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Early Decisions, Long-Lasting Consequences: The Macroeconomics of School Tracking*

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Abstract

School tracking—sorting students into different school programs—is widespread and shapes how students accumulate skills. We study its aggregate, distributional, and intergenerational effects in a general-equilibrium overlapping-generations model with endogenous school-track choices and a novel skill formation technology trading off learning gains from homogeneous groups against the risk of misallocating children whose skills are still developing. Calibrated to Germany’s early-tracking system, the model shows that delaying tracking from age 10 to 14 improves intergenerational mobility and reduces inequality but generates modest efficiency losses. These losses arise only in general equilibrium, as a falling college wage premium dampens college expansion.

Keywords: Intergenerational Mobility, School Tracking, Human Capital, Inequality, Efficiency

JEL Codes: E24, I24, J24, I28, J62

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1 Introduction

In most countries, children are sorted into distinct secondary school programs at some point, a practice known as school tracking.¹ Tracking by skill aims to create more homogeneous classrooms and facilitate more tailored instruction (Hanushek and Wößmann, 2006). At the same time, it places students on distinct paths that determine how intensively they are taught, with whom they learn, and what opportunities they have after leaving school (Dustmann et al., 2017). Tracking policies thus shape both the level and distribution of human capital even before higher education and labor-market entry. Differences in human capital at this early stage have been shown to be a central determinant of aggregate productivity, lifetime inequality, and intergenerational mobility in the macroeconomic literature (Becker, 1964; Keane and Wolpin, 1997; Huggett et al., 2011; Lee and Seshadri, 2019; Abbott et al., 2019; Krueger et al., 2025). Moreover, because track assignment is often strongly related to students' socioeconomic background (Dustmann, 2004), tracking may contribute to the intergenerational persistence of economic status, much like other forms of early human-capital investment (Yum, 2023; Daruich, 2026). Despite its prevalence and these potentially far-reaching consequences, school tracking has received little attention in macroeconomics.

We study the long-run consequences of school tracking in a new overlapping-generations life-cycle model with heterogeneous agents and endogenous school-track choices. The model links tracking and skill formation during school to higher education choices, labor-market outcomes, and the intergenerational transmission of human capital. At its core is a novel skill-formation technology that captures two competing forces: more homogeneous groups allow instruction to be better tailored to students' skills, but assigning children to tracks before their skills are fully developed creates a risk of mismatch. We use the model, calibrated to Germany's early-tracking system, to quantify how tracking reforms affect aggregate efficiency, inequality, and intergenerational mobility, complementing the applied literature on tracking (e.g., Meghir and Palme, 2005; Pekkarinen et al., 2009; Duflo et al., 2011; Dustmann et al., 2017) with a quantitative macroeconomic analysis that highlights general-equilibrium adjustments and intergenerational feedback.

In our skill technology, a track affects learning through its pace of instruction and its peer composition. Each child has an individually optimal pace of instruction that increases with her skill level, but each track has a single pace, reflecting the intensity and depth of the

¹In 2018, 27 of 37 OECD countries and 34 of 35 non-OECD countries had an education system with at least two distinct school programs available to 15-year-old students (OECD, 2020). The exact process and age of sorting vary considerably across countries. Germany is among the earliest-tracking countries, selecting students at age 10 (see Appendix B.1 for more details).

curriculum. This pace is set endogenously to maximize aggregate end-of-period skills in the track, and thus depends on its skill composition. In addition, children benefit directly from higher-skilled peers. In a simplified setting without later skill shocks, we show analytically that skill-based tracking maximizes aggregate skills but increases skill inequality: homogeneous groups allow instruction to be better tailored to students’ skill levels (Duflo et al., 2011), while direct peer effects amplify learning differences across tracks, with the lower-pace track bearing a larger share of the resulting losses. When skills evolve unpredictably, however, early track assignments may become mismatched with later-realized skills and delaying tracking can improve matching and, potentially, aggregate learning. Track mismatch may be further compounded when assignment depends on parental background rather than students’ skills alone. The technology thus captures both the efficiency gains from homogeneous grouping and the distributional costs emphasized in the empirical literature.

We embed this skill technology in a quantitative general-equilibrium overlapping-generations model with incomplete markets and endogenous saving, labor supply, and college decisions, building on Daruich (2026). In line with the German education system, children are sorted into a *vocational* or an *academic* track at age 10, after four years of comprehensive primary school. Altruistic parents choose the track, subject to a taste shock correlated with parental education. Tracks endogenously shape children’s skills, and the academic track also offers easier access to college, reflecting limited second-chance opportunities. End-of-school skills and college education determine adult human capital, which evolves stochastically over the working life and is imperfectly transmitted to the next generation of children. Finally, equilibrium wages and the interest rate, which parents anticipate, depend on the distribution of human capital across college and non-college workers and on aggregate savings.

We solve the model numerically and calibrate it to Germany using the National Educational Panel Study (NEPS, Blossfeld et al., 2019) and the Socio-Economic Panel (SOEP, Goebel et al., 2019). We estimate some parameters from the data, including those of the skill formation technology, following a latent-variable framework (Cunha et al., 2010; Agostinelli et al., 2026), and calibrate the remaining ones by simulated method of moments. The model matches the data well, including track choices by skill and parental background, intergenerational mobility, and cross-sectional inequality. As a further validation test, a model-based analogue of the natural experiment in Dustmann et al. (2017), in which children on either side of the school-entry cutoff end school with similar skills, reproduces a sizable gap in initial track attendance but much smaller gaps in college graduation and earnings. These small gaps for children at the margin do not mean that the schooling years matter little for

inequality: a variance decomposition in the spirit of [Huggett et al. \(2011\)](#) shows that lifetime inequality takes shape early, much of it during secondary school, as parental characteristics and child skills explain about one-third of the variance in lifetime income at age 10, when tracking occurs, and nearly 70% by age 18.

Our main policy experiment delays school tracking by four years, from age 10 to 14, approximately the average tracking age across OECD countries ([OECD, 2020](#)). We find that this policy reduces cross-sectional economic inequality and raises intergenerational mobility. Specifically, the Gini coefficient of wages falls by around 1% and the association between parental and child earnings, as measured by the rank-rank coefficient, drops by 3.6%. These distributional effects arise because keeping all children together for four additional years reduces the dispersion of end-of-school skills and weakens the influence of parental background on track choice. In particular, children of non-college-educated parents—who have lower skills on average—benefit from longer comprehensive schooling, whereas children of college-educated parents experience relative learning losses. This narrows parental background gaps in academic-track attendance and college graduation, ultimately reducing labor market inequality and increasing intergenerational mobility.

The policy also improves student-track matching. The model allows us to measure mismatch directly, as the share of children attending a track other than the one that maximizes their end-of-school skills given track sizes.² In the baseline, 27% of children are mismatched, only partly because of unpredictable skill development, and mismatch differs by background mainly in direction: children of college-educated parents are misplaced toward the academic track, those of non-college-educated parents away from it. Delaying tracking lowers mismatch to 21% and raises the share of academic-suited children of non-college-educated parents who attend the academic track from 64% to 75%.

Despite better matching, however, delaying tracking reduces aggregate human capital and output, revealing a long-run trade-off between efficiency, on the one hand, and equality and mobility, on the other.³ Average end-of-school skills remain nearly unaffected as the early learning losses from four extra years of comprehensive schooling in heterogeneous peer groups are almost fully offset by more efficient learning between ages 14 and 18. However, the policy compresses the upper tail of the skill distribution, generating learning losses among

²Parents choose tracks to maximize their expected utility, which need not coincide with maximizing their child’s realized end-of-school skills.

³A similar trade-off has been highlighted in the literature on the effects of economic segregation on growth and inequality (see [Bénabou, 1996](#)), and more recently by [Arenas and Hindriks \(2021\)](#), where efficiency gains from unequal school opportunities arise because of positive assortative matching between parents who invest more in their children and better schools.

the high-skilled students who are most likely to attend college. Because end-of-school skills and college education are complementary in the production of adult human capital, these losses reduce aggregate human capital and ultimately output by approximately 0.3%.

Importantly, this trade-off depends on how wages and college attendance respond in general equilibrium. When factor prices are held fixed, the increase in academic-track attendance induced by later tracking also raises the college share by 1.2 percentage points, resulting in higher, not lower, aggregate human capital and output relative to the baseline, by 0.1% and 0.3%, respectively. When factor prices adjust, however, the initial expansion in college-educated labor lowers the college wage premium and weakens incentives to attend college for the next generations, so the college share rises only slightly and fails to compensate for the learning losses among the highest-skilled students. Whether delayed tracking generates an efficiency-mobility trade-off thus depends on this general-equilibrium channel.

Finally, we evaluate three hypothetical benchmark policies: abolishing tracking (*No Tracking*), allowing re-tracking at age 14 (*Flexible Tracking*), and assigning tracks strictly by skill at age 10 (*Skill-Based Tracking*). *No Tracking* amplifies the efficiency-mobility trade-off, with output and human capital falling by around 1% and the income rank-rank coefficient by 13%. *Flexible* and *Skill-Based Tracking* instead reverse it: more skill-homogeneous tracks raise output by 0.5% and 1.2%, respectively, but increase inequality and reduce mobility. The reversal is starkest under *Skill-Based Tracking*, where the rank-rank coefficient rises by 8.8% and the wage Gini by 3.4%. Sorting strictly by skill raises average skills but benefits most those with high skills at age 10, who are disproportionately children of college-educated parents. The resulting skill gaps by parental background widen human capital differences among adults, which are passed on to the next generation and, with track assignment determined solely by skills, translate more directly into track attendance and later outcomes. As a result, an ostensibly meritocratic rule can increase the persistence of economic status across generations.⁴

Related Literature This paper primarily contributes to the quantitative macroeconomic literature on human-capital formation, inequality, and intergenerational mobility, following the seminal contributions of [Becker and Tomes \(1979, 1986\)](#) and [Galor and Zeira \(1993\)](#). This literature studies how parental investments, borrowing constraints, and education policies shape skill formation and educational attainment across generations (e.g., [Fernández and Rogerson, 1998](#); [Restuccia and Urrutia, 2004](#); [Lee and Seshadri, 2019](#); [Caucutt and Lochner,](#)

⁴A related argument appears in [Comerford et al. \(2022\)](#), who show that more accurate rewards for ability can increase inequality and intergenerational persistence when ability is shaped by parental investment.

2020; Yum, 2023; Daruich, 2026; Blandin and Herrington, 2022; Lochner and Monge-Naranjo, 2011; Krueger and Ludwig, 2016; Abbott et al., 2019; Capelle and Matsuda, 2026; Krueger et al., 2025; Jang and Yum, 2024; Fuchs-Schündeln et al., 2022). Its quantitative applications, mostly focused on the United States, span early-childhood investments, public-school finance, instructional time loss after Covid-related school closures, and access to and the financing of higher education.⁵ Comparatively little attention, however, has been paid to the institutional organization of secondary schooling. We fill this gap, and to our knowledge, provide the first quantitative general-equilibrium analysis of broad school-tracking reforms with endogenous family choices and intergenerational linkages.⁶ While we study Germany, one of the earliest-tracking countries in the world, our framework applies more broadly to education systems that sort students across or within schools.

Our theory of skill formation during the school years builds on the literature studying how children’s skills evolve as a function of prior skills, parental and environmental inputs, and schooling inputs (see, e.g., Cunha and Heckman, 2007; Todd and Wolpin, 2007; Cunha et al., 2010; Agostinelli et al., 2025; Bonesrønning et al., 2022; Agostinelli et al., 2026). We introduce two channels through which track composition affects learning: direct peer effects and instructional matching.⁷ Children benefit directly from higher-skilled peers, but also learn more when the track’s instruction pace is better matched to their skills. The latter channel is motivated by evidence that teachers adapt instruction to classroom composition and that teaching practices interact with students’ prior achievement (Duflo et al., 2011; Aucejo et al., 2022). Our contribution is to embed these channels in a dynamic framework with endogenous track choice. Conceptually, this match-based representation differs from models in which schools are vertically differentiated by exogenous quality.⁸

Finally, our paper complements a large empirical literature on school tracking, including Dustmann et al. (2017), drawing on cross-country comparisons and within-country reforms, whose estimates are necessarily context-specific given differences in reform timing, design, and accompanying institutional changes.⁹ Three findings nonetheless recur. First, delaying

⁵An exception is the work on free secondary schooling in Ghana by Fujimoto et al. (2026).

⁶A theoretical literature studies tracking in partial-equilibrium settings, emphasizing peer effects, resource allocation, and ability uncertainty (Epple et al., 2002; Betts, 2011), including the optimal timing of tracking (Brunello et al., 2007, 2012). These frameworks, however, do not jointly incorporate dynamic skill formation, endogenous family choices, general-equilibrium responses, and intergenerational feedback.

⁷See Epple and Romano (2011) for a survey of theoretical models of peer interactions and tracking, and Sacerdote (2001, 2011) for evidence on and reviews of peer effects in education.

⁸Arenas and Hindriks (2021), for example, model unequal access to schools of different quality, with all students benefiting from higher-quality schools.

⁹Some reforms, including Meghir and Palme (2005) and Aakvik et al. (2010), also raised the minimum school-leaving age, so effects cannot be attributed to tracking alone. Cross-country comparisons additionally

tracking tends to raise attainment, though effects on achievement and earnings are mostly null or modest, in Romania, Sweden, Norway, Finland, and the Netherlands (Meghir and Palme, 2005; Aakvik et al., 2010; Malamud and Pop-Eleches, 2011; Pekkala Kerr et al., 2013; Borghans et al., 2020). Second, more rigid tracking raises educational inequality, in both cross-country comparisons (e.g., Hanushek and Wößmann, 2006) and reforms in Germany and the Netherlands, where it generated learning and earnings losses for lower-achievers (Piopiunik, 2014; Borghans et al., 2020; Matthewes, 2021).¹⁰ Third, the benefits of later tracking concentrate among children from lower socioeconomic backgrounds, as shown by reforms in Scandinavia, Romania, and Germany (e.g., Meghir and Palme, 2005; Pekkarinen et al., 2009; Aakvik et al., 2010; Malamud and Pop-Eleches, 2011; Pekkala Kerr et al., 2013; Piopiunik, 2014). Our model, via an entirely different methodology, offers a unified framework whose predictions align with all three findings, while also delivering long-run predictions for aggregate outcomes, inequality, and intergenerational mobility.

The remainder of the paper is organized as follows. Section 2 presents the model and the skill formation technology. Section 3 explains how we parameterize and calibrate the model. It also offers some validation exercises. In Section 4, we use the calibrated model to perform a series of counterfactual experiments. Finally, Section 5 concludes.

2 The Model

Time is discrete, with one model period corresponding to four years. Individuals enter the model at age 2, live for 20 periods, and have one child, so 20 generations overlap at every point in time. We assume a unit mass of individuals in each period.

Figure 1 summarizes the life cycle. Children accumulate skills during school. At age 10, their parents choose one of the two secondary-school tracks, academic or vocational. At age 18, children become independent, receive an inter-vivos transfer, and decide whether to attend college.¹¹ Their skills become adult human capital, which evolves stochastically over the working life. During adulthood, individuals work until retirement at age 66, save, and, once they become parents, make educational and transfer decisions for their child.¹² The following sections describe child skill formation and the associated life-cycle decisions.

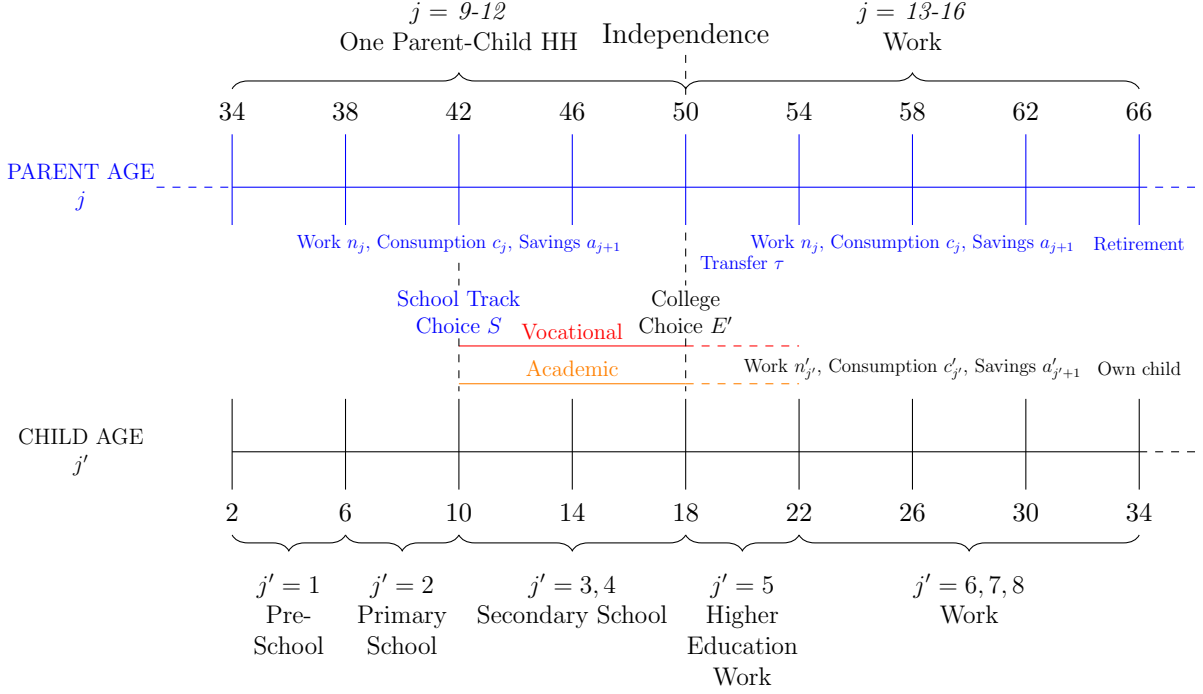
face identification challenges given ambiguous tracking classifications (Betts, 2011).

¹⁰Matthewes (2021) compares two non-academic tracks and finds that two extra years of comprehensive schooling raise achievement specifically among vocational, i.e., lower-achieving, students.

¹¹Appendix B.2 gives an overview of the German education system.

¹²For clarity, we denote all child variables and periods with primes whenever both parental and child states are referenced.

Figure 1: Timeline of Life-cycle Events



2.1 Child Skill Formation

We first introduce the process of child skill formation during school and the endogenous choice of instruction pace. We then derive analytical results to provide intuition on how the allocation and timing of tracking affect aggregate learning and skill inequality in school.

2.1.1 The Technology of Skill Formation

Every child begins schooling at age 6 (period $j = 2$) with an initial skill level θ_2 (in log-arithms), whose intergenerational determination is described in Section 2.2. During the school years $j = 2, 3, 4$, skill formation depends on past skills and parental education—a proxy for the home environment and parental investments (Heckman and Mosso, 2014)—, and the skill levels of a child's peers as well as the match between the child's skills and the pace of instruction in her school track S . Skills are also subject to i.i.d. shocks η_{j+1} each period, capturing unforeseeable influences on skill development, such as health, household, or migration shocks (Cunha et al., 2010). Specifically,

$$\theta_{j+1} = v_0 \theta_j + v_1 \bar{\theta}_j^S + g(\theta_j, P_j^S) + \zeta E + \eta_{j+1}, \quad \eta_{j+1} \sim \mathcal{N}(0, \sigma_{\eta_{j+1}}^2), \quad (1)$$

where v_0 governs skill persistence, v_1 captures direct peer effects from the average skill level $\bar{\theta}_j^S$ in school track S , and ζ captures the effect of parental education E .¹³ Before tracking, $S = C$ denotes comprehensive schooling; afterward, $S \in \{A, V\}$ denotes the *academic* or *vocational* school track.¹⁴ Each track S has a common pace of instruction, P_j^S , capturing differences in the intensity and depth with which the same curricular content is taught.¹⁵ They affect skill formation through

$$g(\theta_j, P_j^S) = v_2 P_j^S + v_3 \theta_j P_j^S - \frac{v_4}{2} (P_j^S)^2. \quad (2)$$

Equation (2) implies an individually optimal instructional pace for each skill level, $P_j^*(\theta_j)$: instruction that is too fast may overwhelm a student, whereas instruction that is too slow may constrain learning. When $v_3 > 0$, this pace increases with the child’s skill, consistent with empirical evidence on positive “match” effects between teaching practices and classroom skill composition (see Dufflo et al. (2011), Aucejo et al. (2022), and references therein).

A policymaker chooses a common instruction pace for each track and period to maximize students’ aggregate end-of-period skills.¹⁶ The optimal pace in each track is therefore the pace preferred by a student with the track’s average skill:

Lemma 1. *The instruction pace in track S that maximizes expected aggregate end-of-period skills is*

$$P_j^S = P_j^*(\bar{\theta}_j^S) = \frac{v_2 + v_3 \bar{\theta}_j^S}{v_4} \quad (3)$$

where $\bar{\theta}_j^S$ is the average skill of children in track S .

Proof. The result follows from the first-order condition of expected aggregate skills with

¹³We view the school track choice as the main margin of parental investment during secondary school and hold other school-age parental inputs, captured by ζ , fixed across policy experiments. Evidence on how parental inputs respond to school assignment is mixed. Empirically, parents in some settings reduce their own inputs when school inputs improve (e.g., Pop-Eleches and Urquiola, 2013; Das et al., 2013), while in others they become more involved, for example after being offered Head Start (Gelber and Isen, 2013) or assigned to the highest school track (Klein and McNamara, 2025). Structural models are similarly divided: parental inputs substitute for school, peer, or neighborhood inputs in some (e.g., Agostinelli et al., 2026; Bellue, 2026) and complement them in others (e.g., Jang and Yum, 2024; Chyn and Daruich, 2025).

¹⁴We restrict our analysis to at most two tracks, the typical number across OECD countries and the number that most German states have converged to (see Appendix B.2).

¹⁵We abstract from within-track heterogeneity and model each track as a representative school, although our estimation exploits classroom variation within schools. Academic-track schools generally cover similar subjects more intensively and comprehensively than vocational-track schools, motivating our representation of tracking as vertical differentiation in instruction pace. This differentiation does not represent differences in teacher quality or school resources. See Appendix B.2 for details.

