ROCKWOOL Foundation Berlin
Institute for the Economy and the Future of Work (RFBerlin)

DISCUSSION PAPER SERIES 306/26

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www.rfberlin.com NOVEMBER 2026

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Reference

JEL Codes: J24, J31, O31

Keywords: Labor Market for Inventors, Firms, Wage Differentials, Early Career, Employer Learning, Incentive Contracts

Recommended Citation: Sabrina Di Addario, Alice H. Wu (2026): Firm Heterogeneity in the Identification and Reward of Inventors. RFBerlin Discussion Paper No. 306/26

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Firm Heterogeneity in the Identification and Reward of Inventors*

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November 12, 2026

Abstract

This paper documents firm heterogeneity in identifying and rewarding inventor talent, using Italian Social Security employment records linked to patent applications. Young new inventors are 20% less likely to file their first patent application at firms in the bottom quartile of the firm wage distribution than similar workers at top-quartile firms. Experienced inventors, however, patent at similar rates across low- and high-wage firms, suggesting that low-wage firms do patent but concentrate their innovation activity among workers of proven ability. Wages rise significantly around a young worker's first patent application, and the increase is largest at low-wage firms. We build a model of employer learning and incentive contracts to explain these findings: low-wage firms invest less in discovering talent but, when firm investment and worker effort are weak complements, set a higher bonus for new inventors.

Keywords: Labor Market for Inventors; Firms; Wage Differentials; Early Career; Employer Learning; Incentive Contracts.

*We thank Federico Barbiellini Amidei, David Card, Federico Cingano, Santiago Pereda Fernandez, Claudia Goldin, Benny Goldman, Larry Katz, Patrick Kline, Fabian Lange, Ethan Lewis, Chris Taber, Roberto Torrini, Na'ama Shenhav, and the seminar audience at the EALE, RFBerlin, IAAE, BLOW, Harvard, Bank of Italy, and the Federal Reserve Bank of San Francisco for helpful comments and discussions. The views expressed by the authors do not represent the views of the Bank of Italy or the Italian Social Security Institute.

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1 Introduction

Expanding the stock of inventor talent is crucial for raising the rate of innovation, a central driver of economic growth (Romer (1990); Aghion and Howitt (1992)). The availability of matched employer-employee data linked to patent records has recently allowed researchers to study the supply of and demand for inventors on a larger scale. A growing literature has used such data to analyze the demographic, socioeconomic, and financial factors behind who becomes an inventor (Bell et al. (2019a); Bell et al. (2019b)), and to characterize inventors' employers (Akcigit and Goldschlag (2025)) and earnings (Aghion et al. (2018); Kline et al. (2019); Toivanen and Väänänen (2012)).

Less attention has been paid to the role of firms in identifying inventors from the pool of their employees, and to firm heterogeneity in the wage returns to patenting. A better understanding of the firms that employ inventors could support the design of more targeted innovation policies aimed at improving productivity (Akcigit and Goldschlag 2025). This paper provides novel evidence on firm heterogeneity in discovering and rewarding inventors in Italy. Italy is the sixth largest economy among OECD members,[1] but it has experienced a prolonged productivity slowdown, during which young workers have faced declining economic opportunities relative to older cohorts (Bianchi et al. 2023; Goldin et al. 2024; Daruich et al. 2023). This backdrop lends urgency to the study of frictions in how the labor market discovers and promotes talent, especially of workers with the potential to become inventors and contribute to innovation.

We document large differences between lower-wage and higher-wage firms in (1) the discovery of new inventors (i.e., the employees who apply for a patent for the first time), and (2) the wage returns to a worker's first patent application. These gaps are smaller for older workers and disappear for experienced inventors. We interpret the findings with a model of employer learning and incentive contracts that endogenizes firms' investment decisions and workers' effort choices. The model helps explain why lower-wage firms set a higher bonus for new inventors than higher-wage firms do, even in the absence of increased poaching after workers become inventors.

[1] Italy's nominal GDP was 2.37 trillion US dollars in 2024, the sixth largest among OECD countries, according to World Bank data.

