function $\phi(N_t)$ is increasing and strictly concave, with $\phi(0) = 0$, $\lim_{N_t \rightarrow 0} \phi'(N_t) = \infty$, and $\lim_{N_t \rightarrow \infty} \phi'(N_t) = 0$. Consequently, for any $N_0 > 0$, the population converges monotonically to the unique positive steady-state level:

$$\bar{N} = \left[\frac{\gamma-\beta}{\rho(1-\beta)} \right]^{1/\alpha} AX > 0. \quad (\text{A53})$$

The population increases if $N_0 < \bar{N}$ and decreases if $N_0 > \bar{N}$.

C.5 Earnings Dynamics

C.5.1 The Time Path of Output Per Worker

Noting (A38) and (A52), the time path of output per worker is:

$$y_{t+1} = \left[\frac{AX}{N_{t+1}} \right]^\alpha = \left[\frac{\rho(1-\beta)}{\gamma-\beta} \right]^\alpha y_t^{1-\alpha} \equiv \psi(y_t). \quad (\text{A54})$$

The function $\psi(y_t)$ is increasing and strictly concave. Output per worker converges to the unique positive steady-state level:

$$\bar{y} = \frac{\rho(1-\beta)}{\gamma-\beta}. \quad (\text{A55})$$

C.5.2 Vanishing Earnings Inequality

Each child inherits b_t units of land, regardless of parental wealth. Since aggregate land bequests equal X ,

$$a_{t+1}^i = b_t = \frac{X}{N_{t+1}} \quad \text{for every individual in period } t+1. \quad (\text{A56})$$

All individuals consequently have identical landholdings from period 1 onward. Their earnings are:

$$I_t^i = w_t + r_t \frac{X}{N_t} = (1-\alpha)y_t + \alpha y_t = y_t, \quad t \geq 1. \quad (\text{A57})$$

Thus, income inequality vanishes after one generation, and earnings converge to the common steady-state level $\bar{I} = \bar{y}$.

Proposition 4 (The Paradox of Inequality with a Land Market). *Suppose individuals have identical labor endowments and preferences. For any initial distribution of landholdings, inherited landholdings and earnings are equal across individuals from period 1 onward. The population converges to a unique positive steady state, and individual earnings converge to*

$$\bar{I} = \frac{\rho(1 - \beta)}{\gamma - \beta}. \quad (\text{A58})$$

Proof: Equation (A47) establishes that every child receives the same land bequest. Market clearing therefore implies (A56), and identical labor endowments imply earnings equality by (A57). Population convergence follows from (A52), while convergence of earnings follows from (A54) and (A55). $\square$

A land market therefore does not prevent the equalization of incomes across individuals. Greater parental wealth raises both the total estate left to children and the number of children among whom it is divided. Although parents can purchase land to expand their estates, their greater reproductive success offsets this expansion, eliminating the inherited economic advantage of their descendants.

D The Paradox of Inequality with Investment in Child Quality

This section establishes that the Paradox of Inequality persists when individuals invest in their children's productive skills. Consider the perfectly competitive overlapping-generations economy developed in Section 2, and suppose that workers and landowners share the same preferences for the quantity and quality of their children. Investment in quality raises workers' labor productivity and landowners' ability to manage their estates. Despite these productive benefits, the income gap between landowners and workers vanishes in the long run.

The society consists of R_t landowners and L_t workers. Each landowner owns an equal share of the fixed land endowment, X , and divides this holding equally among their children. Children inherit their parental class. Workers supply one unit of time to production, while landowners manage their own estates. Productive skills are acquired through parental investment during childhood and remain constant during adulthood.

D.1 Production

Production occurs according to a constant-returns-to-scale technology in effective land services and effective labor. Output in period t , Y_t , is given by

$$Y_t = (Ah_t^R X)^\alpha (h_t^L L_t)^{1-\alpha}, \quad \alpha \in (0, 1), \quad (\text{A59})$$

where h_t^L measures workers' labor productivity and h_t^R measures landowners' management skills. Each worker supplies h_t^L units of effective labor, and each landowner supplies $h_t^R X/R_t$ units of effective land services. Management skills are embodied in the owner, and the return to these skills accrues to the owner through the services supplied by the managed estate. Thus, investment in either group's skills raises output, holding land and population sizes constant. Output per worker, $y_t \equiv Y_t/L_t$, is

$$y_t = (h_t^R)^\alpha (h_t^L)^{1-\alpha} (AX/L_t)^\alpha. \quad (\text{A60})$$

Under perfect competition, payments per unit of effective labor and effective land services are, respectively, $(1 - \alpha)Y_t/(h_t^L L_t)$ and $\alpha Y_t/(h_t^R X)$. Hence, the earnings of each worker, w_t , and the return per unit of owner-managed land, r_t , are

$$w_t = (1 - \alpha)Y_t/L_t; \quad r_t = \alpha Y_t/X. \quad (\text{A61})$$

Total output is therefore divided between workers' earnings, $w_t L_t = (1 - \alpha)Y_t$, and landowners' income, $r_t X = \alpha Y_t$.