¹⁶Individualized instruction would be first-best in the model but is infeasible to implement in practice. In Germany, curricula and educational standards are updated regularly by the federal states and differentiated across school tracks (Kultusministerkonferenz, 2026).

respect to P_j^S . □

Substituting the optimal pace (3) and the pace function (2) into the skill formation technology (1), we obtain the following final skill technology:¹⁷

$$\theta_{j+1} = \frac{v_2^2}{2 v_4} + \left(v_0 + \frac{v_2 v_3}{v_4} \right) \theta_j + v_1 \bar{\theta}_j^S + \frac{v_3^2}{2 v_4} \theta_j^2 - \frac{v_3^2}{2 v_4} (\theta_j - \bar{\theta}_j^S)^2 + \zeta E + \eta_{j+1}. \quad (4)$$

The final term involving $(\theta_j - \bar{\theta}_j^S)^2$ captures instructional mismatch: when $v_4 > 0$, skill gains decline with the distance between a child's skill and the average skill in her track. A lower-skilled child may therefore learn less in a higher-paced track despite benefiting from higher-skilled peers. The model thus distinguishes a positive direct peer effect from an instructional-matching effect that depends on within-track skill homogeneity.

2.1.2 Analytical Insights on the Effects of Tracking

To build intuition about the implications of these forces for school tracking, we first characterize track assignment in a simplified one-period setting without skill shocks, parental inputs ($\zeta = 0$), or track-related costs. Initial skills θ_j are normally distributed with mean 0 and variance $\sigma_{\theta_j}^2$, and end-of-school skills are θ_{j+1} . This simplification isolates the role of instructional mismatch and peer composition; Appendix A.1 provides a more detailed discussion and all proofs. Throughout, we maintain $v_3 > 0$, $v_4 > 0$, and $b \equiv v_0 + v_2 v_3 / v_4 \geq 0$, and consider the direct peer effect v_1 separately.¹⁸

We consider two assignment mechanisms. Under parental choice, each parent selects the track that maximizes her child's expected end-of-school skill, taking track-specific peers and instruction pace as given. Under planner assignment, a social planner allocates all children to maximize aggregate expected end-of-school skills. Proposition 1 shows that, under both mechanisms, track assignment is governed by a skill threshold $\tilde{\theta}$. With positive direct peer effects ($v_1 > 0$), the threshold is lower under parental choice than under planner assignment.¹⁹ The difference arises because the planner internalizes the peer externality while parents do not: better peers for one child come at the expense of worse peers for others, so that gains from direct peer effects are exactly offset in aggregate, leaving only the

¹⁷In the estimation, all parameters are allowed to vary by period. To lighten notation, however, we omit the subscript j here.

¹⁸These restrictions are consistent with our estimates (Section 3.1.1).

¹⁹With a symmetric skill distribution, the planner sets the threshold at the mean, minimizing within-track skill variance and thus the aggregate instructional mismatch.

mismatch channel as a source of efficiency gains from tracking.²⁰ Parents, by contrast, only consider the effect of tracking on their own child: since children with skills just below the planner’s threshold would individually benefit from the academic track’s higher average peer skills, they have an incentive to send such children there, thus lowering the skill threshold.

Proposition 1. *Suppose that θ_j is normally distributed. Under both parental choice and planner assignment, children are sorted by a skill threshold $\tilde{\theta}$: those below $\tilde{\theta}$ attend one track—vocational—and children with skills above $\tilde{\theta}$ the other—academic.*

1.1. *Under planner assignment, the optimal threshold equals the average initial skill level, $\tilde{\theta}^P = \mathbb{E}[\theta_j] = 0$.*

1.2. *Under parental choice, the equilibrium threshold $\tilde{\theta}^M$ depends on the direct peer effect. If $v_1 > 0$, then $\tilde{\theta}^M < \tilde{\theta}^P$.*

We next compare tracking with comprehensive schooling in the same one-period setting. Proposition 2.1 shows that tracking weakly raises expected end-of-school skills, regardless of the assignment threshold. By reducing within-track skill variance, tracking brings students on average closer to their track’s instruction pace, raising learning efficiency.²¹ These gains increase with the skill-instruction complementarity v_3 . The planner’s threshold maximizes this gain by minimizing within-track variance.

Proposition 2.

2.1. *Expected end-of-school skills are weakly higher under tracking than under comprehensive schooling. The gain from tracking is*

$$\mathbb{E}(\theta_{j+1}|T) - \mathbb{E}(\theta_{j+1}|C) = \frac{v_3^2}{2v_4} \left(\sigma_{\theta_j}^2 - \mathbb{E}(\text{Var}[\theta_j|S]) \right) \geq 0,$$

and is strictly positive whenever both tracks have positive mass and different average skill levels.

²⁰This exact offset is a property of linear-in-means direct peer effects and need not hold if peer effects are non-linear. While a large literature documents such non-linearities, there is little consensus on their form or size (Sacerdote, 2011; Carrell et al., 2013). In our model, non-linearities arise through instruction pace, which makes a child’s learning depend on her distance to the track average. Benefits from more homogeneous groups may also arise through other channels, such as more engaged interactions with similar peers (Booij et al., 2017); our mismatch term captures these in reduced form.

²¹By the law of total variance, tracking can never perform worse than comprehensive schooling, since within-track variance is always weakly smaller than the unconditional variance.

2.2. *The variance of end-of-school skills is higher under optimal tracking than under comprehensive schooling:*

$$\text{Var}(\theta_{j+1} | T) - \text{Var}(\theta_{j+1} | C) = \frac{2}{\pi} \sigma_{\theta_j}^2 \left[\underbrace{\frac{v_3^4}{v_4^2} \left(1 - \frac{2}{\pi}\right) \sigma_{\theta_j}^2}_{\text{instructional matching}} + \underbrace{v_1 (v_1 + 2b)}_{\text{direct peer effects}} \right] > 0,$$

where b is the coefficient on θ_j in (4).

2.3. *Optimal tracking benefits some children and harms others. With positive direct peer effects ($v_1 > 0$), learning losses are concentrated among vocational-track students.*

While optimal tracking increases aggregate learning, it also widens skill inequality (Proposition 2.2), through two channels. The first is instructional matching itself: under tracking, a child's skill gain rises with her distance from the assignment threshold, since children at either end are taught at a pace much closer to their own, while those near the threshold are not. These uneven gains add dispersion even without direct peer effects, and more so the stronger the complementarity between skills and instruction and the more heterogeneous initial skills are.²² The second channel is direct peer effects: academic-track students benefit from higher-skilled peers, whereas vocational-track students are exposed to lower-skilled ones, which further widens the gap between tracks.

Not all children gain from tracking, as it necessarily leaves some—those with initial skills near the threshold—worse off than under comprehensive schooling, where instruction is closer to their individually optimal pace (Proposition 2.3). Without direct peer effects, these losses are symmetric across tracks. With positive peer effects, however, losses concentrate among vocational-track children, who face both a less favorable instructional match and lower-skilled peers.

Underlying these results is the assumption that skill development through the remaining schooling years is fully predictable at the time of track assignment. We now introduce uncertainty over skills in a simplified two-period setting to compare *early tracking* (*ET*) with *late tracking* (*LT*). Under *ET*, children are assigned using the mean threshold from Proposition 1.1 at the beginning of secondary school, before an intermediate i.i.d. skill shock, η_{j+1} , is realized, and remain there through the end of school. Under *LT*, they initially

²²Without direct peer effects, the gain from tracking relative to comprehensive schooling depends only on $|\theta_j|$, which is uncorrelated with θ_j under a symmetric skill distribution. The variance of end-of-school skills therefore equals the variance under comprehensive schooling plus the variance of these gains. See Appendix A.1.

attend comprehensive school and are optimally assigned to tracks only after the shock.²³ Late tracking therefore improves the information available at assignment but forgoes the initial learning gains from homogeneous grouping and tailored instruction.

Proposition 3. *Expected end-of-school skills are higher under optimal late tracking than under mean-split early tracking if and only if $\frac{\sigma_{\eta_{j+1}}^2}{\sigma_{\theta_j}^2}$ exceeds a critical value that is increasing in the direct peer effect v_1 and in the strength of instructional matching $v_3^2/(2v_4)$.*

The variance ratio measures how much uncertainty about students' relative skills is resolved after the early tracking decision. When it is high, skill rankings change substantially, making early assignments increasingly mismatched with later skills and delaying tracking more valuable. When it is low, initial skills remain predictive and early tracking preserves the benefits of homogeneous peers and tailored instruction. Stronger direct peer effects and skill-instruction complementarities increase these benefits and thus favor earlier tracking.

Taken together, these results show that the effects of tracking on learning depend critically on its timing. Tracking too early risks subsequent mismatch, whereas tracking too late forgoes earlier gains from homogeneous grouping and tailored instruction. The magnitude of these effects depends on the strength of direct peer spillovers and skill-instruction complementarities relative to uncertainty in skill development. The skill technology can therefore accommodate the mixed evidence on average achievement and the more consistent finding that tracking increases educational inequality and concentrates losses among lower-skilled students (see [Related Literature](#)). The analytical setting, however, considers only skill formation during school. It abstracts from how non-skill factors shape track choices, how tracks affect higher education and labor-market opportunities, and how the resulting changes feed back into educational decisions across generations. We now incorporate these forces into the full model by defining the recursive decision problems at every life stage.

2.2 Life-cycle Decisions

2.2.1 Model Fundamentals

In each period j , preferences over consumption c_j and labor hours n_j are represented by the utility function $u\left(\frac{c_j}{q}, n_j\right)$, where q is a consumption equivalence scale, larger than one when a child is present in the household, and equal to one otherwise. The period budget

²³End-of-school skills are now θ_{j+2} , with an intermediate period in which skills θ_{j+1} are affected by i.i.d. shocks $\eta_{j+1} \sim \mathcal{N}(0, \sigma_{\eta_{j+1}}^2)$, as in the full model.

constraint in all working periods ($j = 5$ to $j = 16$) is

$$c_j + a_{j+1} = y_j + (1 + r)a_j - T(y_j, a_j), \quad (5)$$

where y_j is gross labor income, r is the period interest rate on the risk-free asset a_j , and $T(y_j, a_j)$ denotes taxes net of lump-sum transfers, comprising labor income and capital taxes. In addition, consumption, labor hours, and asset choices are subject to standard time ($n_j \in [0, 1]$) and zero borrowing ($a_{j+1} \geq 0$) constraints.

Gross labor income is $y_j = w_E h_j n_j$, where w_E is the equilibrium wage per efficiency unit of human capital h_j and $E \in \{0, 1\}$ indicates college education. Following Yum (2023), working-life human capital evolves according to

$$h_{j+1} = \gamma_{j,E} h_j \varepsilon_{j+1}, \quad \log \varepsilon_{j+1} \sim \mathcal{N}(0, \sigma_\varepsilon^2), \quad (6)$$

where $\gamma_{j,E}$ are age- and education-specific deterministic growth rates, and ε_{j+1} are i.i.d market luck shocks, as in Huggett et al. (2011). Agents discount future utility by β , taking expectations over child skill shocks while a child is present in the household and over market-luck shocks throughout working life.

2.2.2 The School Track Decision

When the child turns 10, the parent chooses between the vocational or academic track, $S' \in \{V, A\}$.²⁴ The child then remains in that track for two periods, until the end of secondary school at age 18.²⁵ At the time of the track decision, corresponding to $j = 11$, the parent's state space consists of education E , human capital h_j , assets a_j , and her child's skill level $\theta'_{j'}$, where $j' = 3$. The values are then given by

$$V_{11}(E, h_{11}, a_{11}, \theta'_3) = \max_{S' \in \{V, A\}} \{W_{11}(E, h_{11}, a_{11}, \theta'_3, S' = V), W_{11}(E, h_{11}, a_{11}, \theta'_3, S' = A) - \chi\},$$

²⁴This choice is unconstrained by education policy in most of Germany; see Appendix B.2 for more details.

²⁵We abstract from track switches during secondary school. Switches during the first stage of secondary school are rare: about 2.5% of children switched school tracks in 2010/11 (Bellenberg and Forell, 2012). Transitions at the end of secondary school are captured by the college decision at age 18 (see Appendix B.2).

where interim values for track S' are given by

$$W_{11}(E, h_{11}, a_{11}, \theta'_3, S') = \max_{c_{11}, a_{12} \geq 0, n_{11} \in [0,1]} \left\{ u\left(\frac{c_{11}}{q}, n_{11}\right) + \beta \mathbb{E} V_{12}(E, h_{12}, a_{12}, \theta'_4, S') \right\}$$

s.t. (4), (5), and (6).

The academic-track taste shock, or utility cost, $\chi \sim H^E(\chi)$ captures unmodeled idiosyncratic heterogeneity in track choices. Its unconditional mean captures how demanding the two periods in the academic track are relative to the vocational alternative, with a positive value indicating that the academic track is more demanding or psychologically taxing. Its distribution H^E depends on parental education, capturing family-background-related factors beyond child skills that matter throughout the child's time in the track, such as the support children receive at home, the family's familiarity with the academic environment, or a preference for children to follow their parents' path (Doepke and Zilibotti, 2017).²⁶

2.2.3 College Choice

After secondary school, at age 18 ($j = 5$), the young adult becomes independent and chooses whether to go to college. Her state space at this stage consists of end-of-school skills θ_5 , secondary school track S , and parental education E° . We assume that she has zero initial wealth, but expects to receive a monetary transfer τ from her parent at the end of this period, after the college decision has been made.²⁷ She therefore decides whether to attend and graduate from college ($E = 1$) or not ($E = 0$) by solving the problem

$$V_5(\theta_5, \tau, S, E^\circ) = \max_{E \in \{0,1\}} \{W_5(E = 0, \theta_5, \tau), W_5(E = 1, \theta_5, \tau) - \nu\},$$

²⁶See Dustmann (2004); Hanushek and Wößmann (2006); Pekkarinen et al. (2009); Falk et al. (2026) for evidence of parental influence on school track choice. In NEPS data, children deviating from the teacher's recommendation typically follow their parent's own track. If parents overrode teachers purely because they knew their child's potential better, deviators should perform like the typical child in the track they enter. Instead, those who move up to the academic track perform below its average, and those who move down to the vocational track above it, as the recommendation would have predicted. This suggests that deviations do not purely reflect parental private information about the child's potential (see Appendix Figure B.3).

²⁷Wealth at age 18 is negligible in the data, but young adults may still expect inter-vivos transfers from their parents. Paying the transfer before the college decision would lead children of wealthy parents, who expect large transfers, to increasingly forgo college, at odds with the data.

where interim values conditional on education E are given by

$$\begin{aligned} W_5(E, \theta_5, \tau) &= \max_{c_5, a_6 \geq 0, n_5 \in [0, \bar{n}(E)]} \{u(c_5, n_5) + \beta \mathbb{E} V_6(E, h_6, a_6 + \tau)\} \\ \text{s.t. } h_6 &= \gamma_{5,E} \exp(\theta_5)^{\xi_E} \varepsilon_6, \quad \log \varepsilon_6 \sim \mathcal{N}(0, \sigma_\varepsilon^2), \\ c_5 + a_6 &= w_0 \exp(\theta_5) n_5 - T(y_5, 0). \end{aligned}$$

College education and end-of-school skills affect initial human capital, h_6 , through $\gamma_{j,E}$ and ξ_E . Following [Daruich \(2026\)](#), ξ_E allows college and end-of-school skills to be complementary, making college more beneficial for higher-skilled students. College students may work at the non-college wage w_0 , subject to a tighter time constraint: $\bar{n}(E = 1) < \bar{n}(E = 0) = 1$. Consistent with the German system, college is tuition-free.

Mirroring the academic-track choice, and in line with the literature (e.g., [Daruich, 2026](#); [Fuchs-Schündeln et al., 2022](#)), college attendance is also affected by a stochastic taste shock $\nu \sim G^{E^\circ, S}(\nu)$, capturing unmodeled heterogeneity in college costs, akin to the “psychic” cost in that literature. Its mean captures how difficult attending college is relative to working directly, with a positive value indicating that college is more demanding or psychologically taxing. Its distribution depends on parental education E° , capturing similar family-background-related factors to the ones described above for χ , and on the secondary school track S , capturing the academic track’s easier access to college.²⁸

2.2.4 Parenthood, Remaining Work Life and Retirement

Parenthood At age 50 ($j = 13$), altruistic parents choose an inter-vivos transfer τ , taking into account the child’s expected continuation value upon independence, $V_{j'=5}$. Following [Daruich \(2026\)](#), parents observe their current market-luck shock and the child’s final skill shock, but not the child’s college taste shock or college choice. The parent’s value at the beginning of period 13 is

$$V_{13}(E, h_{13}, a_{13}, \theta'_5, S') = \max_{\tau \geq 0} \left\{ \tilde{V}_{13}(E, h_{13}, \tilde{a}_{13}) + \Lambda \mathbb{E}_{\nu'} V_{j'=5}(\theta'_5, \tau, S', E) \right\},$$

²⁸Academic-track graduates are significantly more likely to attend college, partly because they automatically obtain a college-entrance qualification in Germany, unlike vocational-track graduates, who can still qualify later through “second-chance” routes (see [Dustmann et al., 2017](#)).

where Λ governs the strength of parental altruism. The value for a parent with remaining assets $\tilde{a}_{13} = a_{13} - \tau$, after the inter-vivos transfer has been made, $\tilde{V}_{13}$, is then

$$\begin{aligned} \tilde{V}_{13}(E, h_{13}, \tilde{a}_{13}) &= \max_{c_{13}, a_{14} \geq 0, n_{13} \in [0,1]} \{u(c_{13}, n_{13}) + \beta \mathbb{E} V_{14}(E, h_{14}, a_{14})\} \\ \text{s.t. } c_{13} + a_{14} &= y_{13} + (1+r)\tilde{a}_{13} - T(y_{13}, \tilde{a}_{13}), \text{ and (6),} \end{aligned}$$

where $\tau \geq 0$, so parents cannot borrow against the future income of their child.²⁹

In the remaining periods with a child at home, $j \in \{9, 10, 12\}$, parents choose consumption, savings, and labor supply while anticipating the child's skill evolution. For $j = 10, 12$, corresponding to the child's primary school years (ages 6–9) and the second half of secondary school ($j' = 4$, ages 14–17), their problem is

$$\begin{aligned} V_j(E, h_j, a_j, \theta'_{j'}, S') &= \max_{c_j, a_{j+1} \geq 0, n_j \in [0,1]} \left\{ u\left(\frac{c_j}{q}, n_j\right) + \beta \mathbb{E} V_{j+1}(E, h_{j+1}, a_{j+1}, \theta'_{j'+1}, S') \right\} \\ \text{s.t. } &\text{(4), (5) and (6).} \end{aligned}$$

In period $j = 9$, corresponding to early childhood (ages 2–5), the problem is identical, except that the child's initial school-entry skill is given by

$$\theta'_2 = \iota \log(h_9) + \eta'_2, \quad \eta'_2 \sim \mathcal{N}(\mu_{\eta_2}, \sigma_{\eta_2}^2),$$

capturing the intergenerational transmission of human capital before primary school.³⁰

Remaining Work Life Outside the parenthood periods— $j = 6, 7$ (ages 22–29) and $j = 14, 15, 16$ (ages 54–65)—working-age adults solve the standard consumption, labor-supply, and savings problem

$$\begin{aligned} V_j(E, h_j, a_j) &= \max_{c_j, a_{j+1} \geq 0, n_j \in [0,1]} \left\{ u\left(\frac{c_j}{q}, n_j\right) + \beta \mathbb{E} V_{j+1}(E, h_{j+1}, a_{j+1}) \right\} \\ \text{s.t. } &\text{(5), (6).} \end{aligned}$$

In period $j = 8$ (ages 30–33), adults anticipate that they will have a child at the beginning

²⁹The transfer is deducted from the parent's beginning-of-period assets and credited to the child at the start of her next period, so it earns no return in period 13, which acts as a proportional cost on inter-vivos transfers.

³⁰We treat this as the initial skill due to a lack of reliable pre-primary-school data. The influence of parental human capital captures factors such as parental investment and the living environment during infancy and early childhood, as well as genetic transmission.

of the next period, so their continuation value depends on the future child's skill level $\theta'_{j'+1}$.

Retirement Everybody retires at age 66 ($j = 17$). Human capital then remains fixed, and retirees receive benefits $\pi_j(h_{16}, E) = \Omega h_{16} w_E$, proportional to final working-life earnings capacity.³¹ For $j = 17, \dots, 20$, retirees solve

$$\begin{aligned} V_j(E, h_{16}, a_j) &= \max_{c_j, a_{j+1} \geq 0} \left\{ u\left(\frac{c_j}{q}, 0\right) + \beta V_{j+1}(E, h_{16}, a_{j+1}) \right\} \\ \text{s.t. } c_j + a_{j+1} &= \pi_j(h_{16}, E) + (1 + r)a_j - T(0, a_j), \end{aligned}$$

and die with certainty after period 20.

2.3 Firm, Government, and Equilibrium

To close the model, we assume a representative firm produces output according to the Cobb-Douglas production function $Y = AK^\alpha H^{1-\alpha}$, where A denotes total factor productivity, K aggregate physical capital stock, and H is aggregate human capital, given by $H = [\varphi H_0^\rho + (1 - \varphi)H_1^\rho]^{\frac{1}{\rho}}$. Here, H_0 and H_1 denote the aggregate labor supply in efficiency units of non-college and college-educated workers, respectively, and ρ governs the constant elasticity of substitution between the two labor types. Physical capital depreciates at a rate of δ .

The government taxes labor income progressively, with after-tax income $y_{net} = \lambda y^{1-\tau_n}$ (Heathcote et al., 2017), and capital income proportionally at rate τ_a . Tax revenue finances retirement benefits π_j and lump-sum social welfare benefits g paid to every household. Any excess is dissipated through wasteful (or utility-separable) government spending, so we do not impose a budget-balancing condition.

We solve for the model's stationary equilibrium and its associated distribution using the numerical strategy in Lee and Seshadri (2019). Stationarity implies that the cross-sectional distribution over all states in every age-period j is constant across cohorts. As is standard, equilibrium requires that households and firms optimize according to their value functions and firm first-order conditions, respectively, with capital and labor prices (r, w_0, w_1) , competitively determined to clear all markets. A special feature of our model is that learning during the school years depends on the distribution of children across tracks. Equilibrium therefore requires that parents' expectations over this distribution coincide with its actual realization. Appendix A.2 provides a detailed equilibrium definition.