Using matched employer-employee data from the Italian Social Security Institute linked to European patent applications by Depalo and Di Addario (2014), we follow the careers of about 15,000 matched inventors and over 1 million non-inventors who enter the labor market at the same firms. Including non-inventors enables us to study who becomes an inventor across firms without conditioning on the outcome of eventually patenting. Because non-inventors initially earn lower wages and are more likely to hold blue-collar jobs and temporary contracts, we reweight them by inverse propensity scores so that the reweighted sample resembles matched inventors at labor market entry.

We classify firms into quartiles of their mean worker wage in the previous year and estimate differences across quartiles in the probability that a worker files her first patent application at her current employer. We find that workers under 35 who have not yet patented are 20% less likely to become inventors at lower-wage firms (quartile 1) than observationally similar workers at higher-wage firms (quartile 4). For older workers the gap between quartiles 1 and 4 shrinks to 11%, suggesting that higher-wage firms are better at identifying new inventors, especially among the young employees. The gap in patent application rates between lower- and higher-wage firms, however, applies only to workers who have not yet patented: once workers have filed an application ("experienced inventors"), the gap becomes insignificant for younger workers and even flips sign for older ones. This contrast suggests that lower-wage firms do engage in patenting, but tend to offer these opportunities to experienced rather than potential inventors.

These patterns are consistent with uncertainty about the ability of workers, especially young ones, which is the starting point of employer learning models (e.g., Altonji and Pierret 2001; Farber and Gibbons 1996). Firms may be reluctant to assign innovation tasks to employees whose ability is not yet known, and more willing to offer the opportunity to experienced inventors. Lower-wage firms thus appear to support experienced inventors as much as their higher-wage counterparts, while overlooking younger workers with the potential to invent.

We also investigate whether firms reward workers for patent applications and how inventors' wage growth varies across firms. We estimate the wage changes around a worker's first patent application by firm quartile, conditioning on person, industry, loca-

tion, and year fixed effects as well as time-varying worker and job characteristics. Younger workers receive a 3–5 log-point wage increase when they file their first application at quartile 4 firms. This is consistent with evidence that wages rise *before* patents are granted (Depalo and Di Addario 2014; Bell et al. 2019a), though existing studies of inventors' wage growth have rarely examined differences across firms. We find that the increase around a younger worker's first application is 6–7 log points higher at lower-wage firms (quartile 1) than at higher-wage firms (quartile 4). Older workers also receive an increase in the year of their first application, although the gap between quartiles 1 and 4 shrinks to 3 log points. In contrast, we do not find a significant wage increase around subsequent applications for experienced inventors. This pattern is also consistent with employer learning: if wages move with employer beliefs about worker ability, a worker's first-ever patent application delivers a larger information shock than later applications, and the wage response is larger for new inventors than for inventors with an established record.

Why is the wage return to a worker's first patent application higher at lower-wage firms in quartile 1? One hypothesis is that the labor market holds a lower prior on the ability of workers at lower-wage firms. Before patenting, there is an 11–14 log-point wage gap between quartiles 1 and 4 for young workers, conditional on unobserved individual heterogeneity and time-varying worker and job characteristics. The first patent application triggers a larger belief update when the prior is low, consistent with a larger wage increase that closes about half of this gap. A second hypothesis is that lower-wage firms face higher turnover than higher-wage companies and must reward new inventors more to retain them after patenting. We find some evidence for this: before becoming inventors, young workers in quartile 1 are more than twice as likely as their counterparts at higher-wage firms to move to a new firm within a year. Filing a first patent application is negatively correlated with changing firm, and in quartile 1 it offsets the higher turnover completely. The higher wage increase at the first application may help lower-wage firms retain new inventors, but the regression estimates cannot tell us whether the return is set for that purpose. It also remains puzzling that, even three years after the first patent application, there is little evidence that new inventors leave their firms or move up the job ladder, as it would be expected if the labor market revised its belief about their ability upward.

Without poaching, employer learning theory alone does not explain why lower-wage firms set a higher bonus for a first patent application.