D.2 Individuals

Individuals live for two periods. During childhood, they are supported by their parents and acquire productive skills. During adulthood, they supply their factor endowment and allocate their income between consumption, childrearing, and investment in their children's quality.

Investment of e_t^i units of the final good in each child of an adult in class $i \in \{L, R\}$ produces skills

$$h_{t+1}^i = (e_t^i)^\eta, \quad \eta \in (0, 1). \quad (\text{A62})$$

Workers and landowners share the same preferences and skill-acquisition technology. Their utility is

$$u_t^i = (1 - \gamma) \ln c_t^i + \gamma \ln n_t^i + \beta \ln h_{t+1}^i, \quad \gamma \in (0, 1), \quad \beta > 0, \quad \beta\eta < \gamma. \quad (\text{A63})$$

Here, c_t^i is consumption, n_t^i is the number of children, and β measures the preference for child quality. The restriction $\beta\eta < \gamma$ ensures a finite interior solution. Individuals maximize utility subject to

$$c_t^i + n_t^i(\rho + e_t^i) \leq I_t^i = \begin{cases} w_t & \text{if } i = L, \\ r_t(X/R_t) & \text{if } i = R, \end{cases} \quad (\text{A64})$$

where $\rho > 0$ is the basic goods cost of raising a child.

The optimality conditions imply

$$c_t^i = (1 - \gamma)I_t^i; \quad n_t^i = \frac{\gamma I_t^i}{\rho + e_t^i}; \quad \frac{\beta\eta}{e_t^i} = \frac{\gamma}{\rho + e_t^i}. \quad (\text{A65})$$

Consequently, investment per child is identical across classes and independent of parental income:

$$e_t^L = e_t^R = e^* \equiv \frac{\beta\eta\rho}{\gamma - \beta\eta}. \quad (\text{A66})$$

The optimal allocation is therefore

$$c_t^i = (1 - \gamma)I_t^i; \quad n_t^i e^* = \beta \eta I_t^i; \quad \rho n_t^i = (\gamma - \beta \eta)I_t^i. \quad (\text{A67})$$

Investment in quality absorbs resources that could otherwise support additional children. However, it does so in the same proportion for both classes. Richer parents have more children, each receiving the same investment. In accordance with the Malthusian paradigm, fertility remains proportional to income:

$$n_t^i = \frac{\gamma}{\rho + e^*} I_t^i = \frac{\gamma - \beta \eta}{\rho} I_t^i. \quad (\text{A68})$$

From period 1 onward, both groups have the same productive skills,

$$h_t^L = h_t^R = \bar{h} \equiv (e^*)^\eta, \quad t \geq 1, \quad (\text{A69})$$

and production becomes $Y_t = \bar{h}(AX)^\alpha L_t^{1-\alpha}$. Initial adult skills, h_0^L and h_0^R , may be any positive values.

D.3 Population Dynamics

At time 0, there are $L_0 > 0$ workers and $R_0 > 0$ landowners, with

$$L_0 > [(1 - \alpha)/\alpha]R_0. \quad (\text{A70})$$

Since $I_0^R/I_0^L = [\alpha/(1 - \alpha)]L_0/R_0$, landowners initially earn more than workers.

D.3.1 Population Dynamics: Workers

The working population evolves according to

$$L_{t+1} = n_t^L L_t = (1 - \alpha) \frac{\gamma}{\rho + e^*} Y_t. \quad (\text{A71})$$

From period 1 onward, its time path is governed by

$$L_{t+1} = (1 - \alpha) \frac{\gamma \bar{h}}{\rho + e^*} (AX)^\alpha L_t^{1-\alpha} \equiv \phi(L_t). \quad (\text{A72})$$

The function ϕ is increasing and strictly concave, with $\phi(0) = 0$, $\lim_{L \rightarrow 0} \phi'(L) = \infty$, and $\lim_{L \rightarrow \infty} \phi'(L) = 0$. Thus, for any $L_1 > 0$, the working population converges monotonically to the unique positive steady state

$$\bar{L} = \left[(1 - \alpha) \frac{\gamma \bar{h}}{\rho + e^*} \right]^{1/\alpha} AX. \quad (\text{A73})$$

In particular, $\ln(L_{t+1}/\bar{L}) = (1 - \alpha) \ln(L_t/\bar{L})$ for $t \geq 1$. The population increases if $L_1 < \bar{L}$ and decreases if $L_1 > \bar{L}$.