³¹As is common in the literature, benefits depend on human capital to proxy for lifetime earnings, the actual basis of pension benefits.

3 Model Calibration and Validation

We calibrate the model to Germany in two steps. We first estimate a set of parameters, including those of the child skill formation technology, directly from the data, and set others to standard values from the literature. We then calibrate the remaining parameters by simulated method of moments. We validate the model using non-targeted moments and a model-based analogue of the natural experiment in [Dustmann et al. \(2017\)](#), and conclude this section with a variance decomposition examining how much of lifetime inequality becomes predictable during childhood and adolescence.

3.1 Model Calibration

Our main data sources are the German National Educational Panel Study (NEPS) and the German Socio-Economic Panel (SOEP), supplemented by official education statistics. NEPS follows nationally representative samples of students starting in 2010 and provides repeated standardized assessments of mathematics, reading, science, and grammar competencies ([Blossfeld et al., 2019](#)). We use Cohorts 2, 3, and 4.³² SOEP is an annual representative household survey providing information on education, labor supply, and income ([Goebel et al., 2019](#)). We use the 2010–2018 waves to construct the working-life moments, retaining workers and non-workers but excluding workers with hourly wages below the 1st or above the 99th percentile. All monetary variables are expressed in 2015 euros.

3.1.1 Externally Estimated Parameters

Child Skill Technology As is common in the literature ([Cunha et al., 2010](#); [Agostinelli and Wiswall, 2025](#)), we treat test scores as imperfect measures of children’s latent skills, adopting a latent variable approach with a log-linear measurement system based on a series of test scores in each period. Following [Cunha et al. \(2010\)](#), we use covariances to identify the scales and loadings of each measure, and to account for measurement error, aggregate the measures into a composite skill index θ_j using Bartlett factor scores, as in [Agostinelli et al. \(2026\)](#).³³

We specify the empirical analog of child i ’s (log) skill technology (4) as

$$\theta_{i,j+1} = \omega_{0,j} + \omega_{1,j} \theta_{i,j} + \omega_{2,j} \bar{\theta}_{-i,j}^S + \omega_{3,j} (\theta_{i,j} - \bar{\theta}_j^S)^2 + \omega_{4,j} \theta_{i,j}^2 + \omega_{5,j} E_i + \eta_{i,j+1},$$

³²Appendix [B.3](#) describes the assessments, samples, and variable construction.

³³Appendix Section [B.4](#) details skill measurement and estimation.

Table 1: Externally Calibrated and Estimated Parameters

Panel A: Child Skill Technology				
Parameter	$j = 2$	$j = 3$	$j = 4$	Interpretation
$\omega_{1,j}$	0.76	0.77	0.90	Own-skill persistence
$\omega_{2,j}$	0	0.15	0.13	Direct peer effect
$\omega_{3,j}$	0	−0.13	−0.19	Squared distance to track mean
$\omega_{5,j}$	0.07	0.05	0.05	Parental education
Panel B: Human-Capital Growth Profiles (SOEP)				
Parameter	Values for $j = 5, \dots, 15$			
$\{\gamma_{j,0}\}_{j=5}^{15}$	(1.00, 0.96, 1.09, 1.10, 1.04, 1.02, 1.00, 1.01, 0.99, 1.01, 0.99)			
$\{\gamma_{j,1}\}_{j=5}^{15}$	(1.00, 1.15, 1.19, 1.11, 1.06, 1.01, 0.99, 0.97, 0.98, 0.99, 1.01)			
Panel C: Remaining Parameters				
Parameter	Value	Description (source)		
σ	1.5	Inverse EIS (Lee and Seshadri, 2019)		
ϕ	0.5	Frisch elasticity (Fuchs-Schündeln et al., 2022)		
α	1/3	Capital share (standard value)		
ρ	1/3	CES curvature, elasticity of substitution 1.5 (Ciccone and Peri, 2005)		
δ	4%	Annual depreciation		
λ	0.679	Labor-tax scale (Kindermann et al., 2020)		
τ_n	0.128	Labor-tax progressivity (Kindermann et al., 2020)		
τ_a	0.25	Capital-income tax rate (German withholding tax)		
g	0.06	Lump-sum transfers (6% of average labor earnings)		
q	1.56	Household equivalence scale (Jang and Yum, 2024)		
$\bar{n}(E = 1)$	0.40	Maximum work time during college (40 hours of study per week)		
ξ_0	1	Non-college return to school skills (normalization)		

Notes: Panel A reports the child skill technology parameters for primary school ($j = 2$, NEPS Cohort 2, Grades 1–4), early secondary school ($j = 3$, Cohort 3, Grades 5–9), and late secondary school ($j = 4$, Cohort 4, Grades 9–12); the constants $\omega_{0,j}$ are normalized to zero. Panel B reports four-year human-capital growth factors for non-college-educated ($E = 0$) and college-educated ($E = 1$) individuals.

where E_i equals one if at least one of child i 's parents holds a college degree, $\bar{\theta}_j^S$ is the average skill of all children in track S , and $\bar{\theta}_{-i,j}^S$ is the average skill of child i 's classroom peers. The two averages play different roles. The pace of instruction is set at the track level, because it reflects curricula that depend on the skill distribution of the whole track. Direct peer effects, by contrast, depend on who a child learns with, which varies across classrooms within a track. While the model abstracts from classroom heterogeneity and represents each track by a single class, the estimation uses classroom-level peer skills to exploit this variation.

Appendix Tables B.5 and B.6 present four specifications (with or without school fixed effects and weights) for grades 5–9 and 9–12 ($j = 3, 4$) using NEPS Cohorts 3 and 4. Estimates are stable across specifications, and we cannot reject $-\hat{\omega}_{3,j} = \hat{\omega}_{4,j}$, consistent with our model. Because instruction pace is unobserved, we interpret $\omega_{3,j}$ through the lens of the model, as a non-linear interaction between a child's skill and the track's instruction pace. The positive

peer effects fall within the wide range in the literature (see Table 4.2 in [Sacerdote, 2011](#)), and the squared-distance estimates imply that an increase of 0.01 in the squared distance to a track’s average skill lowers grade-9 skills by 0.07–0.18% and grade-12 skills by 0.08–0.32%, consistent with negative effects of classroom heterogeneity on achievement.³⁴

Panel A of Table 1 presents the calibrated parameters. We impose $-\omega_{3,j} = \omega_{4,j}$ and set $\omega_{2,j}$ and $\omega_{3,j}$ to their average estimates across the four specifications in period-3 and period-4, and we normalize the constants $\omega_{0,j}$ to zero. Before tracking (period 2) there is a single comprehensive track, so $\omega_{2,2} = \omega_{3,2} = 0$. The parameters $\omega_{1,j}$ and $\omega_{5,j}$ come from a separate regression of future skills on current skills and parental education (Appendix Table B.7), whose point estimates we use for $j = 2, 3, 4$.³⁵ The implied persistence of 0.76–0.90 is in line with standard estimates (e.g., [Cunha et al., 2010](#); [Agostinelli et al., 2025](#)).

Human Capital Growth We estimate the deterministic human capital growth profiles $\gamma_{j,E}$ for $j = 5, \dots, 15$ using wage regressions on SOEP data, following [Lagakos et al. \(2018\)](#) (details in Appendix B.6). We set $\{\gamma_{j,E}\}_{j=5}^{15}$ to the estimated experience-wage growth rates for four-year experience bins, expressed relative to the previous bin (Panel B of Table 1).

Remaining Parameters We specify adults’ preferences over consumption and labor supply in each period using the utility function:

$$u\left(\frac{c_j}{q}, n_j\right) = \frac{(c_j/q)^{1-\sigma}}{1-\sigma} - b \frac{n_j^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}}.$$

We set the inverse elasticity of intertemporal substitution to $\sigma = 1.5$ ([Lee and Seshadri, 2019](#)) and the Frisch elasticity of labor supply to $\phi = 0.5$ ([Fuchs-Schündeln et al., 2022](#)). The disutility-of-work parameter b is calibrated internally. On the production side, we set the capital share to $\alpha = 1/3$ and $\rho = 1/3$, implying an elasticity of substitution of 1.5 between college and non-college labor ([Ciccone and Peri, 2005](#)). Capital depreciates at an annual rate of $\delta = 4\%$. For the tax and transfer system, we set the labor income tax scale to $\lambda = 0.679$ and progressivity to $\tau_n = 0.128$, following [Kindermann et al. \(2020\)](#), and the linear capital tax to $\tau_a = 0.25$, corresponding to Germany’s final withholding tax on capital gains, interest, and dividends. The lump-sum government transfer is set to $g = 0.06$, equivalent to 6% of

³⁴[Duflo et al. \(2011\)](#), for instance, attribute tracking’s achievement gains to indirect peer effects via teaching adjustments; direct comparisons are difficult, however, since our estimates capture heterogeneity across tracks rather than within classrooms.

³⁵The exception is $\hat{\omega}_{1,4}$, which we take from Appendix Table B.6, since the corresponding point estimate in Appendix Table B.7 exceeds 1.

average labor earnings in equilibrium. Finally, the household equivalence scale is $q = 1.56$ (Jang and Yum, 2024), and since full-time study takes about 40 hours per week—roughly 60% of the maximum weekly work hours—we set the maximum working time during higher education to $\bar{n}(E = 1) = 0.40$.³⁶

3.1.2 Internally Calibrated Parameters

We internally calibrate the remaining 21 parameters via the simulated method of moments, targeting 21 data moments. Although all parameters jointly influence all moments, we provide intuition on which moments primarily identify each one. Table 2 presents the parameters, their estimated values, and the model-implied and target moments.

Preferences and Educational Taste Shocks We set the discount factor β so that the equilibrium interest rate equals 4% annually, as is standard. The labor supply disutility shifter b targets the average time worked in the SOEP data, 0.36 of total available time (normalized to 1). The altruism parameter Λ is calibrated so that the model’s ratio of average parent-to-student transfers to average labor income of parents at the transfer age matches its empirical counterpart.³⁷

The academic-track taste shock follows $\chi \sim H^E \equiv \mathcal{N}(\mu_\chi^E, \sigma_\chi^2)$, with a parental-education-specific mean that deviates symmetrically from an unconditional mean: $\mu_\chi^{E=1} = \mu_\chi + \Delta(\mu_\chi^E)$ and $\mu_\chi^{E=0} = \mu_\chi - \Delta(\mu_\chi^E)$. We calibrate μ_χ to the overall academic-track share and $\Delta(\mu_\chi^E)$ to the academic-track shares by parental education. The variance σ_χ^2 targets the R^2 from a regression of school-track attendance on end-of-primary-school skills and parental education.

The college taste shock follows $\nu \sim G^{E,S'}(\nu) \equiv \mathcal{N}(\mu_\nu^{E,S'}, \sigma_\nu^2)$, whose mean combines a symmetric deviation by parental education with a common shift for vocational-track graduates: $\mu_\nu^{E=1,S'} = \mu_\nu + \Delta(\mu_\nu^E) + \mathbb{1}_{\{S'=V\}}\mu_\nu^V$ and $\mu_\nu^{E=0,S'} = \mu_\nu - \Delta(\mu_\nu^E) + \mathbb{1}_{\{S'=V\}}\mu_\nu^V$. The mean μ_ν and the vocational shift μ_ν^V match college-attendance shares of academic- and vocational-track graduates, respectively, and $\Delta(\mu_\nu^E)$ matches the ratio of college shares between children of college-educated and non-college-educated parents. The variance σ_ν^2 targets the R^2 from a regression of college education on grade 9 skills and parental education.

³⁶The average study length in Germany is eight semesters, or four years.

³⁷Average monthly transfers from private households to students were 460 euros in 2018 (Statistisches Bundesamt (Destatis), 2020), about 20.7% of the labor income of 50–53-year-old parents, the age at which children in our model become independent.

Table 2: Internally Calibrated Parameters

Param.	Value	Description	Target	Data	Model
Preferences					
β	0.954	Discount Factor	Annl. Interest Rate	0.04	0.04
b	22.1	Labor Disutility	Avrg. Labor Supply	0.36	0.36
Λ	0.383	Parental Altruism	Transfer/Income	0.21	0.21
Academic-Track Taste Shock					
μ_χ	0.284	Intercept	Share A-Track	0.44	0.44
$\Delta(\mu_\chi^E)$	0.05	Diff in χ by E	$\frac{\text{Share in A from CL HH}}{\text{Share in A from NCL HH}}$	2.05	2.04
σ_χ^2	0.039	Var. A-Track Cost Shock	Reg. S' on θ'_3 and E : R^2	0.30	0.34
College Taste Shock					
μ_ν	1.032	Intercept	Share in CL from A-Track	0.46	0.47
μ_ν^V	0.114	Add. Costs for V-Track	Share in CL from V-Track	0.14	0.13
$\Delta(\mu_\nu^E)$	0.12	Diff. in Means by E	$\frac{\text{Share in CL from CL HH}}{\text{Share in CL from NCL HH}}$	2.57	2.60
σ_ν^2	0.21	Var. CL Taste Shock	Reg. E' on θ'_4 & E : R^2	0.18	0.14
Child Skills and Human Capital					
ι	0.389	Coefficient on $\log h_9$	Initial Skill Gap by Parental Education (in sd)	0.63	0.63
μ_{η_2}	-0.107	Mean of Initial Ability	Normalization	0.00	0.00
$\sigma_{\eta_2}^2$	0.077	Var. of Initial Skills	Normalization	0.10	0.10
$\sigma_{\eta_3}^2$	0.04	Var. Learning Shock $j = 3$	Rank $_{j=2}$ -Rank $_{j=3}$	0.75	0.76
$\sigma_{\eta_4}^2$	0.035	Var. Learning Shock $j = 4$	Rank $_{j=3}$ -Rank $_{j=4}$	0.79	0.79
$\sigma_{\eta_5}^2$	0.038	Var. Learning Shock $j = 5$	Rank $_{j=4}$ -Rank $_{j=5}$	0.74	0.77
ξ_1	1.09	CL Returns to Human Capital	Reg. E' on θ'_5 : coef. θ'_5	0.51	0.47
σ_ε^2	0.009	Var. Market Luck Shock	Increase in Wage Gini (54-26)	0.04	0.04
Miscellaneous					
Ω	0.117	Pension Anchor	Replacement Rate	0.48	0.47
A	3.186	TFP	Avrg. Labor Earnings	1.0	1.0
φ	0.604	Weight Non-CL H_0	CL to Non-CL Earnings	1.50	1.52

Notes: This table presents the internally calibrated parameters, targeted moments, and their model-generated counterparts. See Appendix B.3 for details on data sources.

Child Skills and Human Capital The coefficient on parental human capital, ι , is calibrated to match the difference in average skills at age 6 by parental education. The parameters μ_{η_2} and $\sigma_{\eta_2}^2$ of the intergenerational skill shock are set so that initial log skills have mean zero and variance of 0.1, consistent with our data normalization.³⁸ The shock variances over the remaining childhood periods, $\sigma_{\eta_{j+1}}^2$ for $j = 2, 3, 4$, target the correlation of a child's skill percentile rank across periods, which captures changes in relative position not explained by the deterministic components of skill formation or by track assignment.

We normalize $\xi_0 = 1$ for non-college human capital and calibrate ξ_1 , the analogous coefficient for college graduates, to match the coefficient on end-of-school skills from a regression of

³⁸We re-normalize skills to this variance in every period, for consistency with the data.

college graduation on skills. The estimated $\xi_1 > 1$ implies a complementarity between school skills and college education. After college, adults face idiosyncratic market luck shocks that accumulate in human capital over the life cycle (Yum, 2023). We calibrate their variance σ_ϵ^2 to match the difference in the Gini coefficient of wages between ages 54-57 and ages 26-29.

Miscellaneous Following Lee and Seshadri (2019), we set the weight on non-college human capital in the firm’s CES aggregator, φ , to the college earnings premium in the SOEP data. The TFP parameter A is calibrated so that average labor income equals 1, and the pension scale parameter Ω to match an average replacement rate of 48% (Mahler and Yum, 2024).

3.2 Model Validation

We assess the model’s validity in two ways. First, we compare targeted and non-targeted moments from the model-simulated data to their empirical counterparts. Second, we conduct a model-based analogue of the natural experiment in Dustmann et al. (2017), who use school-entry age cutoffs to estimate the effects of initial track attendance on later outcomes.

3.2.1 Model Fit of Targeted and Non-targeted Moments

Table 2 shows that the model fits the targeted moments well, including the academic-track share (44%) and transitions from each track into college, which together imply an aggregate college graduation rate of 28%, as well as differences in educational attainment by parental background. The model slightly understates the relationship between end-of-school skills and college graduation, although its coefficient lies within the empirical confidence interval. Matching the persistence of skill ranks across school stages requires skill-shock variances of 0.035–0.04.

Table 3 shows that the model also performs well on non-targeted measures of intergenerational mobility and inequality. It generates an intergenerational labor income rank elasticity of 0.35 (children aged 30–34 relative to their parents at ages 46–50), close to the SOEP-based estimate of 0.34 in Baarck et al. (2025). It also broadly reproduces the relationship between parental income and academic-track graduation in Dodin et al. (2024): a ten-percentile increase in parental income rank raises graduation by 4 percentage points in the model versus 5.2 in the data, and the Q5/Q1 ratio is 2.16 versus 2.13. The wage Gini at age group 57–60 is similarly close to its empirical counterpart.

The model also matches non-targeted moments on child skills and education choices, the core mechanism of our model. It closely reproduces skill gaps by parental education at

Table 3: Non-targeted Moments

Moment	Data	Model
Intergenerational Mobility and Inequality		
Rank-rank coefficient (Baarck et al., 2025)	0.34	0.35
Parental Income Gradient (Dodin et al., 2024)	0.52	0.40
Q5/Q1 A-track on Income (Dodin et al., 2024)	2.13	2.16
Q1 A-track on Income (Dodin et al., 2024)	0.34	0.28
Wage Gini (57-60)	0.27	0.26
Child Skills and Education Choices		
<i>Differences in Average Skills by Parental Education (in Standard Deviation)</i>		
Beginning Secondary School	0.67	0.69
Middle Secondary School	0.73	0.73
<i>Differences in Average Skills by School Track (in Standard Deviation)</i>		
Beginning Secondary School	1.01	1.12
Middle Secondary School	1.15	1.02
<i>Rank-Rank Coefficients</i>		
Rank _{j=2} – Rank _{j=3} if S = A*	0.67	0.70
Rank _{j=3} – Rank _{j=4} if S = A	0.70	0.73
Rank _{j=2} – Rank _{j=3} if S = V*	0.68	0.70
Rank _{j=3} – Rank _{j=4} if S = V	0.74	0.70

Notes: This table presents non-targeted moments and their model-generated counterparts.

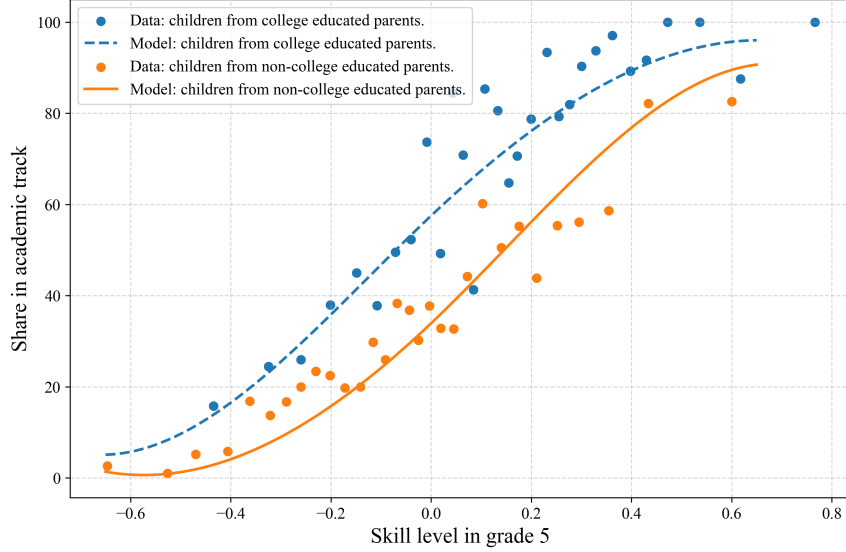
* We exploit the panel structure of the data and group students by their future school track, though the Cohort 2 sample is skewed toward academic-track students (see Appendix Table B.3).

the start of secondary school (0.69 standard deviations in the model vs. 0.67 in the data) and in mid-secondary school (0.73 in both), skill gaps across tracks of around one standard deviation at both stages, and within-track skill-rank persistence. Figure 2 shows that, for both parental-education groups, the model produces S-shaped curves that closely follow the shape of the data across the skill distribution. Appendix Tables B.5–B.7 further show that child skill regressions estimated on 100 model samples sized to match our data closely reproduce the empirical estimates.

3.2.2 The Effects of Track Choice on Students at the Margin

We next conduct a model-based analogue of the natural experiment in Dustmann et al. (2017), which compares children born just before and just after Germany’s July 1 school-entry cutoff. Children born just before the cutoff, who are among the youngest in their cohort, are about 5.2 percentage points less likely to enter the academic track. Despite this early disadvantage, they find no corresponding gap in college completion or in earnings at age 30, consistent with the maturity gap narrowing over time and with later *second-chance opportunities* that allow vocational-track students to enter higher education.

Figure 2: Academic Track Attendance by Skills and Parental Education



Notes: This figure shows the share of children attending the academic track as a function of their skills. Blue and orange dots represent data moments for children from college- and non-college-educated backgrounds, respectively. Lines show the model-simulated counterparts. Data source: NEPS Cohort 3. All observations are weighted using NEPS weights and our own track-share weights (see Appendix B.3).