To reconcile these findings, we build a two-period model of the labor market for potential inventors: firms post contracts that comprise a base wage, a bonus for a patent application, and an investment in the worker's innovation activity, and workers choose employers and innovation effort accordingly. Information on worker ability is imperfect but symmetric across firms and workers, and a patent application fully reveals the worker as a high-ability inventor. In equilibrium, more productive firms set a higher base wage, which justifies ranking firms by their mean wages in the empirical analysis. Effort on innovation is not contractible. Workers choose effort to maximize their expected utility from wages net of the cost of effort today, plus their option value on the labor market next period, which rises if they are recognized as an inventor. When new inventors have fewer opportunities to move between firms, the increase in option value is smaller, and firms rely more on a bonus contingent on invention to elicit effort (Harris and Raviv 1979; Holmström 1979) .

The model predicts that more productive firms invest more in a worker's innovation activity when firm investment and worker effort are complementary in the production of patents and the production function is concave in effort.[2] This prediction is consistent with the finding that workers are more likely to become inventors at higher-wage firms. A higher investment has two opposing effects on the bonus. First, it raises the probability that a worker patents, and when this probability is low, raising the bonus becomes a costlier way to elicit effort. Second, because investment and effort are complementary, a higher investment makes the bonus more effective at eliciting effort. When the complementarity is weak, the higher cost of the bonus dominates, and more productive firms set a lower bonus than their less productive counterparts. Through the lens of incentive contracts, we account for the higher wage return to patenting at lower-wage firms, a pattern that employer learning alone cannot explain given the absence of poaching by higher-wage

[2]We find empirical evidence for this prediction: among the 7% of firms in our data that we match to the annual Survey of Industrial and Service Firms (INVIND), higher-wage firms invest more in research-related activities. The R&D investment variable is missing for 90% of firm-year observations, but mean investment in several categories indicates that lower-wage firms invest less in innovation-related activities (Appendix Table A.2).

firms.

1.1 Contribution to the Literature

This paper contributes to several strands of literature. First, we add to the recent literature on individual returns to patents. Linking inventors to administrative employer-employee data, Aghion et al. (2018) find that inventors in Finland receive, on average, a reward equivalent to 5 percent of their annual income after a patent application, while Toivanen and Väänänen (2012) show a 3 percent premium once the patent is granted. Similarly, Depalo and Di Addario (2014) estimate that inventors in Italy receive on average a 3.5 percent wage premium at the time of application. In the US, Kline et al. (2019) find that patent allowance decisions have a stronger effect on inventors' earnings than on those of non-inventors, while Akcigit, Grigsby, and Nicholas (2017) find that immigrant inventors earn 5 percent less on average than native-born inventors. In contrast to these papers, we estimate the returns to a first-ever patent application separately from the returns to subsequent applications, and show that the former are higher across firms and age groups. The first patent application is a particularly informative event: it reveals previously unobserved inventive ability to employers. Moreover, first-time inventors are younger on average, which makes them a particularly relevant group for studying the discovery of talent and the emergence of new ideas.

Second, our paper relates to the studies on the factors that determine who becomes an inventor. Bell et al. (2019b) merge USPTO records with US individual tax data to study the life cycle of inventors and the returns to invention. In line with Aghion, Akcigit, Hyytinen, and Toivanen (2023), they find that patenting is strongly correlated with parental income and education. Bell et al. (2019b,a) point to the importance of early exposure to innovation, the lack of which contributes to the under-representation of women, minorities, and individuals from low-income families among inventors. Our analysis of the heterogeneity in the discovery of new inventors between firms suggests that exposure to firms that engage younger workers in patenting is another channel through which the labor market recognizes potential inventors. At lower-wage firms, experienced inventors patent at similar rates as at higher-wage firms, but younger workers are less likely to become an

inventor.

Third, our emphasis on firm heterogeneity relates to the literature on the characteristics of firms that employ inventors. Akcigit and Goldschlag (2025) document substantial heterogeneity in inventor productivity across firms: although inventors have become increasingly concentrated in large, established firms, those at younger firms tend to generate innovations with greater impact. We characterize firms by their mean wages, which are positively related to productivity and to investment in innovation-related activities. Workers at lower-wage firms are less likely to become inventors, but the wage return to a first patent application is higher there. Importantly, the contrast between lower-wage and higher-wage firms in the discovery and reward of inventors applies to first-time inventors alone. Experienced inventors are as likely to patent again at lower-wage firms as at higher-wage firms. Our model of employer learning and incentive contracts helps explain these patterns.