D.3.2 Population Dynamics: Landowners

The landowning population evolves according to

$$R_{t+1} = n_t^R R_t = \alpha \frac{\gamma}{\rho + e^*} Y_t. \quad (\text{A74})$$

Hence, from period 1 onward,

$$R_{t+1} = \alpha \frac{\gamma \bar{h}}{\rho + e^*} (AX)^\alpha L_t^{1-\alpha} \equiv R(L_t), \quad (\text{A75})$$

which is independent of R_t . As L_t converges to $\bar{L}$, the landowning population converges to

$$\bar{R} = \frac{\alpha}{1-\alpha} \bar{L} = \alpha(1-\alpha)^{(1-\alpha)/\alpha} \left(\frac{\gamma \bar{h}}{\rho + e^*} \right)^{1/\alpha} AX. \quad (\text{A76})$$

D.4 Earnings Dynamics

D.4.1 The Time Path of Output Per Worker

From period 1 onward, output per worker is $y_t = \bar{h}(AX/L_t)^\alpha$. Since $n_t^L = [\gamma/(\rho + e^*)](1-\alpha)y_t$, its time path satisfies

$$y_{t+1} = \bar{h} \left(\frac{AX}{n_t^L L_t} \right)^\alpha = \left[\frac{\rho + e^*}{(1-\alpha)\gamma} \right]^\alpha y_t^{1-\alpha} \equiv \psi(y_t), \quad t \geq 1. \quad (\text{A77})$$

The function ψ is increasing and strictly concave. Output per worker converges to the unique positive steady-state level

$$\bar{y} = \frac{\rho + e^*}{(1-\alpha)\gamma} = \frac{\rho}{(1-\alpha)(\gamma - \beta\eta)}. \quad (\text{A78})$$

The productive benefits of education enter population dynamics through $\bar{h}$. The expenditure required to educate each child also raises the income necessary for replacement fertility.

D.4.2 Vanishing Earnings Inequality between Social Classes

The steady-state income of each landowner is

$$\bar{I}^R = \alpha \bar{y} \bar{L} / \bar{R} = \frac{\rho + e^*}{\gamma} = \frac{\rho}{\gamma - \beta\eta}, \quad (\text{A79})$$

while the steady-state income of each worker is

$$\bar{I}^L = (1-\alpha)\bar{y} = \frac{\rho + e^*}{\gamma} = \frac{\rho}{\gamma - \beta\eta}. \quad (\text{A80})$$

Thus, investment in productive child quality does not prevent the equalization of incomes between landowners and workers.

Proposition 5 (The Paradox of Inequality with Child Quality). *Suppose workers and landowners share the preferences and skill-acquisition technology specified above. For any initial earnings inequality such that $I_0^R > I_0^L$, the earnings of landowners and workers are equal from period 1 onward and converge to a common steady-state level. In particular,*

$$\lim_{t \rightarrow \infty} (I_t^R - I_t^L) = 0. \quad (\text{A81})$$

Proof: Equations (A71) and (A74) imply $R_{t+1}/L_{t+1} = \alpha/(1-\alpha)$ for every $t \geq 0$. Therefore,

$$\frac{I_{t+1}^R}{I_{t+1}^L} = \frac{\alpha}{1-\alpha} \frac{L_{t+1}}{R_{t+1}} = 1. \quad (\text{A82})$$

Convergence to the common steady-state income follows from (A77)–(A80). $\square$

The quantity–quality trade-off changes investment, productive skills, population sizes, and the common steady-state income. It does not overturn the Paradox of Inequality: under common preferences and investment opportunities, landowners' initial affluence generates greater reproductive success, diluting inherited landholdings and eliminating their earnings advantage.

E The Endogenous Emergence of Economic Classes

This section establishes that economic classes can emerge from initially equal land and labor endowments when individuals differ in their predisposition toward conspicuous consumption. Individuals with a stronger taste for displays devote fewer resources to reproduction, leaving larger inheritances to each child. These differences generate a relatively wealthy class with larger landholdings and a poorer class with smaller landholdings, sustaining income inequality across generations.