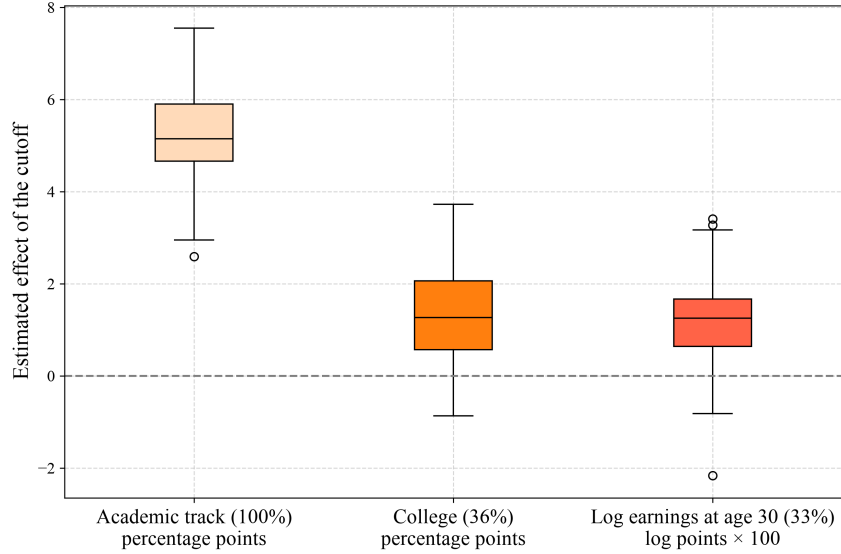
Because the model does not include birth dates, we use the timing of skill shocks as a reduced-form proxy for maturity. These shocks capture any factor beyond ability, parental input, schooling, and peers that influences skill acquisition: large early shocks represent earlier maturation, as for children born after July 1, and large later shocks represent later maturation, as for children born before July 1. This embeds the catch-up process that Dustmann et al. (2017) emphasize, namely that initially less mature children develop faster later on. We define the maturity scale

$$x = \eta_2 + \eta_3 - \kappa(\eta_4 + \eta_5) + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2),$$

where the skill shocks η_j are standardized and ϵ captures other factors that influence skill shocks besides date of birth. We choose σ_ϵ^2 to match the 5.2 percentage-point gap in academic-track access. We set $\kappa = 0.575$ so that, within each track, children on either side of the cutoff end school with approximately the same expected end-of-school skills. This mirrors the exclusion restriction of Dustmann et al. (2017), namely that school starting age affects long-term outcomes solely through track assignment.³⁹ After trimming the top

³⁹ κ is the average of $(b_2 + b_3)/(b_4 + b_5)$ across the two tracks, 0.61 (academic) and 0.54 (vocational), where b_j is the coefficient on the standardized shock η_j in a regression of end-of-school skills among children

Figure 3: Model-Simulated Effects of School-Entry Timing on Track, College, and Earnings Outcomes



Notes: This figure shows, across 100 Monte Carlo replications, the distribution of estimated treatment effects on academic-track attendance, college graduation, and earnings at age 30, obtained by applying [Dustmann et al. \(2017\)](#)’s regression discontinuity design to model-simulated data. In each replication, we compare model-simulated agents whose skill-shock draws place them just below and just above the synthetic July 1 school-entry cutoff—mimicking children born just before and just after the cutoff in the original study. Estimates are bias-corrected local-linear RD estimates, with robust bias-corrected p-values ([Calonico et al., 2014](#)). Effects on academic-track attendance and college graduation are in percentage points, and effects on earnings are in percent (log points $\times$ 100). Each replication uses about 225,000 simulated observations after trimming. The frequency of estimates significant at the 5% level is in parentheses.

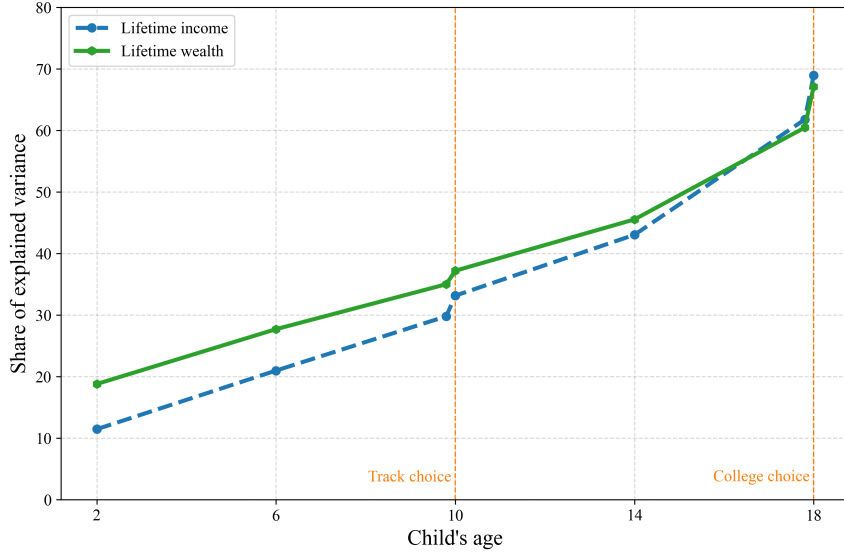
and bottom 5% of the scale, we divide the simulated population into 365 bins representing days of birth, with July 1 as the highest bin and June 30 as the lowest, and apply the same cutoff design as in the data.

Figure 3 summarizes, in boxplots, 100 Monte Carlo replications of a regression discontinuity design at the synthetic July 1 cutoff, each with a new draw of ϵ . By construction, the effect on academic-track attendance is centered around 5.2 percentage points. The corresponding effects on college graduation and earnings at age 30 are much smaller (medians of 1.3 percentage points and 1.3 percent, respectively) and statistically insignificant at the 5% level in 64% and 67% of replications, respectively.⁴⁰ A sizable gap in initial track attendance across the cutoff thus translates into small and mostly statistically insignificant differences in later-life outcomes, as in [Dustmann et al. \(2017\)](#).

attending that track.

⁴⁰Appendix Figure B.2 illustrates the regression discontinuity design for one draw.

Figure 4: Share of Lifetime Income and Wealth Variance Explained, by Age



Notes: This figure shows the share of variation in lifetime labor income (blue dashed line) and wealth (green solid line) explained at each stage of early life. Vertical lines at ages 10 and 18 mark the school-tracking and college choices, respectively. Just to the left of each line, we recompute the explained variance share excluding that decision (and, at age 18, the expected transfer as well).

3.2.3 Decomposition of Lifetime Inequality

That track choice has little effect on later outcomes for students at the margin does not imply that tracking, or the schooling years more broadly, is unimportant for the distribution of outcomes across the population. Following [Huggett et al. \(2011\)](#), [Lee and Seshadri \(2019\)](#), and [Bolt et al. \(2025\)](#), we decompose the variance of lifetime labor income and wealth into variation explained by agents' state variables at successive childhood stages. At each age, the explained share is the variance of conditional mean outcomes across groups defined by the state variables observed at that stage, relative to total outcome variance.⁴¹

Figure 4 shows that parental characteristics before birth explain 11% of the variance in lifetime income and 19% of the variance in lifetime wealth, indicating that family background is already informative about later outcomes before children enter school. By age 10, when track assignment occurs, these shares have risen to 33% and 37%, respectively. Conditional on the other state variables observed at that age, adding the realized track choice increases the explained share by about three percentage points. This increment should not be interpreted as the causal effect of track attendance, because track choice is jointly determined

⁴¹Continuous state variables are discretized into three bins. For example, the age-ten state includes child skills and track, as well as parental human capital, education, and assets.

with child skills and family background. In fact, variation in child skills accounts for most of the explained variation at that age.

By age 18—once young adults are independent and have made their college decision—close to 70% of the variation in lifetime income and wealth is already explained, with the remainder stemming primarily from market luck shocks during the working years.⁴² Skill differences again drive most of this explained heterogeneity. Excluding the college decision lowers the explained share by just over four percentage points, and excluding expected parental transfers by about three more. As at age 10, these increments are not causal effects, since the choices are correlated with skills and family background. Overall, this decomposition shows that much of lifetime inequality becomes predictable during childhood and adolescence, underscoring the importance of modeling child skill accumulation and the policies that shape it, such as tracking.

4 The Long-Run Effects of Tracking Policies

In this section, we use the calibrated and validated model to quantify the long-run aggregate and distributional effects of four counterfactual tracking policies: delaying tracking, abolishing it, making it more flexible, and imposing a strict skill threshold.

4.1 The Effects of Delaying School Tracking

We first evaluate a four-year delay in tracking, from age 10 to 14, approximately the average tracking age across OECD countries (OECD, 2020). Under this *Late Tracking* (*LT*) policy, children remain in comprehensive school until age 14 and then enter the academic or vocational track, with track-specific instruction paces adjusting endogenously to each track’s student composition. Because the academic track now lasts one model period rather than two, we rescale its taste shock to the costs of a single period.⁴³ We solve for the new stationary equilibrium under full price adjustment. To isolate the role of general-equilibrium price responses, we also consider a counterfactual in which the efficiency wages w_0 and w_1 and the interest rate r remain fixed at their baseline values. We first examine the policy’s effects on child skills, before turning to its later-life aggregate, distributional and intergenerational outcomes and, finally, the role of student-track misallocation.

⁴²This share is broadly comparable to estimates for the United States and the United Kingdom (Keane and Wolpin, 1997; Huggett et al., 2011; Lee and Seshadri, 2019; Bolt et al., 2025).

⁴³Specifically, we divide χ by $1 + \beta$, treating the shock as a per-period cost that is incurred in each period the child attends the academic track.

4.1.1 The Effects on Child Skills

To better understand the forces driving the aggregate results, we first examine the direct effects of LT on skill gains in each period. To this end, Panel A of Table 4 reports average child skill gains across periods under LT relative to the baseline economy, as a percentage of a baseline standard deviation. The results differ little between the fixed-price counterfactual (Columns 1–3) and the new stationary equilibrium (Columns 4–6), so price adjustment has little direct effect on skill formation in school. Moreover, LT affects skill gains mostly from age 10 onward, since learning gains are essentially unchanged before that.⁴⁴

Between ages 10 and 14, four additional years of comprehensive schooling lower average skill gains by about 1% of a baseline standard deviation, because the more heterogeneous peer group does not allow for tailored instruction (see Section 2.1.2). Between ages 14 and 18, however, skill growth under LT outpaces the baseline by about the same margin, as more skill uncertainty has resolved, yielding more homogeneous classrooms with better-matched instruction (Proposition 3 in Section 2.1.2). These later gains almost fully offset the earlier losses, leaving average end-of-school skills close to the baseline (Panel B in Table 4).⁴⁵

These aggregate numbers, however, mask substantial heterogeneity by parental background. Between ages 10 and 14, children of non-college-educated parents gain about 0.4% (Column 6), whereas children of college-educated parents lose about 5% (Column 5). Children near the center of the skill distribution are well matched to the comprehensive school's pace of instruction, and those who would have attended the vocational track also gain better peers. Children in the upper tail, who would have been more likely to attend the academic track, face a slower pace and less skilled peers. Children of non-college-educated parents are more likely to be among the former, and children of college-educated parents among the latter. After age 14, children of college-educated parents recover about 2%, while the gain of the other group is close to zero. Yet this does not offset the earlier loss: by the end of school, skills are about 1% higher for children of non-college-educated parents and 3% lower for children of college-educated parents. LT thus narrows the skill gap by parental education.

Overall, and in line with Proposition 2.2 in Section 2.1.2, Panel B of Table 4 shows that LT compresses the skill dispersion, and thus skill inequality. Skill variance at age 14 falls by 6.8% because more years of comprehensive schooling expose would-be vocational-track

⁴⁴Earlier skill gains can arise due to changes in the distribution of inherited skills coming from intergenerational feedback, which we discuss below.

⁴⁵Because the gains over the two secondary-school periods nearly offset each other, long-run differences in average end-of-school skills largely mirror differences in average skills at school entry. These arise through the intergenerational feedback discussed in the next subsection.

Table 4: Effects of Late Tracking (LT) on Child Skills

	Baseline Prices			Prices Adjust		
	All (1)	CL (2)	NCL (3)	All (4)	CL (5)	NCL (6)
<i>Panel A: Skill Gains Relative to the Baseline Economy</i>						
Gains btw. Ages 6 and 10	0.0	0.0	+0.1	0.0	0.0	+0.1
Gains btw. Ages 10 and 14	-0.8	-4.8	+0.5	-0.9	-5.0	+0.4
Gains btw. Ages 14 and 18	+0.8	+2.2	+0.3	+0.8	+2.1	+0.3
<i>Panel B: Changes in the Skill Distribution</i>						
<i>Average Skills</i>						
Initial	+0.7	-0.2	-0.1	-0.0	-0.2	-0.1
End-of-School	+0.7	-2.9	+0.9	-0.2	-3.1	+0.8
... if College-Grad.	-0.5			-0.6		
... if Non-College-Grad.	-0.1			-0.2		
<i>Skill Variance (% change)</i>						
Initial	0.0			-0.6		
At Age 14	-6.8			-6.8		
End-of-School	-2.5			-3.1		

Notes: Panel A presents average skill gains under LT across periods, relative to baseline gains. Panel B presents changes in the distribution of child skills under LT , relative to the baseline economy. "CL" and "NCL" denote children of college-educated and non-college-educated parents, respectively. The "if College-Grad." and "if Non-College-Grad." rows report end-of-school skills by the child's own subsequent graduation status rather than by parental education, and are shown only for the full population. Skill variance rows are likewise reported only for the full population. Columns (1)–(3) show the results when the efficiency wages w_0 , w_1 , and the interest rate r are fixed at baseline levels; columns (4)–(6) show the effects in the new general equilibrium. Unless otherwise specified, effects are expressed as percentage of a baseline standard deviation.

children to better peers, and would-be academic-track children to lower-skilled peers. Once tracking occurs, uncertainty over skills has largely resolved, allowing more precise sorting into distinct groups, which partly reverses this compression. Even so, end-of-school variance remains about 3% below its baseline level.

4.1.2 Later-Life Aggregate and Distributional Effects

With prices fixed, LT raises aggregate human capital by 0.3% and output by 0.1% (Column 1 of Table 5), despite no net change in average skill gains during school. The academic-track share increases by 9 percentage points. Part of this reflects the shorter academic track (four years rather than eight), which we model by scaling the entire distribution of χ , including its

mean, its parental-education difference, and its standard deviation.⁴⁶ Part reflects delayed sorting: children of non-college-educated parents, who are more numerous, gain relatively more before tracking. The parental-education ratio in academic-track attendance falls by about 17%. Since academic-track graduation facilitates college access, the college share rises by 1.2 percentage points, though less than one-for-one: the compression of the skill distribution thins the upper tail of students most likely to attend college and lowers the average skills of college graduates by about half a percent of a baseline standard deviation. At fixed wages, however, the larger college share dominates these losses, because college graduates' human capital grows faster. This in turn also raises the initial skills of the next generation through intergenerational human capital transmission.

This picture reverses when prices adjust. In the new stationary equilibrium, aggregate human capital and output fall by about 0.3% (Column 2 of Table 5). Although the academic-track share still rises (+8.5pp), the college share increases by only 0.2 percentage points. The reason is standard general equilibrium forces in the labor market: a larger supply of college-educated labor lowers the college wage premium, which dampens the incentive to attend college, so that in the new stationary equilibrium the long-run college share remains close to its baseline level. Even though average end-of-school skills are barely affected, college attainment thus rises too little to offset the roughly half a percent skill loss among predominantly college-bound students after the reform. As a consequence, aggregate human capital falls. Moreover, with little change in the share of college-educated parents, intergenerational feedback also leaves average skills at school entry essentially unchanged (Column 4 in Panel B of Table 4).

The effects of delaying tracking on aggregate efficiency thus crucially depend on the relative demand for college- and non-college labor, which governs how strongly wages respond to the reform-induced change in relative labor supply. Forces that shift demand toward college labor, such as skill-biased technical change, or knowledge spillovers from a more educated workforce, could dampen the fall in the college wage premium and bring the general equilibrium closer to the fixed-price counterfactual.

Unlike the efficiency effect, delaying tracking increases both mobility and equality regardless of whether prices adjust, with somewhat larger effects in general equilibrium (Column (2) of Table 5). Because children of non-college-educated parents benefit more from longer

⁴⁶As noted in Section 2, we treat the taste shock as a per-period burden, so its mean, family differences, and dispersion all scale with duration. The scaling lowers the taste-driven component of the parental-education gap but also raises the weight of skills in the track choice, and skills differ by parental education, so its net effect on the gap is a priori ambiguous.

Table 5: Aggregate and Distributional Effects of Late Tracking (*LT*)

	Baseline Prices (1)	Prices Adjust (2)
<i>Educational Outcomes</i>		
% Academic Track (in pp.)	+9.0	+8.5
Parental-Education Ratio in Academic-Track Attendance	-17.2	-17.1
% College (in pp.)	+1.2	+0.2
Parental-Education Ratio in College Graduation	-7.5	-6.0
Reg. College on θ_5 : Coef. θ_5	+4.4	+4.4
<i>Efficiency</i>		
Output (Y)	+0.1	-0.3
Human Capital (H)	+0.3	-0.3
<i>Cross-sectional Inequality</i>		
Wage Gini (54-57)	-0.8	-1.1
Standard Deviation of Wealth	-1.1	-1.7
College/Non-College Earnings	-0.3	-0.4
<i>Mobility</i>		
Income Rank-Rank Coefficient	-2.7	-3.6
A-track on Income (Dodin et al., 2024)	-13.0	-14.0

Notes: This table presents changes in outcomes relative to the baseline economy under *LT*. Column (1) holds the efficiency wages w_0 , w_1 , and the interest rate r fixed at their baseline levels; Column (2) allows them to adjust to clear markets. Unless otherwise specified, effects are expressed as percentage changes.

comprehensive schooling, parental-background gaps in end-of-school skills, academic-track attendance, and college graduation narrow. *LT* lowers the parental-education ratio in college graduation by about 6%. The association between parental income and academic-track attendance falls by 14%, and the income rank-rank coefficient by 3.6%, signifying greater intergenerational mobility in labor incomes. Together with the compression of the skill distribution, these gains reduce cross-sectional inequality: the wage Gini among 54–57-year-olds falls by 1.1%, and the standard deviation of wealth by 1.7%.

In sum, *LT* generates a long-run efficiency-mobility trade-off in general equilibrium: delaying tracking by four years modestly reduces aggregate human capital and output while increasing equality and intergenerational mobility.⁴⁷ With prices fixed, the stronger college-attainment response reverses the efficiency loss, so efficiency and mobility rise together.

⁴⁷[Bénabou \(1996\)](#) similarly shows that mobility-enhancing policies may entail growth costs, while [Arenas and Hindriks \(2021\)](#) find that equalizing school opportunities raises mobility at a modest cost in human capital. In our model, the trade-off arises from the interaction of track-specific skill formation, college choice, and general-equilibrium wage adjustment.

Despite its different methodology, our model thus rationalizes three recurring but scattered findings in the empirical literature (see [Related Literature](#)): mixed effects of delaying tracking on aggregate learning and earnings, higher educational inequality under more rigid tracking, and benefits concentrated among children from lower socioeconomic backgrounds.

4.1.3 The Effects on Track Misallocation

Delaying tracking changes how students are matched to tracks, raising the question of whether it also affects the share of students who attend a track other than the one that best fits their skills. This type of student-track mismatch or track misallocation, while central to the tracking literature, is difficult to measure empirically, in part because it can stem from factors known at the time of the track decision, such as socioeconomic differences, but also from factors unknown at that time, such as later-emerging developmental differences.⁴⁸

The model, however, allows us to measure and decompose this mismatch directly. We define a child's *skill-suited track* as the one that would have maximized her end-of-school skills, holding track-specific instruction pace and peer skills fixed, so that we consider only the effect of an individual switch, not a change in the whole population's sorting. Because the academic track carries a positive average taste cost, we require the skill gain from the academic track to exceed a cutoff above zero, chosen so that the share of children classified as suited to the academic track equals the actual academic-track share in each economy.⁴⁹ We classify children who attend a track other than their skill-suited one as mismatched. In the baseline economy, we separate the two sources of mismatch using two thought experiments. First, we compute end-of-school skills under the realized shocks η_4 , asking whether each child's chosen track was *ex-post* suited given her actual skill development.⁵⁰ Second, we set the skill shock after tracking to zero, $\eta_4 = 0$, asking whether the chosen track was *ex-ante* suited given the predictable component of skill development at the time of the track decision. The difference captures mismatch arising from unforeseen skill shocks. Under *LT*, by contrast, this uncertainty has resolved by the time tracking occurs, so there is no ex-post

⁴⁸Existing tracking research often relies on indirect evidence, offering only suggestive conclusions. [Carlana et al. \(2022\)](#) and [Falk et al. \(2026\)](#) find disproportionate vocational enrollment among immigrants in Italy and low-socioeconomic-status children in Germany, respectively, suggesting potential misallocation rather than differences in underlying potential.

⁴⁹Under a skills-only definition, with no constraint on the number of academic places, about 90% of children would gain skills in the academic track at baseline, and almost all mismatch (46%, against 0.3% in the opposite direction) consists of vocational-track children who would gain from the academic track. Since the academic track carries a positive taste cost, a skills-only definition is a learning benchmark and not a welfare criterion.

⁵⁰Only η_4 matters here: η_5 enters end-of-school skills additively and identically in both tracks, so it does not affect which track maximizes them.

mismatch by construction. Remaining misallocation therefore reflects other determinants of track choice, such as the academic-track taste shock χ and the track's value for college access.

Table 6: Track Mismatch by Parental Education

	All	CL parents	NCL parents
<i>Panel A: Baseline economy</i>			
Total mismatch	27.0	27.4	26.9
Misplaced toward academic	13.5	15.5	12.7
Misplaced toward vocational	13.5	11.9	14.1
Share suited to the academic track	43.8	65.5	35.3
Suited to academic: share attending academic	69.2	81.8	59.9
Suited to vocational: share attending vocational	76.0	55.1	80.4
<i>Panel B: Baseline economy, zero shocks</i>			
Total mismatch	24.5	24.8	24.3
Misplaced toward academic	12.2	13.1	11.9
Misplaced toward vocational	12.2	11.7	12.4
Share suited to the academic track	43.8	67.6	34.4
Suited to academic: share attending academic	72.1	82.7	63.8
Suited to vocational: share attending vocational	78.2	59.4	81.9
<i>Panel C: Late Tracking (LT)</i>			
Total mismatch	20.8	18.8	21.6
Misplaced toward academic	10.4	10.2	10.5
Misplaced toward vocational	10.4	8.6	11.1
Share suited to the academic track	52.4	72.5	44.5
Suited to academic: share attending academic	80.2	88.1	75.1
Suited to vocational: share attending vocational	78.2	63.0	81.1

Notes: Percentages. CL and NCL denote children of college- and non-college-educated parents. The first three rows are percentages of all children in the group. The share suited to the academic track is a percentage of the group, and the last two rows are the percentage of children suited to each track who attend it. The cutoff defining the skill-suited track is set in each economy so that the share classified as academic-suited equals the actual academic-track share, which makes the two directions of mismatch equal in the full population by construction. Under *LT* there are no unforeseen skill shocks, so Panel C is comparable to Panel B. Group sizes differ slightly across economies.