Finally, by comparing the career progression of younger and older workers, we contribute to the growing literature on the career challenges faced by younger workers in Italy. A series of reforms aimed at increasing labor market flexibility led to a high share of temporary contracts and to wage depression that hurt younger workers disproportionately (Daruich et al. 2023). An aging population and delayed retirement further reduce the opportunities available to younger workers (Bianchi and Paradisi 2023; Bianchi et al. 2023). We show that there is a substantial gap between lower-wage and higher-wage firms in discovering inventors among younger workers. Slow discovery of younger inventors can be particularly costly, given that inventors face human capital depreciation and that innovation productivity often peaks in a worker's early 40s (Bell et al. 2019a; Kaltenberg et al. 2023).

2 Data

We build a panel data on the employment and innovation history of more than 100,000 potential inventors in Italy, using the database in Depalo and Di Addario (2014) that matched the employer-employee data from the Italian Social Security Institute (INPS)

with the Worldwide Patent Statistical Database (PATSTAT). This database contains the employment and patent records for matched Italian inventors and the employment history of their coworkers across employers from 1987 to 2009.

In this section, we first summarize the matching between PATSTAT and INPS originally done by Depalo and Di Addario (2014), and then describe our sample of potential inventors: matched inventors and their coworkers, whom we reweight by the estimated propensity of ever inventing.

2.1 INPS-PATSTAT Matched Database

PATSTAT contains the universe of patent applications ever submitted to the European Patent Office (EPO). It provides detailed information both on the inventors listed in the applications, and on the assignees, or owners of the patent, which are typically the inventors' employers. Depalo and Di Addario (2014) selected all the patent applications submitted by any firm located in Italy, as recorded in PATSTAT 2009.[3]

After cleaning the names of inventors and firms applying for patents, Depalo and Di Addario (2014) asked INPS to match (by name and location) the assignees with the employers present in their administrative data, and to match the inventors with the employees (by name and municipality of residence). INPS returned a de-identified database of about 16,000 matched inventors (as well as 4.5 million coworkers employed in the same company) and all the firms in which these inventors had transited in our observational period, even before or after submitting a patent. We refer to Depalo and Di Addario (2014) for further details on the matching process.

The matched INPS-PATSTAT database contains workers' annual wages at their employers in the private sector, the type of contract (permanent or temporary), the occupation group (blue or white collar), and basic demographic information such as gender or year of birth. INPS also provided information at the establishment level, such as size, sector, location, and dates of business opening and closure. The matching between patent appli-

[3]This restriction excludes patent applications by individuals, universities, or public entities which could not be matched with INPS data, which only covers the private sector in Italy. State-owned enterprises, which undertook a large amount of R&D between 1950 and 1994 (Antonelli et al. 2014), are not included in our data.

cations and employment records also allows us to build a measure of worker's on-the-job innovation outputs, and identify the employer at which a worker files her first patent application ever, if it occurs during the sample period.

2.2 Matched Inventors and Non-Inventors

In this paper, workers are defined as "matched inventors" if they are listed as an inventor on a patent application filed by their employer at the time of the initial filing, where the patent assignees or applicants are matched to employers in INPS.[4] The majority of matched inventors are males, white-collar workers, and hold a permanent job (Table 1).

The matched INPS–PATSTAT data allow us to trace not only the job histories of matched inventors but also those of the workers who are, at some point, employed at the same firm. Observing these coworkers enables us to study questions such as who becomes an inventor and where, without conditioning on future patenting activity, as papers on inventors typically do (e.g., Bell et al. 2019a). The challenge, however, is that the vast majority of non-inventors do not work in research-related jobs that could lead to a patent. For example, an inventor at a large automobile company in Italy has thousands of coworkers on the factory floor rather than in the R&D department.

We therefore focus on workers who (i) spent at least half of their working years in the sample in white-collar positions, (ii) entered the INPS data between ages 14 and 55, and (iii) have at least five years of employment records in INPS. The first restriction excludes the vast majority of blue-collar workers, who are rarely involved in patenting.[5] The second removes the workers who were close to retirement at the beginning of the sample year. The third ensures that we can follow each worker for a sufficiently long period to track changes in wages and job mobility.