Consider a perfectly competitive overlapping-generations economy with the production technology and childrearing costs specified in Section 2. All adults supply one unit of labor and own land. Land cannot be traded, and each parent divides their holdings equally among their children.

At time 0, there are $N_0 > 0$ adults, each holding $a_0 = X/N_0$ units of land. A fraction $\omega \in (0, 1)$ has a predisposition toward displays of affluence, while the remaining fraction does not. Preference type is transmitted from parent to child. Initially, preference type is therefore unrelated to landownership or income.

Let R_t denote the population with a predisposition toward displays and L_t the population without this predisposition, where $N_t = R_t + L_t$. These labels distinguish preference types; they do not describe initial economic differences. Since land passes only to descendants, the aggregate land endowments of the two groups remain:

$$X^R = \omega X; \quad X^L = (1 - \omega)X. \quad (\text{A83})$$

Individuals within each group hold equal amounts of land. Their holdings are therefore:

$$a_t^R = \frac{\omega X}{R_t}; \quad a_t^L = \frac{(1 - \omega)X}{L_t}. \quad (\text{A84})$$

E.1 Production

Production occurs according to a constant-returns-to-scale technology. Output in period t , Y_t , is given by:

$$Y_t = (AX)^\alpha N_t^{1-\alpha}; \quad \alpha \in (0, 1). \quad (\text{A85})$$

Output per worker, $y_t \equiv Y_t/N_t$, is therefore:

$$y_t = (AX/N_t)^\alpha. \quad (\text{A86})$$

Under perfect competition, the wage, w_t , and the rental rate on land, r_t , are:

$$w_t = (1 - \alpha)Y_t/N_t; \quad r_t = \alpha Y_t/X. \quad (\text{A87})$$

Since all individuals supply labor and own land, the income of an individual of type $i \in \{L, R\}$ is:

$$I_t^i = w_t + r_t a_t^i. \quad (\text{A88})$$

E.2 Individuals

Individuals live for two periods. During childhood, they are supported by their parents. During adulthood, they supply labor and land to production and allocate their income among consumption, displays of affluence, and childrearing.

Preferences are as specified in Section 3:

$$u_t^i = (1 - \gamma) \ln(c_t^i) + (1 - \gamma)\theta^i \ln(x_t^i) + \gamma \ln(n_t^i), \quad \gamma \in (0, 1), \quad (\text{A89})$$

where $\theta^R = 1$ and $\theta^L = 0$. When $\theta^i = 0$, the display term is absent. Individuals maximize utility subject to:

$$c_t^i + x_t^i + \rho n_t^i \leq I_t^i. \quad (\text{A90})$$

Let $D \equiv 2 - \gamma > 1$. The optimal fertility choices are:

$$n_t^L = (\gamma/\rho)I_t^L; \quad n_t^R = \frac{\gamma}{\rho D}I_t^R. \quad (\text{A91})$$

Individuals of type L choose $x_t^L = 0$, while individuals of type R devote a fraction $(1 - \gamma)/D$ of their income to displays. At a given income, individuals of type R therefore have fewer children.

Since each parent divides their land equally among their children,

$$a_{t+1}^i = \frac{a_t^i}{n_t^i}. \quad (\text{A92})$$

Initially, both groups have identical landholdings and incomes:

$$a_0^R = a_0^L = a_0; \quad I_0^R = I_0^L = y_0. \quad (\text{A93})$$

Nevertheless, $n_0^R = n_0^L/D$. Consequently,

$$a_1^R = Da_1^L, \quad (\text{A94})$$

and $I_1^R > I_1^L$. Income inequality thus emerges after one generation, despite initially equal endowments.

E.3 Population Dynamics

The populations of the two preference types evolve according to:

$$L_{t+1} = n_t^L L_t; \quad R_{t+1} = n_t^R R_t. \quad (\text{A95})$$

Using (A84), (A87), and (A91), these equations become:

$$L_{t+1} = (\gamma/\rho)Y_t \left[(1 - \alpha) \frac{L_t}{N_t} + \alpha(1 - \omega) \right], \quad (\text{A96})$$

$$R_{t+1} = \frac{\gamma}{\rho D}Y_t \left[(1 - \alpha) \frac{R_t}{N_t} + \alpha\omega \right]. \quad (\text{A97})$$

E.3.1 The Relative Sizes of the Two Groups

Let $z_t \equiv R_t/L_t$. Dividing (A97) by (A96) gives:

$$z_{t+1} = \frac{\alpha\omega + (1 - \alpha + \alpha\omega)z_t}{D[1 - \alpha\omega + \alpha(1 - \omega)z_t]} \equiv F(z_t). \quad (\text{A98})$$

The function $F(z)$ is strictly increasing. Its fixed points satisfy:

$$D\alpha(1-\omega)z^2 + [D(1-\alpha\omega) - (1-\alpha+\alpha\omega)]z - \alpha\omega = 0. \quad (\text{A99})$$

Since the leading coefficient is positive and the constant term is negative, this equation has exactly one positive root, $\bar{z}$. Moreover, $F(z) > z$ for $0 < z < \bar{z}$ and $F(z) < z$ for $z > \bar{z}$. Strict monotonicity of F therefore implies monotonic convergence to $\bar{z}$.