Table 6 shows substantial mismatch. In the baseline economy (Panel A), 27% of children attend a track other than their skill-suited one, split equally between the two directions by construction. Of the children suited to the vocational track, 76% attend it, versus 69% of those suited to the academic track. Removing skill-shock-based mismatch (Panel B) lowers total mismatch to 25% and raises these attendance shares to 78% and 72%, so unforeseen skill shocks account for only part of the mismatch, with the remainder driven by other forces. *LT* (Panel C) further lowers total mismatch from 25% (Panel B) to 21% and raises the

share of academic-suited children who attend the academic track to 80%, while the group of academic-suited children grows from 44% to 52% of all children. This suggests that delaying tracking improves matching beyond the resolution of skill uncertainty alone.

The CL and NCL columns of Table 6 show that mismatch differs by parental education more in direction than in size. In the baseline economy, total mismatch is similar for both groups (27%), but children of college-educated parents are misplaced toward the academic track and children of non-college-educated parents away from it: among children suited to the vocational track, only 55% of those with college-educated parents attend it, and among children suited to the academic track, only 60% of those with non-college-educated parents do. Removing skill-shock uncertainty lowers total mismatch only modestly, to 25% and 24%, respectively. *LT* lowers it further, to 19% for children of college-educated parents and 22% for children of non-college-educated parents, while the direction of mismatch persists. *LT* also raises the share of academic-suited children who attend the academic track, from 64% to 75% among children of non-college-educated parents and from 83% to 88% among children of college-educated parents, while the group of academic-suited children among the former grows from 34% to 44%. This improvement in the academic-suited match rate is larger for children of non-college-educated parents, consistent with their larger skill gains from the additional years of comprehensive schooling documented above.

Delaying tracking thus improves student-track matching, especially access to the academic track for suited children of non-college-educated parents. Better matching, however, does not by itself determine the policy’s aggregate efficiency effects, which also depend on the distribution of learning gains and losses and on the subsequent response of college choices and labor-market prices.

4.2 The Effects of No, Flexible, and Skill-Based Tracking

The *LT* experiment reveals a long-run trade-off between aggregate efficiency and both equality and intergenerational mobility. To examine how this trade-off varies with the extent and precision of student sorting, we consider three additional benchmark policies. The first, *No Tracking* (*NT*), keeps students in comprehensive school through age 18.⁵¹ The second, *Flexible Tracking* (*FT*), keeps the baseline tracking age of 10 but allows re-tracking at age 14,

⁵¹Since the track-specific component of the college taste shock is no longer defined, we set its mean to the baseline average, $\mu_\nu + s^V \mu_\nu^V$, where s^V is the baseline vocational-track share. This keeps the average non-pecuniary cost of college unchanged, so that *NT* does not mechanically lower college barriers for former vocational-track students.

so that mismatch arising from skill uncertainty can be corrected.⁵² The third, *Skill-Based Tracking* (*ST*), also keeps the baseline tracking age of 10 but assigns tracks by a fixed skill threshold, chosen to keep the academic-track share at its baseline level, which removes taste shocks and parental discretion from track choice. While various recent German reforms go in related directions, we view these policies as stylized benchmarks that bound the effects of more incremental reforms.⁵³ In all scenarios, we solve for the new stationary distribution, with general equilibrium price adjustment and endogenous instruction pace.

4.2.1 The Effects on Child Skills

Panel A of Table 7 shows that skill gains fall by roughly 1% of a baseline standard deviation in each secondary-school period (ages 10–14 and 14–18) under *NT*, while they rise under *FT* and *ST*, most under *ST* (below +1% per period under *FT* and about +1% under *ST*).⁵⁴ As for *LT*, skill homogeneity drives these patterns: *NT* prevents instruction from being tailored to distinct skill groups, whereas *FT* and *ST* create more homogeneous tracks through re-tracking and skill-based assignment, respectively.

The gains under *FT* are concentrated in the first period. Although re-tracking also makes second-period peer groups more homogeneous, the resulting gain is close to zero and well below the roughly +1% under *LT*, suggesting that the model’s richer sorting mechanisms, not peer homogeneity alone, shape aggregate learning.

Across all three policies, changes in learning gains between ages 10 and 14 are larger for children of college-educated parents: losses are greater for them under *NT*, and gains are greater under *FT* and *ST*. Under *NT*, children of non-college-educated parents—more likely to occupy the center of the skill distribution—lose only in the second period (0% then -1%), while children of college-educated parents—more likely to occupy the top of the distribution—lose in both periods (about -3% and -1%). The pattern reverses under *FT* and *ST*: children of college-educated parents benefit most from better sorting on skills in the first period (around +1% per period). Yet better sorting also benefits children of non-college-educated parents, so both groups gain by the end of secondary school.

Panel B of Table 7 shows that the policies also have opposite effects on skill inequality.

⁵²At age 14, after drawing a new taste shock, parents choose the track anew, irrespective of the track attended at ages 10–13. As in *LT*, the academic track costs $\chi/(1 + \beta)$ per period.

⁵³Several German states have expanded integrated secondary schools that separate students later (Maaz et al., 2024). Conversely, since 2024/25, the state of Baden-Württemberg requires a pedagogical assessment and a centralized test for academic-track admission (KM BW, 2025).

⁵⁴Since all three policies leave schooling before age 10 unchanged, the changes in skill gains between ages 6 and 10 reflect only long-run changes in parental characteristics and thus initial skills.

Table 7: Effects of Benchmark Policies on Child Skills

	<i>No Tracking (NT)</i>			<i>Flexible Tracking (FT)</i>			<i>Skill-Based Tracking (ST)</i>		
	All (1)	CL (2)	NCL (3)	All (4)	CL (5)	NCL (6)	All (7)	CL (8)	NCL (9)
<i>Panel A: Skill Gains Relative to the Baseline Economy</i>									
Gains btw. Ages 6 and 10	+0.3	+2.1	-0.4	-0.1	-1.3	+0.4	-0.4	-1.8	+0.3
Gains btw. Ages 10 and 14	-0.7	-3.1	+0.0	+0.7	+1.0	+0.5	+1.0	+1.3	+0.8
Gains btw. Ages 14 and 18	-0.9	-0.8	-1.0	+0.2	+0.8	-0.1	+1.1	+0.8	+1.0
<i>Panel B: Changes in the Skill Distribution</i>									
<i>Average Skills</i>									
Initial	-0.2	-9.3	+2.1	+0.3	+2.2	-0.2	+0.7	+6.4	-0.6
End-of-School	-1.5	-11.2	+0.8	+1.1	+2.7	+0.7	+2.3	+6.6	+1.5
... if College-Grad.	-22.9			+5.5			+14.7		
... if Non-College-Grad.	+5.8			-0.5			-1.5		
<i>Skill Variance (% change)</i>									
Initial	-2.5			+1.2			+2.5		
At Age 14	-8.0			+1.3			+8.3		
End-of-School	-9.2			+3.2			+9.0		

Notes: Panel A presents average skill gains under each policy reform across periods, relative to baseline gains. Panel B presents changes in the distribution of child skills under each policy, relative to the baseline economy. "CL" and "NCL" denote children of college-educated and non-college-educated parents, respectively. Columns (1)–(3), (4)–(6), and (7)–(9) report the *No Tracking* (NT), *Flexible Tracking* (FT), and *Skill-Based Tracking* (ST) policies, respectively. The "if College-Grad." and "if Non-College-Grad." rows report end-of-school skills by the child's own subsequent graduation status rather than by parental education, and are shown only for the full population. Skill variance rows are reported only for the full population. All columns allow prices to adjust to clear markets. Unless otherwise specified, effects are expressed as a percentage of a baseline standard deviation.

Eliminating tracking under *NT* reduces skill variance at age 18 by 9.2%, while *FT* and *ST* increase it by 3.2% and 9.0%, respectively. As in Proposition 2 (Section 2.1.2), better sorting on skills exposes vocational-track children to lower-skilled peers and academic-track children to higher-skilled ones, stretching skill differences between tracks. This effect is strongest under *ST*, which removes discretionary and idiosyncratic variation from track assignment and so creates the most sharply differentiated learning environments.

4.2.2 Later-Life Aggregate and Distributional Effects

Under *NT*, the efficiency-mobility trade-off observed under *LT* becomes significantly more pronounced (Column (1) of Table 8). The learning losses described above lower end-of-

school skills by 1.5% of a baseline standard deviation, particularly for prospective college graduates.⁵⁵ Despite a 1.8pp rise in the college share rises, aggregate human capital and output fall by 1.1%. At the same time, the sharply reduced skill inequality under *NT* goes along with narrower skill differences by parental background, driven by substantial declines among children of college-educated parents (by around 11% of a baseline standard deviation), while children of non-college-educated parents gain slightly (Panel B of Table 7). As a result, the college-graduation ratio falls by 22.5%. As these children become parents, the resulting compression in human capital also narrows differences in the initial skills of the next generation. Together, these changes generate larger mobility and equality effects than under *LT*: the income rank-rank coefficient falls by 13.1%, the wage Gini by 4.6%, and the standard deviation of wealth by 5.0%.

Table 8: Aggregate and Distributional Effects of the Benchmark Policies

	<i>NT</i> (1)	<i>FT</i> (2)	<i>ST</i> (3)
<i>Educational Outcomes</i>			
% Academic Track (<i>in pp.</i>)	-	+10.8	+0.1
Parental-Education Ratio in Academic-Track Attendance	-	-18.0	-9.2
% College (<i>in pp.</i>)	+1.8	-0.1	-0.8
Parental-Education Ratio in College Graduation	-22.5	-4.1	-2.3
Reg. College on θ_5 : Coef. θ_5	-31.1	+11.1	+11.1
<i>Efficiency</i>			
Output (<i>Y</i>)	-1.1	+0.5	+1.2
Human Capital (<i>H</i>)	-1.1	+0.5	+1.2
<i>Cross-Sectional Inequality</i>			
Wage Gini (54-57)	-4.6	+1.1	+3.4
Standard Deviation of Wealth	-5.0	+0.6	+2.8
College/Non-College Earnings	-8.3	+1.3	+4.7
<i>Mobility</i>			
Income Rank-Rank Coefficient	-13.1	+1.2	+8.8
A-Track on Income (Dodin et al., 2024)	-	-7.5	+39.8

Notes: This table presents changes in outcomes relative to the baseline economy under *NT* (Column (1)), *FT* (Column (2)), and *ST* (Column (3)). All columns allow the efficiency wages w_0 , w_1 , and the interest rate r to adjust to clear markets. Unless otherwise specified, effects are expressed as percentage changes.

FT reverses the trade-off (Column 2 of Table 8). Improved sorting raises average end-

⁵⁵The drop in the correlation between end-of-school skills and college graduation also partly reflects that, in the calibrated model, academic-track attendance, not skills alone, eases college access. Since track and skills are correlated, removing tracks weakens the link between skills and college graduation.

of-school skills, which translates into aggregate efficiency gains: output and human capital increase by 0.5%. During school, the policy appears mobility-enhancing: with the academic-track share up 10.8pp, the parental-education ratio in academic-track attendance falls by 18%, and the association between parental income and track attendance declines by 7.5%. However, the learning gains are concentrated among children of college-educated parents, whose higher skills raise adult human capital and, once they become parents, the initial skills of their children (by around +2%). This intergenerational feedback ultimately dominates the more equal track assignment: the income rank-rank coefficient rises by 1.2%, the wage Gini by 1.1%, and wealth dispersion by 0.6%.

Under *ST*, the larger skill gains translate into stronger aggregate efficiency effects: output and human capital rise by 1.2% despite a slight drop in the share of college graduates (-0.8pp, Column 3 of Table 8). These gains, however, come with greater inequality than under *FT*: the wage Gini rises by 3.4%, wealth dispersion by 2.8%, and the college earnings premium by 4.7%. Skills rise for both parental-education groups, but disproportionately for children of college-educated parents, which raises human-capital inequality among future parents and, through intergenerational transmission, widens the gap in initial skills in the next generation. Moreover, because *ST* assigns tracks purely on skills, removing the idiosyncratic and discretionary factors that attenuate this link under the baseline, and skills at age 10 remain closely tied to parental human capital, track attendance becomes more strongly related to parental background. Thus, despite modest declines in the parental-education ratios in academic-track attendance and college graduation (by 9.2% and 2.3%), the association between parental income and academic-track attendance rises by 39.8% and the income rank-rank coefficient by 8.8%. Perhaps surprisingly, then, an ostensibly meritocratic assignment rule can lead to an equilibrium featuring higher intergenerational persistence of economic status.

In sum, the benchmark policies show how the efficiency-mobility trade-off depends on the extent and precision of student sorting. Eliminating tracking reduces skill inequality and raises mobility at the cost of aggregate efficiency, whereas more flexible or strictly skill-based sorting raises efficiency but lowers mobility and increases inequality, most sharply under *ST*. Whether such unequally distributed gains are socially desirable requires a welfare analysis, which we leave for future research.

5 Conclusion

This paper develops a quantitative macroeconomic framework for studying how school tracking shapes the long-run distribution of human capital, aggregate efficiency, inequality, and intergenerational mobility. At its core is a novel skill formation technology during school that trades off the learning gains from instruction tailored to homogeneous groups against the risk of track mismatch from sorting children before their skills have fully developed. Our analytical results show that, absent skill uncertainty, sorting students by skill maximizes aggregate learning but widens skill inequality, and that delaying tracking becomes more valuable when skills evolve unpredictably. We embed this technology in an overlapping-generations life-cycle model, calibrated to Germany, in which parents choose their children’s track, so that track composition, and with it peers and instruction pace, is determined endogenously. School skills, in turn, shape higher education choices, labor-market outcomes, and ultimately the transmission of human capital across generations.

We find that delaying tracking from age 10 to 14 reduces the dispersion of end-of-school skills, weakens the link between parental background and both track attendance and college attainment, and raises intergenerational mobility. These gains are concentrated among children of non-college-educated parents, who benefit relatively more from longer comprehensive schooling and improved student-track matching. Whether these gains come at a cost to aggregate efficiency depends critically on labor-market adjustment. When factor prices adjust in general equilibrium, increased mobility and equality are accompanied by modest losses in aggregate human capital and output. When factor prices are held fixed, college attainment responds more strongly and this efficiency-mobility trade-off disappears. Abolishing tracking strengthens this trade-off, whereas flexible re-tracking and strictly skill-based assignment reverse it, raising efficiency at the cost of higher inequality and lower mobility.

Taken together, our findings show that tracking policies that shape the sorting of students in secondary school have non-trivial consequences for the long-run distribution and intergenerational transmission of human capital. This illustrates that studying education policy during secondary school is an important complement to research on early-childhood ([Daruich, 2026](#)) or higher education policies ([Abbott et al., 2019](#)). Our findings also highlight that evaluating the long-run effects of such policies requires looking beyond learning at school. In particular, the efficiency consequences of tracking reforms may depend on forces shaping the demand for human capital, including technical change, occupational structure, and labor-market institutions. Incorporating these forces more fully into models of education and mobility is a promising avenue for future research.

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A Model Appendix

A.1 Details on Analytical Model and Proof of Propositions

The analytical model in Section 2.1.2 is designed to develop some intuition about the implications of our formulation of the child skill formation technology on the effects of school tracking. To that end, the analytical model focuses exclusively on the secondary schooling years (periods 3 and 4), and we ignore transitions to higher education and the labor market. Moreover, we simplify parents' preferences, such that they only care about their child's expected end-of-school skills and have no other preferences regarding the school track choice. Finally, we assume for simplicity that there are no direct parental influences, $\zeta = 0$, nor stochastic track costs, $\chi = 0$.

All other assumptions are maintained. In particular, paces of instruction are chosen to maximize the aggregate skill level of all students by the end of the period, such that the pace-setting rule in Lemma 1 holds. Moreover, for analytical practicality, we assume that the distribution of child skills at the beginning of secondary school is normal and centered around 0.

Recall that substituting the pace-setting rule (3) into (1) and (2), and adding and subtracting $\frac{v_3^2}{2v_4}\theta_j^2$, gives the child skill formation technology (4):

$$\theta_{j+1} = \frac{v_2^2}{2v_4} + \left(v_0 + \frac{v_2v_3}{v_4}\right)\theta_j + v_1\bar{\theta}_j^S + \frac{v_3^2}{2v_4}\theta_j^2 - \frac{v_3^2}{2v_4}(\theta_j - \bar{\theta}_j^S)^2 + \eta_{j+1}.$$

Proposition 1

We first show that any optimal allocation chosen by the planner must be characterized by a skill threshold. From equation (4), expected next-period skills of a child with skill θ_j assigned to track S can be written as

$$\mathbb{E}[\theta_{j+1} \mid \theta_j, S] = K(\theta_j) + v_1\bar{\theta}_j^S - \frac{v_3^2}{2v_4}(\theta_j - \bar{\theta}_j^S)^2, \quad (\text{A.1})$$

where

$$K(\theta_j) = \frac{v_2^2}{2v_4} + \left(v_0 + \frac{v_2v_3}{v_4}\right)\theta_j + \frac{v_3^2}{2v_4}\theta_j^2$$

collects all terms that do not depend on the assigned track. Hence, for given track means, expected skill accumulation depends on track assignment only through

$$v_1 \bar{\theta}_j^S - \frac{v_3^2}{2v_4} (\theta_j - \bar{\theta}_j^S)^2.$$

The aggregate contribution of the direct peer-effect term is independent of the allocation rule because

$$\mathbb{E}[\bar{\theta}_j^S] = \Pr(S = A) \bar{\theta}_j^A + \Pr(S = V) \bar{\theta}_j^V = \mathbb{E}[\theta_j].$$

The terms collected in $K(\theta_j)$ are likewise independent of track assignment. The planner's allocation therefore affects aggregate expected skills only through total squared mismatch.

Suppose, by contradiction, that an optimal allocation contains a crossing. In particular, consider two equal, arbitrarily small masses of children with skills $\theta_j^L < \theta_j^H$, where the lower-skill children are assigned to the academic track and the higher-skill children are assigned to the vocational track. Let the associated track means satisfy $\bar{\theta}_j^A > \bar{\theta}_j^V$.

Holding the track means fixed at their values under the original allocation, the total squared mismatch of these children is

$$(\theta_j^L - \bar{\theta}_j^A)^2 + (\theta_j^H - \bar{\theta}_j^V)^2.$$

Exchanging the two equal masses across tracks changes their total squared mismatch to

$$(\theta_j^L - \bar{\theta}_j^V)^2 + (\theta_j^H - \bar{\theta}_j^A)^2.$$

The difference between the mismatch under the original and exchanged assignments is

$$\begin{aligned} & \left[(\theta_j^L - \bar{\theta}_j^A)^2 + (\theta_j^H - \bar{\theta}_j^V)^2 \right] \\ & - \left[(\theta_j^L - \bar{\theta}_j^V)^2 + (\theta_j^H - \bar{\theta}_j^A)^2 \right] \\ & = 2 (\theta_j^H - \theta_j^L) (\bar{\theta}_j^A - \bar{\theta}_j^V) > 0. \end{aligned}$$

Thus, even holding the original track means fixed, assigning the lower-skill children to the lower-mean track and the higher-skill children to the higher-mean track strictly reduces total squared mismatch. After the exchange, allowing the track means to adjust to the new allocation can only reduce total squared mismatch further, since the mean of each track minimizes the sum of squared deviations within that track.

Consequently, an allocation containing such a crossing cannot be optimal. It follows that

any non-degenerate optimal allocation is monotone in skills and is therefore characterized by a threshold $\tilde{\theta}^P$: children with $\theta_j < \tilde{\theta}^P$ attend the vocational track, while children with $\theta_j > \tilde{\theta}^P$ attend the academic track.

We next establish the threshold property under parental choice. For given track means, define the gain in expected next-period skills from attending the academic rather than the vocational track as

$$\begin{aligned}\Delta(\theta_j) &\equiv \mathbb{E}[\theta_{j+1} \mid \theta_j, S = A] - \mathbb{E}[\theta_{j+1} \mid \theta_j, S = V] \\ &= v_1 (\bar{\theta}_j^A - \bar{\theta}_j^V) - \frac{v_3^2}{2v_4} \left[(\theta_j - \bar{\theta}_j^A)^2 - (\theta_j - \bar{\theta}_j^V)^2 \right].\end{aligned}\tag{A.2}$$

The gain from the academic track is strictly increasing in the child's skill level. In particular,

$$\begin{aligned}\frac{\partial \Delta(\theta_j)}{\partial \theta_j} &= -\frac{v_3^2}{2v_4} [2(\theta_j - \bar{\theta}_j^A) - 2(\theta_j - \bar{\theta}_j^V)] \\ &= \frac{v_3^2}{v_4} (\bar{\theta}_j^A - \bar{\theta}_j^V) > 0,\end{aligned}$$

where the inequality follows from $v_3 \neq 0$, $v_4 > 0$, and $\bar{\theta}_j^A > \bar{\theta}_j^V$.

Thus, parental track preferences satisfy a single-crossing property: if a parent whose child has skill θ_j prefers the academic track, then every parent whose child has skill $\theta'_j > \theta_j$ also prefers the academic track. Consequently, any non-degenerate equilibrium under parental choice is characterized by a threshold $\tilde{\theta}^M$ such that

$$S(\theta_j) = \begin{cases} V, & \theta_j < \tilde{\theta}^M, \\ A, & \theta_j > \tilde{\theta}^M. \end{cases}$$

A parent whose child has skill exactly $\tilde{\theta}^M$ is indifferent between the two tracks. Together with the preceding planner argument, this establishes that both the planner's optimal allocation and any non-degenerate parental-choice equilibrium are characterized by a skill threshold.

Next, we characterize the planner's threshold $\tilde{\theta}^P$. Taking unconditional expectations in (A.1) yields

$$\mathbb{E}[\theta_{j+1}] = \mathbb{E}[K(\theta_j)] + v_1 \mathbb{E}[\bar{\theta}_j^S] - \frac{v_3^2}{2v_4} \mathbb{E}[(\theta_j - \bar{\theta}_j^S)^2].\tag{A.3}$$

The aggregate contribution of direct peer effects is independent of the allocation rule because

$$\begin{aligned}\mathbb{E}[\bar{\theta}_j^S] &= \Pr(S = A)\bar{\theta}_j^A + \Pr(S = V)\bar{\theta}_j^V \\ &= \Pr(S = A)\mathbb{E}[\theta_j \mid S = A] + \Pr(S = V)\mathbb{E}[\theta_j \mid S = V] \\ &= \mathbb{E}[\theta_j],\end{aligned}$$

where the last equality follows from the law of iterated expectations. Moreover, $\mathbb{E}[K(\theta_j)]$ is independent of track assignment.

The planner's problem can therefore be written as

$$\max_{\tilde{\theta}} \mathbb{E}[\theta_{j+1}] = \max_{\tilde{\theta}} \left\{ \mathbb{E}[K(\theta_j)] + v_1 \mathbb{E}[\theta_j] - \frac{v_3^2}{2v_4} \mathbb{E}[(\theta_j - \bar{\theta}_j^S)^2] \right\}. \quad (\text{A.4})$$

Since $v_3 \neq 0$ and $v_4 > 0$, this is equivalent to

$$\min_{\tilde{\theta}} \mathbb{E}[(\theta_j - \bar{\theta}_j^S)^2] = \min_{\tilde{\theta}} \mathbb{E}[\text{Var}(\theta_j \mid S)], \quad (\text{A.5})$$

where the equality follows from $\bar{\theta}_j^S = \mathbb{E}[\theta_j \mid S]$.

Thus, maximizing expected end-of-period skills is equivalent to minimizing expected within-track skill variance. For a normally distributed skill variable and a threshold allocation into two tracks, expected within-track variance is minimized by setting the threshold equal to the mean of the skill distribution.⁵⁶ Therefore, $\tilde{\theta}^P = \mu_{\theta_j} = \mathbb{E}[\theta_j]$. This proves part 1.1.

⁵⁶By the law of total variance,

$$\mathbb{E}[\text{Var}(\theta_j \mid S)] = \text{Var}(\theta_j) - \text{Var}(\mathbb{E}[\theta_j \mid S]).$$

It is therefore sufficient to maximize the variance of the conditional means. Suppose that $\theta_j \sim \mathcal{N}(\mu_{\theta_j}, \sigma_{\theta_j}^2)$ and define $z = \frac{\tilde{\theta} - \mu_{\theta_j}}{\sigma_{\theta_j}}$. Under a threshold allocation with $S = V$ if $\theta_j \leq \tilde{\theta}$ and $S = A$ otherwise, the conditional means are

$$\mathbb{E}[\theta_j \mid S = V] = \mu_{\theta_j} - \sigma_{\theta_j} \frac{\phi(z)}{\Phi(z)}$$

and

$$\mathbb{E}[\theta_j \mid S = A] = \mu_{\theta_j} + \sigma_{\theta_j} \frac{\phi(z)}{1 - \Phi(z)}.$$

Consequently,

$$\text{Var}(\mathbb{E}[\theta_j \mid S]) = \sigma_{\theta_j}^2 \frac{\phi(z)^2}{\Phi(z)[1 - \Phi(z)]}.$$

This expression is symmetric around $z = 0$ and uniquely maximized at $z = 0$. Hence expected within-track variance is uniquely minimized when $\tilde{\theta} = \mu_{\theta_j}$.

Next, we characterize the equilibrium threshold arising in the parental allocation game, $\tilde{\theta}^M$. In a non-degenerate equilibrium, a parent whose child has skill level $\tilde{\theta}^M$ must be indifferent between the two tracks:

$$\mathbb{E}[\theta_{j+1} \mid \theta_j = \tilde{\theta}^M, S = A] = \mathbb{E}[\theta_{j+1} \mid \theta_j = \tilde{\theta}^M, S = V].$$

For a candidate threshold t , define the associated track means as

$$\mu_A(t) \equiv \mathbb{E}[\theta_j \mid \theta_j > t], \quad \mu_V(t) \equiv \mathbb{E}[\theta_j \mid \theta_j \leq t].$$

Using equation (A.1), the difference in expected next-period skills between the academic and vocational tracks for a child with skill θ_j is

$$\begin{aligned} \Delta(\theta_j; t) &\equiv \mathbb{E}[\theta_{j+1} \mid \theta_j, S = A] - \mathbb{E}[\theta_{j+1} \mid \theta_j, S = V] \\ &= v_1 [\mu_A(t) - \mu_V(t)] - \frac{v_3^2}{2v_4} [(\theta_j - \mu_A(t))^2 - (\theta_j - \mu_V(t))^2]. \end{aligned} \quad (\text{A.6})$$

The equilibrium threshold is therefore characterized by

$$\Delta(\tilde{\theta}^M; \tilde{\theta}^M) = 0.$$

Using the identity

$$(t - \mu_A(t))^2 - (t - \mu_V(t))^2 = [\mu_A(t) - \mu_V(t)] [\mu_A(t) + \mu_V(t) - 2t],$$

and noting that $\mu_A(t) > \mu_V(t)$, the equilibrium indifference condition can be written as

$$2\tilde{\theta}^M - \mu_A(\tilde{\theta}^M) - \mu_V(\tilde{\theta}^M) = -\frac{2v_1v_4}{v_3^2}. \quad (\text{A.7})$$

For $\theta_j \sim \mathcal{N}(0, \sigma_{\theta_j}^2)$, the conditional track means associated with threshold t are

$$\mu_A(t) = \sigma_{\theta_j} \frac{\phi(t/\sigma_{\theta_j})}{1 - \Phi(t/\sigma_{\theta_j})}$$

and

$$\mu_V(t) = -\sigma_{\theta_j} \frac{\phi(t/\sigma_{\theta_j})}{\Phi(t/\sigma_{\theta_j})}.$$

Hence, $\tilde{\theta}^M$ is characterized as the solution to

$$2\tilde{\theta}^M - \sigma_{\theta_j} \frac{\phi(\tilde{\theta}^M/\sigma_{\theta_j})}{1 - \Phi(\tilde{\theta}^M/\sigma_{\theta_j})} + \sigma_{\theta_j} \frac{\phi(\tilde{\theta}^M/\sigma_{\theta_j})}{\Phi(\tilde{\theta}^M/\sigma_{\theta_j})} = -\frac{2v_1v_4}{v_3^2}. \quad (\text{A.8})$$

The left-hand side of (A.8) is strictly increasing in $\tilde{\theta}^M$ and equals zero at $\tilde{\theta}^M = 0$. Since the planner's threshold is $\tilde{\theta}^P = 0$ under the maintained normalization, it follows that, whenever $v_1 > 0$,

$$\tilde{\theta}^M < 0 = \tilde{\theta}^P.$$

If $v_1 = 0$, equation (A.8) instead implies

$$\tilde{\theta}^M = 0 = \tilde{\theta}^P.$$

Thus, positive direct peer effects lower the equilibrium threshold under parental choice relative to the planner's threshold. The exact parental threshold is defined implicitly by (A.8) and can be computed numerically. This proves part 1.2.

Proposition 2

Let $c \equiv \frac{v_3^2}{2v_4} > 0$ and $b \equiv v_0 + \frac{v_2v_3}{v_4}$. Using equation (A.1), expected next-period skills conditional on initial skills and track assignment can then be written as

$$\mathbb{E}[\theta_{j+1} \mid \theta_j, S] = K(\theta_j) + v_1\bar{\theta}_j^S - c(\theta_j - \bar{\theta}_j^S)^2. \quad (\text{A.9})$$

Because

$$K(\theta_j) = \frac{v_2^2}{2v_4} + b\theta_j + c\theta_j^2,$$

equation (A.9) can equivalently be written as

$$\mathbb{E}[\theta_{j+1} \mid \theta_j, S] = \frac{v_2^2}{2v_4} + b\theta_j + v_1\bar{\theta}_j^S + 2c\theta_j\bar{\theta}_j^S - c(\bar{\theta}_j^S)^2. \quad (\text{A.10})$$

Proof of Proposition 2.1. Under a tracking system, taking unconditional expectations in (A.9) gives

$$\mathbb{E}[\theta_{j+1} \mid T] = \mathbb{E}[K(\theta_j)] + v_1 \mathbb{E}[\bar{\theta}_j^S] - c \mathbb{E}[(\theta_j - \bar{\theta}_j^S)^2].$$

By the law of iterated expectations, $\mathbb{E}[\bar{\theta}_j^S] = \mathbb{E}[\theta_j]$. Moreover, because $\bar{\theta}_j^S = \mathbb{E}[\theta_j \mid S]$, $\mathbb{E}[(\theta_j - \bar{\theta}_j^S)^2] = \mathbb{E}[\text{Var}(\theta_j \mid S)]$. It follows that

$$\mathbb{E}[\theta_{j+1} \mid T] = \mathbb{E}[K(\theta_j)] + v_1 \mathbb{E}[\theta_j] - c \mathbb{E}[\text{Var}(\theta_j \mid S)]. \quad (\text{A.11})$$

Under a comprehensive system, all children attend the same track, so that $\bar{\theta}_j^C = \mathbb{E}[\theta_j]$. Therefore,

$$\mathbb{E}[\theta_{j+1} \mid C] = \mathbb{E}[K(\theta_j)] + v_1 \mathbb{E}[\theta_j] - c \text{Var}(\theta_j). \quad (\text{A.12})$$

Subtracting (A.12) from (A.11) yields

$$\begin{aligned} \mathbb{E}[\theta_{j+1} \mid T] - \mathbb{E}[\theta_{j+1} \mid C] &= c \{ \text{Var}(\theta_j) - \mathbb{E}[\text{Var}(\theta_j \mid S)] \} \\ &= \frac{v_3^2}{2v_4} \left\{ \sigma_{\theta_j}^2 - \mathbb{E}[\text{Var}(\theta_j \mid S)] \right\}. \end{aligned} \quad (\text{A.13})$$

By the law of total variance,

$$\text{Var}(\theta_j) - \mathbb{E}[\text{Var}(\theta_j \mid S)] = \text{Var}(\mathbb{E}[\theta_j \mid S]) \geq 0.$$

Consequently,

$$\mathbb{E}[\theta_{j+1} \mid T] \geq \mathbb{E}[\theta_{j+1} \mid C].$$

The inequality is strict whenever both tracks have positive mass and different average skill levels, so that

$$\text{Var}(\mathbb{E}[\theta_j \mid S]) > 0.$$

This proves part 2.1.

Proof of Proposition 2.2. Consider optimal tracking. By Proposition 1.1, the tracking threshold equals the mean of the initial-skill distribution. Under the maintained normalization, $\tilde{\theta}^P = \mathbb{E}[\theta_j] = 0$. Because the initial-skill distribution is centered and symmetric, the two tracks have equal probability mass and their average initial skills satisfy

$$\Pr(S = A) = \Pr(S = V) = \frac{1}{2}$$

and

$$\bar{\theta}_j^A = -\bar{\theta}_j^V = m,$$

where, under normality,

$$m = \mathbb{E}[\theta_j \mid \theta_j > 0] = \sigma_{\theta_j} \sqrt{\frac{2}{\pi}}. \quad (\text{A.14})$$

Let

$$R = \begin{cases} 1, & S = A, \\ -1, & S = V, \end{cases}$$

and let

$$X = |\theta_j|.$$

Under the centered normal distribution, R and X are independent,

$$\theta_j = RX, \quad \bar{\theta}_j^S = Rm,$$

and

$$\mathbb{E}[X] = m, \quad \mathbb{E}[X^2] = \sigma_{\theta_j}^2, \quad \text{Var}(X) = \sigma_{\theta_j}^2 - m^2.$$

Using equation (A.10), end-of-period skills under optimal tracking are

$$\begin{aligned} \theta_{j+1}^T &= \frac{v_2^2}{2v_4} + bRX + v_1Rm + 2cmX - cm^2 + \eta_{j+1} \\ &= \frac{v_2^2}{2v_4} - cm^2 + R(bX + v_1m) + 2cmX + \eta_{j+1}. \end{aligned} \quad (\text{A.15})$$

The key observation is that the instructional-matching term, $2cmX$, depends only on a child's distance from the threshold, $X = |\theta_j|$, and not on which side of it she is: children far from the threshold in either direction gain from instruction closer to their own skill level, while those near it lose. Moreover, because R is independent of X , $\mathbb{E}[R] = 0$, and the skill shock is independent of both R and X ,

$$\text{Cov}(R(bX + v_1m), 2cmX) = 0.$$

Therefore,

$$\begin{aligned} \text{Var}(\theta_{j+1} \mid T) &= \mathbb{E}[(bX + v_1m)^2] + 4c^2m^2 \text{Var}(X) + \sigma_{\eta_{j+1}}^2 \\ &= b^2\sigma_{\theta_j}^2 + 2bv_1m^2 + v_1^2m^2 + 4c^2m^2(\sigma_{\theta_j}^2 - m^2) + \sigma_{\eta_{j+1}}^2. \end{aligned} \quad (\text{A.16})$$

Under comprehensive schooling, the average skill of the comprehensive track is zero.

Equation (A.10) therefore reduces to

$$\theta_{j+1}^C = \frac{v_2^2}{2v_4} + b\theta_j + \eta_{j+1}.$$

Hence,

$$\text{Var}(\theta_{j+1} \mid C) = b^2\sigma_{\theta_j}^2 + \sigma_{\eta_{j+1}}^2. \quad (\text{A.17})$$

Subtracting (A.17) from (A.16) gives

$$\begin{aligned} & \text{Var}(\theta_{j+1} \mid T) - \text{Var}(\theta_{j+1} \mid C) \\ &= m^2 \left[v_1^2 + 2bv_1 + 4c^2 \left(\sigma_{\theta_j}^2 - m^2 \right) \right]. \end{aligned} \quad (\text{A.18})$$

Substituting $m^2 = (2/\pi)\sigma_{\theta_j}^2$ and $c = v_3^2/(2v_4)$ gives the expression in Proposition 2.2. Consider first the case without direct peer effects, $v_1 = 0$. The right-hand side of (A.18) then reduces to $4c^2m^2(\sigma_{\theta_j}^2 - m^2)$, which is strictly positive: $c > 0$ because $v_3 \neq 0$ and $v_4 > 0$, $m > 0$, and $\sigma_{\theta_j}^2 - m^2 = \text{Var}(X) = (1 - 2/\pi)\sigma_{\theta_j}^2 > 0$. Hence, even without direct peer effects, the variance of end-of-period skills is higher under optimal tracking than under comprehensive schooling. As (A.15) shows, this is because the instructional-matching term $2cmX$ makes skill gains increase with a child's distance from the threshold, adding dispersion that is uncorrelated with the remaining skill components.

With positive direct peer effects, $v_1 > 0$, the bracket in (A.18) contains the additional terms $v_1^2 + 2bv_1 = v_1(v_1 + 2b)$, which are strictly positive under the maintained assumption $b \geq 0$. Direct peer effects therefore amplify the increase in skill variance, and more so the stronger they are, since the derivative of (A.18) with respect to v_1 , $2m^2(v_1 + b)$, is positive. Intuitively, higher-skilled peers raise the skills of academic-track children and lower-skilled peers depress those of vocational-track children, widening the gap between tracks on top of the dispersion created by instructional matching. This proves part 2.2.

Proof of Proposition 2.3. Continue to consider optimal tracking of a centered skill distribution, so that

$$\tilde{\theta}^P = 0, \quad \bar{\theta}_j^A = m, \quad \bar{\theta}_j^V = -m,$$

where $m > 0$.

For a child with initial skill θ_j , define the expected gain from tracking relative to com-

prehensive schooling as

$$G_S(\theta_j) \equiv \mathbb{E}[\theta_{j+1} \mid \theta_j, S, T] - \mathbb{E}[\theta_{j+1} \mid \theta_j, C].$$

Using equation (A.10), the gain for a child assigned to the academic track is

$$\begin{aligned} G_A(\theta_j) &= v_1 m + 2cm\theta_j - cm^2 \\ &= m(v_1 + 2c\theta_j - cm), \quad \theta_j > 0. \end{aligned} \tag{A.19}$$

Hence, an academic-track child loses from tracking if and only if

$$0 < \theta_j < \frac{cm - v_1}{2c}.$$

This interval is nonempty only if $v_1 < cm$.

For a child assigned to the vocational track, the corresponding gain is

$$\begin{aligned} G_V(\theta_j) &= -v_1 m - 2cm\theta_j - cm^2 \\ &= m(-v_1 - 2c\theta_j - cm), \quad \theta_j < 0. \end{aligned} \tag{A.20}$$

Hence, a vocational-track child loses from tracking if and only if

$$-\frac{cm + v_1}{2c} < \theta_j < 0.$$

When $v_1 = 0$, the loss intervals are symmetric:

$$-\frac{m}{2} < \theta_j < 0$$

in the vocational track and

$$0 < \theta_j < \frac{m}{2}$$

in the academic track. By symmetry of the initial-skill distribution, these intervals contain equal probability mass.

When $v_1 > 0$, the vocational loss interval expands, whereas the academic loss interval contracts and is empty if $v_1 \geq cm$. More precisely, the respective loss intervals are

$$-\frac{m}{2} - \frac{v_1}{2c} < \theta_j < 0$$

and

$$0 < \theta_j < \max \left\{ \frac{m}{2} - \frac{v_1}{2c}, 0 \right\}.$$

By symmetry of the initial-skill distribution, the vocational loss interval contains strictly greater probability mass than the academic loss interval whenever $v_1 > 0$.

If $v_1 \geq cm$, no academic-track child loses, while the vocational loss interval remains nonempty. Finally, Proposition 2.1 establishes that non-degenerate tracking strictly increases aggregate expected end-of-period skills. Tracking therefore benefits some children while harming others, with learning losses concentrated disproportionately among vocational-track children whenever $v_1 > 0$. This proves part 2.3.

Proposition 3

Proof of Proposition 3. Let θ_j denote skills at the beginning of secondary school, θ_{j+1} intermediate skills, and θ_{j+2} end-of-school skills. Initial skills and the intermediate skill shock satisfy

$$\theta_j \sim \mathcal{N}(0, \sigma_{\theta_j}^2), \quad \eta_{j+1} \sim \mathcal{N}(0, \sigma_{\eta_{j+1}}^2).$$

We abstract from the second-period skill shock, which has zero mean and therefore does not affect expected end-of-school skills.

As before, we define

$$a \equiv \frac{v_2^2}{2v_4}, \quad b \equiv v_0 + \frac{v_2 v_3}{v_4}, \quad c \equiv \frac{v_3^2}{2v_4}.$$

Using the optimal pace-setting rule, the skill technology can then be written as

$$\theta_{t+1} = a + b\theta_t + v_1 \bar{\theta}_t^S + 2c\bar{\theta}_t^S \theta_t - c(\bar{\theta}_t^S)^2 + \eta_{t+1}, \quad t \in \{j, j+1\}. \quad (\text{A.21})$$

For any track assignment S , taking unconditional expectations in (A.21) gives

$$\mathbb{E}[\theta_{t+1}] = a + (b + v_1) \mathbb{E}[\theta_t] + c \mathbb{E}[(\mathbb{E}[\theta_t | S])^2], \quad (\text{A.22})$$

as, $\mathbb{E}[\bar{\theta}_t^S] = \mathbb{E}[\theta_t]$ like before and

$$\mathbb{E}[\bar{\theta}_t^S \theta_t] = \mathbb{E}[\bar{\theta}_t^S \mathbb{E}[\theta_t | S]] = \mathbb{E}[(\mathbb{E}[\theta_t | S])^2].$$

Let $\kappa \equiv \sqrt{\frac{2}{\pi}}$. For a normally distributed random variable with mean μ and variance σ^2 ,

optimally split at its mean, the two conditional means are $\mu - \kappa\sigma$ and $\mu + \kappa\sigma$. Since each group has probability one half,

$$\mathbb{E}[(\mathbb{E}[\theta_t | S])^2] = \mu^2 + \kappa^2 \sigma^2. \quad (\text{A.23})$$

Late tracking. Under late tracking, children attend comprehensive school in the first period. Since $\bar{\theta}_j^C = 0$, equation (A.21) gives

$$\theta_{j+1,LT} = a + b\theta_j + \eta_{j+1}.$$

Therefore,

$$\mathbb{E}[\theta_{j+1,LT}] = a$$

and

$$\text{Var}(\theta_{j+1,LT}) = b^2 \sigma_{\theta_j}^2 + \sigma_{\eta_{j+1}}^2.$$

At the beginning of the second period, children are optimally tracked at

$$\tilde{\theta}_{j+1,LT}^P = \mathbb{E}[\theta_{j+1,LT}] = a.$$

Applying (A.22) and (A.23) yields

$$\mathbb{E}[\theta_{j+2,LT}] = a + (b + v_1)a + c \left[a^2 + \kappa^2 \left(b^2 \sigma_{\theta_j}^2 + \sigma_{\eta_{j+1}}^2 \right) \right]. \quad (\text{A.24})$$

Early tracking. Under early tracking, children are assigned to tracks at the beginning of the first period based on the mean-split threshold $\tilde{\theta}_{j,ET}^P = \mathbb{E}[\theta_j] = 0$ and remain in the same track throughout secondary school. We impose this threshold as a transparent benchmark. It need not maximize expected end-of-school skills in the two-period problem, because the initial threshold also affects the track-specific skill distributions and instruction paces in the second period.