Initially, $z_0 = \omega/(1-\omega)$. Equal initial incomes and different fertility choices imply $z_1 = z_0/D < z_0$. Hence,

$$z_t \longrightarrow \bar{z} < \frac{\omega}{1-\omega}. \quad (\text{A100})$$

The descendants of individuals with a taste for displays constitute a progressively smaller population relative to their share of land. Indeed,

$$\frac{a_t^R}{a_t^L} = \frac{\omega}{1-\omega} \frac{1}{z_t}, \quad (\text{A101})$$

so their relative landholdings increase from initial equality and converge to a value greater than one.

E.3.2 The Total Population

Adding (A96) and (A97) gives:

$$N_{t+1} = (\gamma/\rho)\mu(z_t)(AX)^\alpha N_t^{1-\alpha}, \quad (\text{A102})$$

where

$$\mu(z) = (1-\alpha)\frac{1+z/D}{1+z} + \alpha\left(1-\omega+\frac{\omega}{D}\right) > 0. \quad (\text{A103})$$

Since $z_t \rightarrow \bar{z}$, let $\bar{\mu} \equiv \mu(\bar{z})$. The total population converges to:

$$\bar{N} = [(\gamma/\rho)\bar{\mu}]^{1/\alpha} AX > 0. \quad (\text{A104})$$

To establish convergence, taking logarithms of (A102) relative to (A104) yields:

$$\ln(N_{t+1}/\bar{N}) = (1-\alpha)\ln(N_t/\bar{N}) + \ln[\mu(z_t)/\bar{\mu}]. \quad (\text{A105})$$

The final term converges to zero, and $1-\alpha \in (0,1)$. Hence, $N_t \rightarrow \bar{N}$. Both group populations therefore converge to positive steady-state levels:

$$\bar{L} = \frac{\bar{N}}{1+\bar{z}}; \quad \bar{R} = \frac{\bar{z}\bar{N}}{1+\bar{z}}. \quad (\text{A106})$$

E.4 Persistent Earnings Inequality

Since both group populations converge to positive steady-state levels, their fertility converges to replacement:

$$\bar{n}^L = \bar{n}^R = 1. \quad (\text{A107})$$

The fertility choices in (A91) consequently imply:

$$\bar{I}^L = \rho/\gamma; \quad \bar{I}^R = D\rho/\gamma. \quad (\text{A108})$$

Hence,

$$\frac{\bar{I}^R}{\bar{I}^L} = D = 2 - \gamma > 1. \quad (\text{A109})$$

Both groups receive the same wage. The income difference therefore reflects unequal landholdings:

$$\bar{r}(\bar{a}^R - \bar{a}^L) = (D - 1)\rho/\gamma > 0. \quad (\text{A110})$$

Proposition 6 (The Endogenous Emergence and Persistence of Inequality). *Suppose individuals initially have identical land and labor endowments, while a fraction $\omega \in (0, 1)$ has an inherited predisposition toward conspicuous consumption. Landholding and income inequality emerge after one generation and persist in the long run. The economy converges to a unique positive steady state in which individuals with a taste for displays hold more land and earn $2 - \gamma$ times the income of individuals without this taste.*

Proof: Initial income equality and (A91) imply (A94), establishing income inequality from period 1. The population ratio converges to the unique positive solution of (A99). Total population convergence follows from (A105), implying convergence of both group populations to positive steady-state levels. Replacement fertility then gives (A108) and (A109). Greater steady-state landholdings follow from (A110). $\square$

Unlike Section 3, which begins with inequality between landowners and workers and establishes its persistence, this section begins with identical labor and land endowments. Differences in preferences for conspicuous consumption generate differences in fertility and, consequently, in the land inherited per child. Individuals with a stronger preference for conspicuous consumption have fewer children, whose larger inheritances give rise to a wealthier class. Thus, inequality emerges from an initially equal distribution of resources and persists in the long run.