Define $m \equiv \kappa\sigma_{\theta_j}$, so that the initial track means are $\bar{\theta}_{j,ET}^A = m$, $\bar{\theta}_{j,ET}^V = -m$. Applying (A.22) in the first period gives

$$\mu_{j+1,ET} \equiv \mathbb{E}[\theta_{j+1,ET}] = a + cm^2. \quad (\text{A.25})$$

Moreover, equation (A.21) implies that the intermediate conditional track means are

$$\begin{aligned}\mathbb{E}[\theta_{j+1,ET} \mid S_{ET} = A] &= \mu_{j+1,ET} + (b + v_1)m, \\ \mathbb{E}[\theta_{j+1,ET} \mid S_{ET} = V] &= \mu_{j+1,ET} - (b + v_1)m.\end{aligned}$$

Consequently,

$$\mathbb{E}[(\mathbb{E}[\theta_{j+1,ET} \mid S_{ET}])^2] = \mu_{j+1,ET}^2 + (b + v_1)^2 m^2. \quad (\text{A.26})$$

Applying (A.22) again in the second period yields

$$\mathbb{E}[\theta_{j+2,ET}] = a + (b + v_1)\mu_{j+1,ET} + c [\mu_{j+1,ET}^2 + (b + v_1)^2 m^2]. \quad (\text{A.27})$$

Subtracting (A.27) from (A.24), and using $\mu_{j+1,ET} = a + cm^2$, $m^2 = \kappa^2 \sigma_{\theta_j}^2$, gives

$$\begin{aligned}\mathbb{E}[\theta_{j+2,LT}] - \mathbb{E}[\theta_{j+2,ET}] \\ = c\kappa^2 \left\{ \sigma_{\eta_{j+1}}^2 - \sigma_{\theta_j}^2 \left[b + v_1 + 2bv_1 + v_1^2 + 2ac + c^2 \kappa^2 \sigma_{\theta_j}^2 \right] \right\}.\end{aligned} \quad (\text{A.28})$$

Since $c > 0$ and $\kappa^2 > 0$, expected end-of-school skills are higher under optimal late tracking than under optimal early tracking if and only if

$$\frac{\sigma_{\eta_{j+1}}^2}{\sigma_{\theta_j}^2} > b + v_1 + 2bv_1 + v_1^2 + 2ac + c^2 \kappa^2 \sigma_{\theta_j}^2. \quad (\text{A.29})$$

Substituting the definitions of a , b , and c , together with $\kappa^2 = 2/\pi$, gives

$$\begin{aligned}\frac{\sigma_{\eta_{j+1}}^2}{\sigma_{\theta_j}^2} &> v_0 + \frac{v_2 v_3}{v_4} + v_1 + 2v_1 \left(v_0 + \frac{v_2 v_3}{v_4} \right) + v_1^2 \\ &\quad + \frac{v_2^2 v_3^2}{2v_4^2} + \frac{v_3^4}{2\pi v_4^2} \sigma_{\theta_j}^2.\end{aligned} \quad (\text{A.30})$$

The right-hand side defines the critical value of the ratio of the intermediate skill-shock variance to initial skill variance. Since $b \geq 0$, the right-hand side of (A.29) is increasing in v_1 , with derivative $1 + 2b + 2v_1$, and, holding a and b fixed, in c , with derivative $2a + 2c\kappa^2 \sigma_{\theta_j}^2$. This establishes the condition under which optimal late tracking yields higher expected end-of-school skills than mean-split early tracking in Proposition 3. Note, however, that because any two-period-optimal early assignment weakly dominates this benchmark, the condition is necessary, but not sufficient, for optimal late tracking to outperform fully optimized early

tracking. This proves Proposition 3.

A.2 Equilibrium Definition

We introduce some notation to define the equilibrium more easily. Let $x_j \in X_j$ be the age-specific state vector of an individual of age j , as defined by the recursive representation of the individual's problems in Section 2, and let Θ_j denote the stationary measure over X_j . Θ_j is not normalized within education groups: it integrates to the population mass of age j , so the integrals below are population totals. Let $\Theta \equiv \{\Theta_j\}_j$ and let $J_r = 16$ denote the last working period. Then, a stationary recursive competitive equilibrium for this economy is a collection of: (i) decision rules for college graduation $\{d^E(x_5)\}$, for school track $\{d^S(x_{11})\}$, consumption, labor supply, and assets holdings $\{c_j(x_j), n_j(x_j), a_{j+1}(x_j)\}$, and parental transfers $\{\tau(x_{13})\}$; value functions $\{V_j(x_j)\}$; (ii) aggregate capital and labor inputs $\{K, H_0, H_1\}$; (iii) prices $\{r, w_0, w_1\}$; and (iv) average skill levels among children in school track S $\{\bar{\theta}_{j,S}\}$ for $j = 2, 3, 4$ such that:

1. Given prices and average skill levels among children in each school track, decision rules solve the respective household problems, and $\{V_j(x_j)\}$ are the associated value functions.
2. Given prices, aggregate capital and labor inputs solve the representative firm's problem.
3. Average track skill levels are consistent with parental choices: the track allocation implied by d^S generates average skills in each track equal to the $\{\bar{\theta}_j^S\}$ taken as given by parents and the policymaker.
4. Labor markets for each education level clear:

$$H_0 = \sum_{j=6}^{J_r} \int_{X_j} \mathbf{1}\{E = 0\} n_j(x_j) h_j(x_j) d\Theta_j + \int_{X_5} n_5(x_5) h_5(x_5) d\Theta_5,$$

$$H_1 = \sum_{j=6}^{J_r} \int_{X_j} \mathbf{1}\{E = 1\} n_j(x_j) h_j(x_j) d\Theta_j,$$

where $h_5 \equiv \exp(\theta_5)$, and the second term in H_0 is the non-college-wage labor supplied at age 18 by both future college graduates and non-graduates.

5. The asset market clears: $K = \sum_{j=5}^{20} \int_{X_j} a_j(x_j) d\Theta_j - \int_{X_{13}} \tau(x_{13}) d\Theta_{13}$, where the second term reflects that transfers are deducted from parental assets at the

beginning of period 13 and therefore do not enter productive capital in that period. This implies that the goods market clears.

6. The distribution is stationary: $\Theta = \Gamma(\Theta)$, where Γ is the law of motion induced by the decision rules and the stochastic processes for shocks.

B Empirical and Calibration Appendix

B.1 School Tracking Worldwide

Through its system-level questionnaire, PISA 2018 asked countries to report the school types or distinct education programs available to 15-year-old students and the age at which students are first selected into them. Only 11 education systems offer a single program (10 of 37 OECD members, and 1 of 35 non-OECD members), and in 23 systems, students are first selected at age 14 or younger (13 of 37 OECD members and 10 of 35 non-OECD members).⁵⁷ The countries that select students at the youngest ages are Austria, Germany, and Hungary (age 10), and the Czech Republic, the Slovak Republic, and Turkey (age 11) (Table B.1; source: Table B3.3.3 in [OECD \(2020b\)](#)).

B.2 German Education System

This section outlines the key features of the German education system. For a more extensive description, see [Henninges et al. \(2019\)](#) or [OECD \(2020a\)](#). Figure B.1 illustrates a simplified structure, from Grade 4 through tertiary education.

Schooling is compulsory from age six to eighteen, though the obligation to attend a general school typically ends after Grade 9 or 10, after which it shifts to a vocational training obligation for those not enrolled in upper secondary school. All children attend a comprehensive primary school for the first four grades.⁵⁸ They are then allocated into secondary school, traditionally split into three tracks—a lower and a medium vocational track, and an academic track. Following the “PISA shock” of the early 2000s, however, most federal states have merged the two vocational tracks into one ([Bellenberg and Forell, 2012](#)). Since even where two vocational tracks remain, they are far more similar to each other than to the academic track, we restrict our analysis to two tracks.

⁵⁷Some countries offer more than one program to 15-year-olds but select students only at age 16 or later; these include Finland, Spain, Chile, and Latvia among OECD members, and the Dominican Republic, Moldova, and the Philippines among non-OECD members.

⁵⁸In Berlin and Brandenburg, comprehensive primary school lasts six grades.

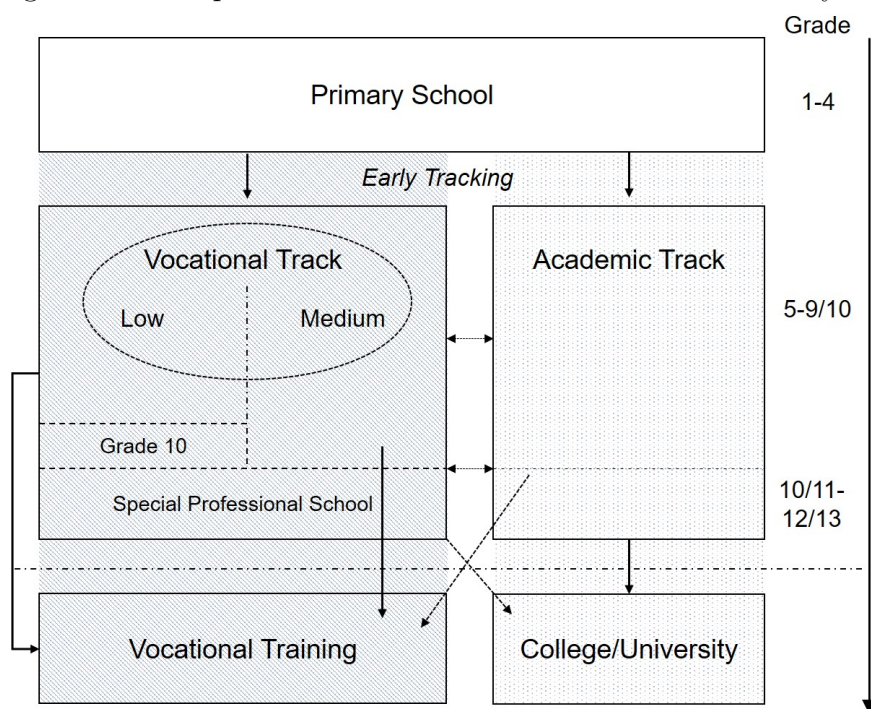
Table B.1: Age at First Selection by Country

10	11	12	13	14
Austria	Czech Republic	Belgium	Bulgaria	Italy
Germany	Luxembourg	Netherlands		Lithuania
Hungary	Slovak Republic	Switzerland		Slovenia
	Turkey	Brunei Darussalam		Argentina
		Indonesia		Croatia
		Lebanon		North Macedonia
		Singapore		Romania
				Ukraine
15		16 or later		
Colombia	Belarus	Macao (China)	Australia	Poland
Denmark	Bosnia and Herzegovina	Montenegro	Canada	Spain
France	Brazil	Panama	Chile	Sweden
Greece	Chinese Taipei	Qatar	Estonia	United Kingdom
Ireland	Costa Rica	Russia	Finland	United States
Israel	Cyprus	Serbia	Iceland	Dominican Republic
Japan	Georgia	Thailand	Latvia	Malta
Korea	Hong Kong (China)	United Arab Emirates	New Zealand	Moldova
Mexico	Jordan	Uruguay	Norway	Philippines
Portugal	Kazakhstan	Viet Nam		
Albania				

Notes: Bold indicates OECD member countries. Azerbaijan, B-S-J-Z (China), Kosovo, Malaysia, Morocco, Peru, and Saudi Arabia are excluded due to missing or inconsistent age data. Source: Table B3.3.3 [OECD \(2020b\)](#).

The tracks differ in curricula, length of study, and end-of-school qualifications. Only the academic track leads to a university entrance qualification, requiring completion of upper secondary school (typically Grades 10/11 through 12/13). Vocational tracks end after Grades 9 or 10, leading either to vocational training or to a professional school preparing students for white-collar, business, or skilled-trade occupations. Different paths into higher education or work exist at this stage: professional degrees can grant access to university studies in select fields; strong students can switch directly into an academic-track school; and, after several years of vocational work experience, some college degrees become accessible. Conversely, academic-track students may switch to vocational training or employment once compulsory education is complete.⁵⁹

Figure B.1: Simplified Structure of the German Education System



Public expenditure per student differs little between the academic track and the vocational track as a whole. Table B.2 reports average per-student expenditures by school type for 2010–2018. Spending is highest in stand-alone lower-vocational-track schools and lowest

⁵⁹In the model, we abstract from potential heterogeneity among the resulting learning environments at the end of secondary school and assume that the representative environment associated with each track remains constant until age 18. The model's vocational track therefore represents the broader nonacademic pathway rather than continued enrollment in the same lower-secondary school type. The NEPS data follow students across these pathways, so our late-secondary skill estimates are not restricted to students who remain in their original school type.

in upper-vocational-track schools, but weighting the vocational school types by their enrollment, the average vocational-track student received about 8,450 euros in 2018, within 4% of the 8,800 euros spent per academic-track student.⁶⁰ About 80% of these expenditures go to teacher pay, with the remainder covering buildings, equipment, and similar investments. This suggests that resource differences across tracks are unlikely to be a major driver of achievement gaps, on average.

Table B.2: Per-Student Public Expenditures across School Types and Years

Year	Primary	Lower Voc.	Upper Voc.	Joint Voc.	Acad.	Compr.
2010	5,200	7,100	5,300	8,000	6,600	6,600
2011	5,500	7,300	5,600	8,000	7,100	7,100
2012	5,400	7,900	5,700	7,700	7,200	7,200
2013	5,600	8,200	5,900	7,700	7,500	7,500
2014	5,900	8,700	6,200	8,000	7,800	7,800
2015	6,000	8,900	6,400	8,000	7,900	8,000
2016	6,200	9,300	6,700	8,100	8,100	8,200
2017	6,400	9,800	7,000	8,300	8,500	8,600
2018	6,700	10,400	7,400	8,700	8,800	9,100

Notes: Source: [Statistisches Bundesamt \(2021\)](#); figures for 2018 by school type are from a special tabulation provided by the Statistisches Bundesamt. All amounts in euros.

Teaching quality could still differ across tracks, for instance, if higher-quality teachers select into academic-track schools. However, becoming a teacher requires a university degree of similar length (7 to 10 semesters) regardless of the track taught, and teacher pay no longer differs significantly by track in many federal states: in most northern and eastern states, for example, tenured teachers at vocational- and academic-track schools are already eligible for the same public pay grade.

B.3 NEPS, Sample Selection and Main Variables

The German National Educational Panel Study (NEPS) is a longitudinal survey of nationally representative pupil samples starting in 2010. We use Starting Cohorts 2, 3, and 4, which include repeated standardized assessments of children’s competencies in mathematics, reading, science, and grammar, along with sociodemographic characteristics such as gender, birth year, and parental education.

⁶⁰Expenditures are from [Statistisches Bundesamt \(2021\)](#); enrollment weights are based on the number of students in lower-vocational, upper-vocational, and joint-vocational schools in the 2018/19 school year ([Statistisches Bundesamt \(Destatis\), \(2019\)](#)).

B.3.1 Main Variables

Competence Scores One of the NEPS study’s core goals is to document the development of individuals’ competencies over their lifespan. Following PISA guidelines, the study administers regular, curriculum-aligned tests across domains—including reading comprehension, mathematical competence, and scientific literacy—to assess both curricular learning and the ability to apply domain knowledge to real-world problems (see [Neumann et al. \(2013\)](#) for test design, and [Pohl and Carstensen \(2012\)](#) for item formats, ranging from simple and complex multiple-choice to short-constructed responses). To convert these test items into latent competency measures, NEPS uses item response theory (IRT) scaling—specifically, the Rasch model for dichotomous items, with the Partial Credit Model for polytomous items ([Pohl and Carstensen, 2013](#); [Rasch, 1960](#); [Masters, 1982](#)). This approach yields an arbitrarily scaled ability estimate for each respondent, which NEPS links across waves and item sets.⁶¹ To construct latent skills and estimate the skill technology, we restrict attention to competence scores estimated using linked item difficulty parameters, ensuring comparability across grades within a cohort. Specifically, we use math, vocabulary, and science scores for Cohort 2 (Grades 1 and 4), and math, reading, and science scores for Cohorts 3 (Grades 5, 9, and 12) and 4 (Grades 9 and 12).

Parental Education NEPS repeatedly surveys one parent at home on the highest educational qualification held by both the respondent and their partner, provided in both ISCED and CASMIN classifications. We classify a household as college-educated if either partner is ever reported, under the ISCED classification, as holding a “Bachelor, Master, Diploma, Magister” or a “doctorate, habilitation” degree in any survey wave—corresponding, under CASMIN, to a “degree from a university of applied science” or a “university degree”—and as non-college-educated otherwise.

Gender and Year of Birth Parents also report their surveyed child’s gender and year of birth, which we use directly.

Track Choice NEPS records students’ current school type. After Grade 5, we assign a student to the academic track if she is ever recorded as attending a “Gymnasium,” or a comprehensive school’s “Gymnasium branch,” and to the vocational track if she is ever recorded as attending a “Hauptschule” or “Realschule,” or the corresponding branch of a

⁶¹[Pohl and Carstensen \(2013\)](#) provide a fuller discussion of the practical challenges in this scaling process, such as handling different response formats, missing responses, adaptive testing, and cross-cohort linking.

comprehensive school. We cannot assign a track if there is never a separation in school branches or if the student switches tracks across waves—though both cases are rare. In Grade 5 of Cohort 3 non-special need sample, for example, only 12.6% cannot be assigned to a track because their track is never specified, and 0.9% because of track switches. Track assignment relying on later separation in school branches affects only a small minority of students—just 1.1% of the sample.

College Graduation In Cohort 4, college graduation status is measured in wave 13, when respondents are 25 years old. A student is classified as college-educated if her highest reported degree is from a university of applied sciences or a traditional university.

B.3.2 Samples and Weights

Our final sample comprises students in the main sample—excluding the special-needs sample—who completed the competence tests, for whom gender and year of birth are known, and, for Cohorts 3 and 4, could be assigned to the academic or vocational track. Since our main regressions include parental education, we further restrict the sample to students for whom this information is available. For the skill technology estimation, students must additionally be assigned to a classroom with at least 10 pupils, from which we compute average peer skills.⁶² While we always present both weighted and unweighted statistics, in the text we refer exclusively to estimates weighted using grade-specific NEPS weights, our preferred specification, since these weights correct for panel attrition.

Table B.3 describes the sample restrictions by cohort and grade. Except for Cohort 2, restricting the sample to students who can be assigned to a track only marginally affects the share of college-educated parents, whereas restricting the sample to those with parental education information tends to increase the academic-track share, suggesting some selection in parental reporting. For Cohort 2, track choice occurs late in the panel, which comes with substantial attrition and sample distortion. For this reason, we restrict the final sample to parental education only, not to track choice.

Track choice and parental education shares vary by cohort and grade, but we need a single target moment for each in the model calibration. Table B.4 presents these targets for school tracks and college education shares. For the academic-track share, we use the official statistic of 44% from the education report, which closely matches the share observed in the

⁶²To maximize the number of observations, we compute average peer skills including all students, even those without information on parental education.

Table B.3: NEPS Mathematic Assessment Samples

	Obs.	% Ac-track		% College Parents		% College	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
<i>Panel A: Full Sample</i>							
G1 (Cohort 2)	5,620	79%	79%	38%	38%		
G4 (Cohort 2)	4,358	79%	79%	39%	40%		
G5 (Cohort 3)	4,420	53%	59%	32%	34%		
G9 (Cohort 3)	4,100	46%	56%	28%	32%		
G9 (Cohort 4)	13,344	43%	42%	28%	28%	35%	34%
G12 (Cohort 4)	4,814	46%	82%	27%	41%	28%	51%
<i>Panel B: If Parental Education</i>							
G1 (Cohort 2)	5,611	79% (77%)	79% (77%)	38% (28%)	38% (28%)		
G4 (Cohort 2)	4,349	79% (76%)	79% (77%)	39% (28%)	40% (28%)		
G5 (Cohort 3)	3,781	57%	62%	32%	34%		
G9 (Cohort 3)	3,432	50%	59%	28%	32%		
G9 (Cohort 4)	7,877	50%	49%	28%	28%	39%	38%
G12 (Cohort 4)	3,258	52%	86%	27%	41%	30%	53%
<i>Panel C: If Parental Education and Track Choice</i>							
G1 (Cohort 2)	2,112	79% (77%)	79% (77%)	53% (42%)	54% (42%)		
G4 (Cohort 2)	1,831	79% (76%)	79% (77%)	54% (42%)	55% (41%)		
G5 (Cohort 3)	3,243	57% (44%)	62% (45%)	33% (28%)	34% (29%)		
G9 (Cohort 3)	3,093	50% (44%)	59% (44%)	29% (28%)	33% (28%)		
G9 (Cohort 4)	7,325	50% (44%)	49% (44%)	28% (28%)	28% (28%)	39% (37%)	38% (37%)
G12 (Cohort 4)	3,100	52% (44%)	86% (44%)	28% (28%)	41% (28%)	31% (29%)	54% (37%)
<i>Panel D: If Parental Education and Track and College Choices</i>							
G9 (Cohort 4)	3,776	58% (52%)	57% (52%)	35% (34%)	34% (34%)	39% (37%)	38% (37%)
G12 (Cohort 4)	1,932	59% (51%)	89% (50%)	29% (30%)	45% (32%)	31% (29%)	54% (37%)
Weights		NEPS	no	NEPS	no	NEPS	no

Notes: This table describes NEPS samples by cohort and grade. Panel A displays the full sample: students who completed the competence tests and for whom gender and year of birth are known. Panel B further restricts the sample to those with parental education information. Panel C restricts to students who can be assigned to a track, and Panel D additionally requires a known college education choice. Column (1) reports the number of observations in each sample; Columns (2)–(3) show the share of academic-track students; Columns (4)–(5) show the share of college-educated parents; and Columns (6)–(7) show the share of college-educated students. Columns (2), (4), and (6) use NEPS grade-specific weights, while Columns (3), (5), and (7) do not. Shares in parentheses additionally use our own weights, and our final samples are shown in bold. Source: NEPS Cohorts 2, 3, and 4.

full sample of NEPS Cohorts 3 and 4. In Cohort 4, children of college-educated parents are 2.05 times as likely to attend the academic track as children of non-college-educated parents, somewhat above the 1.41 reported in [Maaz et al. \(2018, p. 53\)](#).⁶³ In Cohort 4, children of college-educated parents are 2.57 times as likely to graduate from college as children of non-college-educated parents, close to the 2.92 reported in [Maaz et al. \(2018, p. 156, Figure F2-2\)](#).⁶⁴ Finally, we target a college-educated parent share of 28% from NEPS, which, given the model’s stationary equilibrium, implies that 28% of students graduate from college. The relative likelihood of entering college by track and parental education is computed from NEPS Cohort 4, and all remaining shares are set so that the model’s distribution is internally consistent.

To make our descriptive statistics consistent with the targeted model shares, we additionally weight all observations of Cohorts 3 and 4 so that, in every grade and cohort, the share of children in each track matches the targeted values: 28% of parents are college-educated, 44% of students are in the academic track, and the relative likelihood of entering the academic track by parental education is 2.05. These weighted statistics are reported in parentheses in [Table B.3](#), and all data tables and figures in this paper incorporate these additional weights. College graduation share targets, however, are not incorporated into these additional weights, resulting in slightly higher shares in the data (37% using Grade 9, 29% using Grade 12) than in the model (28%, by assumption). For Cohort 2, we re-weight the data to target only the 28% college-educated parent share.

B.4 Details on Child Skills

Following the literature on child skill formation, we model each period’s measures of latent log skills using a linear measurement system:

$$M_{i,k,j} = \mu_{k,j} + \lambda_{k,j}\theta_{i,j} + \epsilon_{i,k,j},$$

where $M_{i,k,j}$ is the k th measure for latent log skills of child i in period j . We observe at least three measures per period—math, vocabulary, and science for Cohort 2 (Grades 1 and 4),

⁶³The 1.41 is the ratio of the share of children with university-educated parents who attended a general education school (76%) to the corresponding share among children whose parents hold a vocational qualification or equivalent (54%): $76/54 \approx 1.41$.

⁶⁴Among 18- to under-25-year-olds, 28% have at least one parent with a higher education degree, and 79% of them start university studies, accounting for 53% of all first-year students. Hence about 41.5% of the age group start university studies. The remaining 72% have no parent with a higher education degree and account for the other 47% of first-year students, so their propensity to start university studies is $0.47 \times 41.5\% / 72\% \approx 27\%$. The ratio is therefore $79/27 \approx 2.92$.

Table B.4: School Tracks and Parental Education Targets

Statistics	Value	Source
% of college parents	28%	NEPS Cohort 4
Track Choice		
% in academic track	44%	Education report p.110
Relative likelihood A track (CL/NCL)	2.05	NEPS Cohort 4
% in ac. track if $E = 1$	70%	Implied
% in ac. track if $E = 0$	34%	Implied
College Graduation		
% who graduate from college	28%	Model assumption
Relative likelihood to enter college (A/V)	3.40	NEPS Cohort 4
% college if academics	46%	Implied
% college if vocational	14%	Implied
Relative likelihood to enter college (CL/NCL)	2.57	NEPS Cohort 4
% college if college parents	50%	Implied
% college if non-college parents	19%	Implied

Notes: This table reports the distribution of students by school track, college education, and parental education, used as target moments, along with the corresponding data sources.

and math, reading, and science for Cohorts 3 (Grades 5, 9, 12) and 4 (Grades 9, 12). The parameters $\mu_{k,j}$ and $\lambda_{k,j}$ denote the location and factor loading of latent log skills, and $\epsilon_{i,k,j}$ is the measurement error.

Following Cunha et al. (2010), we normalize $\mathbb{E}(\theta_j) = 0$, $\lambda_{1,j} = 1$ for all j and $E(\epsilon_{k,j}) = 0$ for all j . The location parameters $\mu_{k,j}$ are then identified from the means of the measures. Assuming measurement errors are independent across measures and of latent skills, and given at least three measures per period, we identify the factor loadings from ratios of measure covariances (as in Agostinelli et al., 2025):

$$\lambda_{k,j} = \frac{Cov(M_{k,j}, M_{k',j})}{Cov(M_{1,j}, M_{k',j})}$$

for all $k, k' > 1$ and $k \neq k'$. Rescaling each measure by its identified location and loading yields error-contaminated latent skill measures:

$$\theta_{i,j} = \frac{M_{i,k,j} - \mu_{k,j}}{\lambda_{k,j}} - \frac{\epsilon_{i,k,j}}{\lambda_{k,j}} = \widetilde{M}_{i,k,j} - \frac{\epsilon_{i,k,j}}{\lambda_{k,j}}.$$

Substituting these into the empirical skill technology yields

$$\begin{aligned}\widetilde{M}_{i,k,j+1} = & \kappa_{0,j} + \kappa_{1,j}\widetilde{M}_{i,k,j} + \kappa_{2,j}\widetilde{M}_{i,k,j}^2 + \kappa_{3,j}\widetilde{\overline{M}}_{-i,j,S} \\ & + \kappa_{4,j}(\widetilde{M}_{i,k,j} - \widetilde{\overline{M}}_{j,S})^2 + \kappa_{5,j}E_i + \zeta_{i,k,j+1},\end{aligned}\tag{B.1}$$

where $\widetilde{\overline{M}}_{-i,j,S}$ is the average value of the k th transformed measure across all children other than i in a classroom in track S and $\widetilde{\overline{M}}_{j,S}$ is the average value of the measures across all children in a school that belongs to track S .

The residual $\zeta_{i,k,j+1}$ contains not only structural skill shocks, $\eta_{i,j+1}$, but also measurement errors, $\epsilon_{i,k,j}$, its interactions with the rescaled measures, and even its variance—so OLS estimation of (B.1) is biased even under mean-independence of η . We therefore follow the literature and use Bartlett factor scores to aggregate the measures into Bartlett factor scores, which are conditionally unbiased predictors of latent skills (Agostinelli et al., 2026). This removes the bias from measurement error in the dependent variable, but not from the remaining error in the regressors. Based on our measurement system, the signal-to-noise ratio of the scores, defined as the share of their variance due to latent skills, $\text{Var}(\theta_j)/\text{Var}(\hat{\theta}_j)$, ranges from 0.77 to 0.86 across grades and is lowest in grade 12. In a linear specification, the remaining noise would attenuate the coefficient on own skills and bias the coefficients on regressors correlated with own skills, such as average peer skills and parental education, away from zero. In our nonlinear specification, the direction of the bias is not determined in general, and we therefore interpret the estimated coefficients as approximations to the true technology.⁶⁵

B.5 Child Skill Technology Estimation

Tables B.5 and B.6 present four specifications (with or without school fixed effects and weights) for grades 5-9 and 9-12 ($j = 3, 4$) using NEPS Cohorts 3 and 4. Estimates are stable across specifications and, consistent with our model, we cannot reject $-\hat{\omega}_{3,j} = \hat{\omega}_{4,j}$. Since instruction pace is not observed, $\omega_{3,j}$ and $\omega_{4,j}$ identify how skill growth varies with a child’s skill relative to her track’s average skill. The model interprets this variation as a non-linear interaction between a child’s skill and the track’s instruction pace, but it may also reflect other track-specific non-linear effects. Table B.7 reports regressions of future skills on current skills alone (Panel A), adding parental education (Panel B), or adding a track-choice

⁶⁵We also explored instrumenting the regressors with scores constructed from the remaining measures; with only three measures per grade, these estimates are too imprecise to be informative.

dummy (Panel C).

Table B.5: Child Skill Technology Estimates in Period 3

Dependent Variable: $\theta_{i,j+1}$	Data				Model
	Grade 5 (cohort 3)				j = 3
	(1)	(2)	(3)	(4)	(5)
$\theta_{i,j}$	0.70 (0.02)	0.72 (0.02)	0.68 (0.02)	0.69 (0.02)	0.74 [0.02, 100%]
$\bar{\theta}_{-i,j}^S$	0.23 (0.03)	0.19 (0.03)	0.11 (0.08)	0.07 (0.08)	0.12 [0.03, 100%]
$(\theta_{i,j} - \bar{\theta}_j^S)^2$	-0.18 (0.08)	-0.16 (0.07)	-0.15 (0.07)	-0.16 (0.07)	-0.11 [0.06, 36%]
$\theta_{i,j}^2$	0.07 (0.05)	0.09 (0.05)	0.07 (0.06)	0.12 (0.05)	0.11 [0.04, 71%]
E_i	0.03 (0.01)	0.03 (0.01)	0.02 (0.01)	0.02 (0.01)	0.05 [0.01, 100%]
Obs.	1,883	1,883	1,883	1,883	1,883
Controls	yes	yes	yes	yes	NA
School FE	no	no	yes	yes	NA
Weights	NEPS	no	NEPS	no	NA

Notes: This table presents coefficients from regressions of grade-9 skills on grade-5 skills, skills squared, average peer skills, squared distance to the track average, and a parental education dummy (see Section 3.1.1). Columns (1) and (3) use longitudinal weights, while Columns (2) and (4) do not. All regressions control for year-of-birth and gender fixed effects. Columns (1)-(2) exclude school fixed effects, while Columns (3)-(4) include them. Robust standard errors are in parentheses. All observations are additionally weighted using our own track-share weights (see Appendix B.3). Source: NEPS Cohort 3. Column (5) reports average regression coefficients across 100 draws of $N = 1,883$ from the baseline model's simulated data, with standard deviations and the frequency of 5% significance in square brackets.

B.6 Estimation of the Human Capital Growth

Concretely, we estimate, separately for each education group, Mincer regressions of log wages on years of schooling and a full set of four-year potential-experience bins, controlling for time and four-year birth-cohort effects:

$$\log w_{ict} = \alpha + \beta s_{ict} + \sum_{k=1}^{10} \delta_k \mathbf{1}\{x_{ict} \in [4k, 4k + 4)\} + \gamma_t + \zeta_c + \epsilon_{ict},$$

where w_{ict} is the wage of individual i , who belongs to birth cohort c and is observed at time t . Wages are defined as total annual labor earnings divided by hours worked. We denote by

Table B.6: Child Skill Technology Estimates in Period 4

Dependent Variable: $\theta_{i,j+1}$	Data				Model
	Grade 9 (cohort 4)				j = 4
	(1)	(2)	(3)	(4)	(5)
$\theta_{i,j}$	0.94 (0.03)	0.96 (0.03)	0.90 (0.04)	0.92 (0.03)	0.73 [0.01, 100%]
$\bar{\theta}_{-i,j}^S$	0.21 (0.04)	0.19 (0.04)	0.05 (0.10)	0.06 (0.09)	0.08 [0.03, 85%]
$(\theta_{i,j} - \bar{\theta}_j^S)^2$	-0.32 (0.10)	-0.28 (0.10)	-0.31 (0.11)	-0.29 (0.10)	-0.16 [0.06, 82%]
$\theta_{i,j}^2$	0.10 (0.08)	0.08 (0.08)	0.14 (0.10)	0.12 (0.09)	0.17 [0.04, 97%]
E_i	0.04 (0.01)	0.04 (0.01)	0.04 (0.01)	0.03 (0.01)	0.05 [0.01, 100%]
Obs.	2,757	2,757	2,757	2,757	2,757
Controls	yes	yes	yes	yes	NA
School FE	no	no	yes	yes	NA
Weights	NEPS	no	NEPS	no	NA

Notes: This table presents coefficients from regressions of grade-12 skills on grade-9 skills, skills squared, average peer skills, squared distance to the track average, and a parental education dummy (see Section 3.1.1). Columns (1) and (3) use longitudinal weights, while Columns (2) and (4) do not. All regressions control for year-of-birth and gender fixed effects. Columns (1)-(2) exclude school fixed effects, while Columns (3)-(4) include them. Robust standard errors are in parentheses. All observations are additionally weighted using our own track-share weights (see Appendix B.3). Source: NEPS Cohort 4. Column (5) reports average regression coefficients across 100 draws of $N = 2,757$ from the baseline model's simulated data, with standard deviations and the frequency of 5% significance in square brackets.

Table B.7: Child Skill Evolution in Period 2, 3 and 4

Dep. Var.: $\theta_{i,j+1}$	Data Grade 1		Model j = 2	Data Grade 5		Model j = 3	Data Grade 9		Model j = 4
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
<i>Panel A: Future Skills on Current Skills</i>									
$\theta_{i,j}$	0.79 (0.01)	0.80 (0.01)	0.77 [0.01, 100%]	0.79 (0.02)	0.80 (0.01)	0.80 [0.01, 100%]	1.02 (0.02)	1.03 (0.02)	0.79 [0.01, 100%]
Obs.	4,196	4,196	4,196	2,037	2,037	2,037	2,905	2,905	2,905
<i>Panel B: Future Skills on Current Skills and Parental Education</i>									
$\theta_{i,j}$	0.76 (0.01)	0.77 (0.01)	0.74 [0.01, 100%]	0.77 (0.02)	0.78 (0.01)	0.78 [0.01, 100%]	1.00 (0.02)	1.01 (0.02)	0.76 [0.01, 100%]
E_i	0.07 (0.01)	0.08 (0.01)	0.07 [0.01, 100%]	0.05 (0.01)	0.04 (0.01)	0.06 [0.01, 100%]	0.05 (0.01)	0.05 (0.01)	0.06 [0.01, 100%]
Obs.	4,196	4,196	4,196	2,037	2,037	2,037	2,905	2,905	2,905
<i>Panel C: Future Skills on Current Skills and Track Choice</i>									
$\theta_{i,j}$				0.72 (0.02)	0.73 (0.02)	0.76 [0.01, 100%]	0.96 (0.02)	0.97 (0.02)	0.75 [0.01, 100%]
S_i				0.09 (0.01)	0.08 (0.01)	0.06 [0.01, 100%]	0.11 (0.02)	0.11 (0.02)	0.04 [0.01, 99%]
Obs.				2,037	2,037	2,037	2,905	2,905	2,905
Controls	yes	yes	NA	yes	yes	NA	yes	yes	NA
Weights	NEPS	no	NA	NEPS	no	NA	NEPS	no	NA

Notes: Panel A presents coefficients from regressions of future skills on current skills, and Panel B adds a parental education dummy (see Section 3.1.1). Panel C presents coefficients from regressions of future skills on current skills and a track-choice dummy. Columns (1), (4), and (7) use longitudinal weights, while Columns (2), (5), and (8) do not. All regressions control for year-of-birth and gender fixed effects. Columns (1)–(2) regress grade-4 skills on grade-1 skills (cohort 2), Columns (4)–(5) regress grade-9 skills on grade-5 skills (cohort 3), and Columns (7)–(8) regress grade-12 skills on grade-9 skills (cohort 4). Robust standard errors are in parentheses. All observations are additionally weighted using our own track-share weights (see Appendix B.3). Source: NEPS Cohorts 2, 3, and 4. Columns (3), (6), and (9) report average regression coefficients across 100 draws from the baseline model’s simulated data, with standard deviations and the frequency of 5% significance in square brackets.

s_{ict} the years of schooling and by x_{ict} potential work experience, which is defined as

$$\begin{aligned} x_{ict} &= age_{ict} - 18 \text{ if } s_{ict} < 12 \\ x_{ict} &= age_{ict} - s_{ict} - 6 \text{ else,} \end{aligned}$$

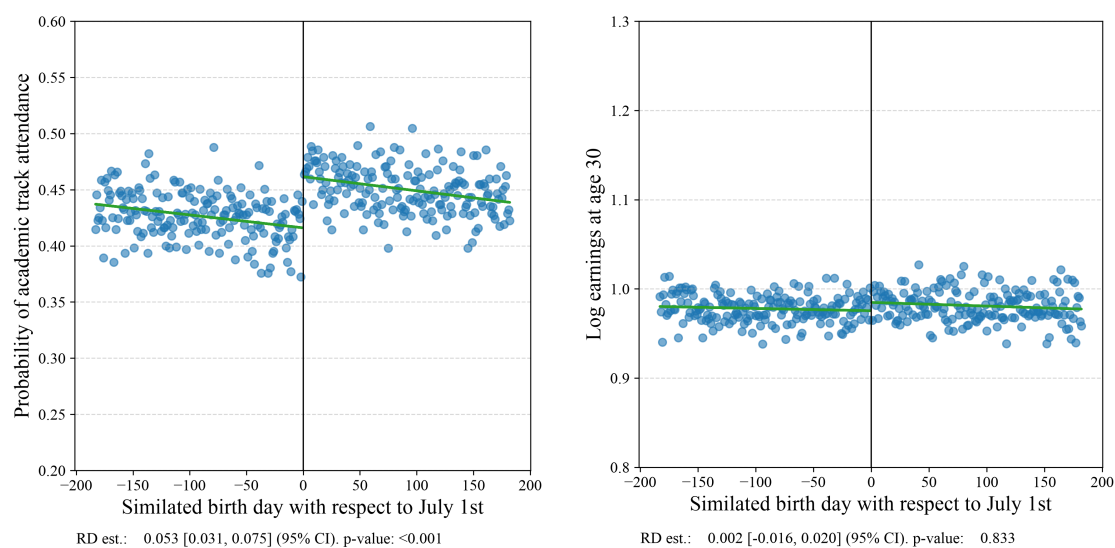
and censored at 44 years; the first experience bin ($k = 0$) is the omitted category. Because experience, time, and cohort effects are collinear, we follow the HLT approach in [Lagakos et al. \(2018\)](#) and assume no net experience effect on wage growth in the last eight years of work, which pins down the trend in the time effects. The four-year growth factors reported in Panel B of Table 1 are the ratios of consecutive experience-bin profiles, $\exp(\delta_{k+1} - \delta_k)$. The resulting ten growth factors apply from model period $j = 6$ onward for both education groups ($\gamma_{5,E} = 1$), so that experience profiles are aligned with labor-market entry after college.

B.7 Additional Figures

Figure B.2 illustrates one draw of the Monte Carlo simulation of the regression discontinuity design described in Section 3.2.2. Consistent with [Dustmann et al. \(2017\)](#)’s findings, the model predicts, in this draw, a gap of 4.9 percentage points in academic-track access between the most and least mature children (by construction, close to the targeted 5.2 percentage points), which does not translate into a corresponding earnings gap: the effect on labor earnings at age 30 is close to zero and not statistically significant.

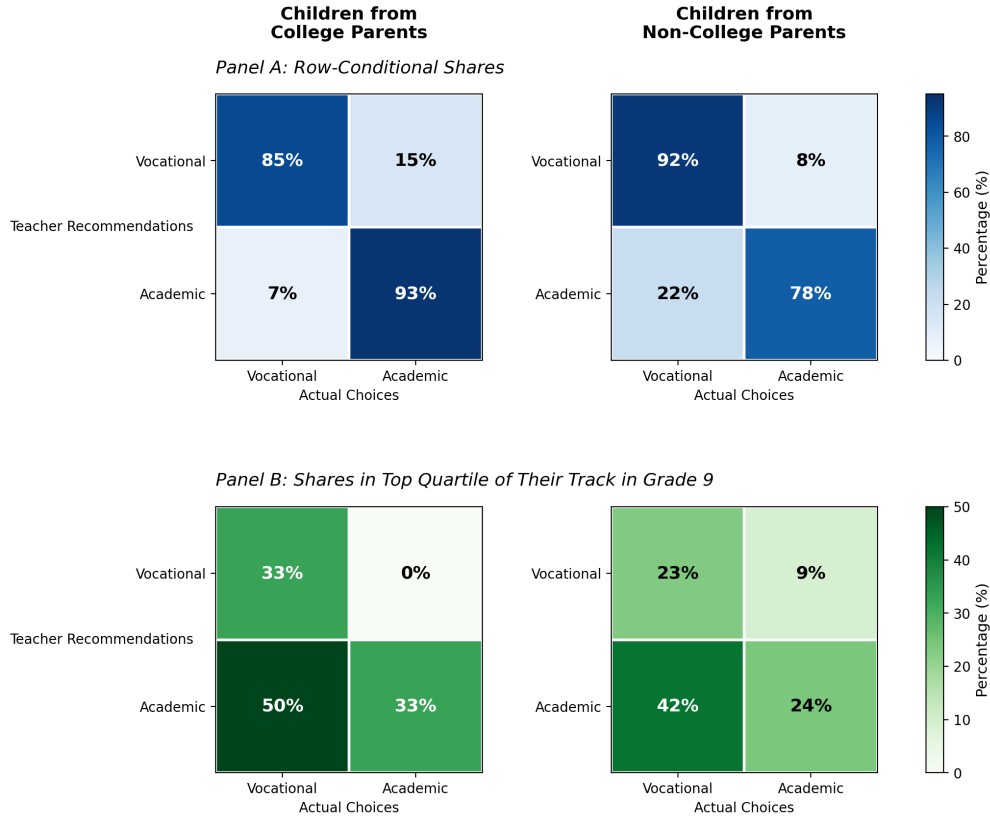
Panel A of Figure B.3 shows that children who deviate from the teacher’s recommendation typically follow their parent’s own track. Among children of college-educated parents, 15% deviate after receiving a vocational-track recommendation, compared to only 7% after receiving an academic-track recommendation. The pattern is similar but reversed for children of non-college-educated parents: 8% deviate after a vocational-track recommendation, versus 22% after an academic-track recommendation. Panel B shows that children who deviate from the recommendation later perform differently from their track peers. Among children of college-educated parents, vocational-to-academic movers underperform academic-track averages: none are in the top quartile of the grade-9 skill distribution, compared to 33% of those who followed the academic-track recommendation. Conversely, among children of non-college-educated parents, academic-to-vocational movers outperform vocational-track averages: 42% are in the top quartile of the grade-9 skill distribution, compared to 24% of those who followed the vocational-track recommendation. This suggests that most of these deviations do not reflect parental private information about the child’s true potential.

Figure B.2: A Regression Discontinuity Design at the Maturity Cutoff



Notes: This figure illustrates the model's regression discontinuity design results at the synthetic July 1 maturity cutoff, for academic-track attendance and labor income at age 30. $N = 225,000$.

Figure B.3: Track Recommendations vs. Actual Choices



Notes: This figure displays matrices summarizing teachers' track recommendations against students' actual track choices (Vocational vs. Academic). Rows of panels correspond to: Panel A, row-conditional shares; and Panel B, the percentage of students who ranked in the top quartile of the grade 9 skill distribution for their eventually chosen track. Columns correspond to subgroups: Children of College-Educated Parents, and Children of Non-College-Educated Parents. Within each matrix, rows indicate the teacher's recommended track and columns indicate the actual choice. Each cell reports the percentage in bold. Data source: NEPS Cohort 3. All observations are weighted using NEPS weights and our own track-share weights (see Appendix B.3).

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