Compared to matched inventors, non-inventors are more than three times as likely to be female, are 4 percentage points (p.p.) less likely to hold white-collar jobs, and are 9 p.p. less likely to hold permanent positions (Table 1). On average, they also earn monthly wages that are 39 log points lower. The wage gap between matched inventors and

[4]Note that the patent applications submitted by firms report the names of each inventor who contributed to the invention. We do not consider the patent applications by individuals.

[5]This restriction does not drop individuals who work as blue-collar workers in some years.

non-inventors is already 18 log points at age 30 and widens as workers gain experience.

We construct a person × year panel for the analysis in Sections 3–4. When a worker has multiple employers in a year, we keep the employer that pays her the highest annual earnings and refer to it as her primary employer, denoted by $j(i,t)$ for worker i in year t.

2.2.1 Propensity Scores

Given these large differences in observables, we reweight non-inventors using inverse propensity score weights (DiNardo et al. 1996; Hirano et al. 2003; Rosenbaum and Rubin 1983) so that their characteristics at labor market entry resemble those of matched inventors. We model the propensity of ever inventing with a logistic cumulative distribution function, conditional on worker characteristics at labor market entry, including the first job observed in the data:

$$Pr\left(Inv_i | X_i, j(i,0)\right) = \frac{\exp\left(X_i'\lambda + \phi_{j(i,0)}\right)}{1 + \exp\left(X_i'\lambda + \phi_{j(i,0)}\right)} \tag{2.1}$$

where Inv_i indicates whether worker i is a "matched inventor" as defined above. The individual-level controls, X_i, include the age at which i enters the labor market, as a proxy for her education level. X_i also include gender, the occupation category (white versus blue collar) of the first job, and interactions between entry age and an indicator for employment records left-truncated at 1987.[6]

Importantly, we include the first employer fixed effects ($\phi_{j(i,0)}$), to absorb employer-level differences in the propensity to hire future inventors. These fixed effects account for sorting on unobservable worker characteristics to the extent that it operates through the choice of first employer: starting at a firm that hires many future inventors predicts more patenting opportunities than starting at one that hires very few. Estimating (2.1) requires each first employer to have at least one matched inventor and one non-inventor at entry.

Estimates of regression (2.1) are reported in Table A.1.[7] Being female reduces the

[6] For workers already employed before 1987, when the INPS records begin, we observe only the age at the first *observed* job, which overstates the true age at labor market entry. The interaction with the left-truncation indicator allows the relationship between entry age and inventing to differ for these workers.

[7] Column 1 of Table A.1 reports logit estimates of (2.1) with industry and province fixed effects; column 2 reports logit estimates with firm fixed effects, bias-corrected following Fernández-Val (2009) and implemented with the `bife` package in R (Stammann, Heiss, and McFadden, 2016).

Table 1: Sample Overview - Person Level

	Matched Inventors		Non-Inventors			
			Unweighted		Weighted	
	mean	sd	mean	sd	mean	sd
Demographics						
Female	0.091	0.288	0.305	0.460	0.096	0.294
Yr of Birth	1958	10.803	1958	12.025	1958	10.826
INPS Sample (left-censored at 1987)						
First Yr in INPS	1990	4.883	1990	5.134	1990	4.927
Present in INPS in 1987	0.577	0.494	0.582	0.493	0.578	0.494
Patent Applications						
Any Patent App 1987-2009	1.000	0.000	0.000	0.000	0.000	0.000
Any Patent App Per Year	0.169	0.162	0.000	0.000	0.000	0.000
Any Patent App by Age 30	0.433	0.495	0.000	0.000	0.000	0.000
... by Age 35	0.667	0.471	0.000	0.000	0.000	0.000
... by Age 40	0.760	0.427	0.000	0.000	0.000	0.000
... by Age 45	0.828	0.377	0.000	0.000	0.000	0.000
... by Age 50	0.889	0.315	0.000	0.000	0.000	0.000
Job Characteristics						
Num. Employers (Firms)	2.113	1.399	1.841	1.215	2.076	1.308
Blue-Collar	0.016	0.062	0.048	0.117	0.025	0.086
White-Collar	0.983	0.065	0.944	0.138	0.970	0.099
Permanent Contracts	0.527	0.256	0.440	0.318	0.471	0.296
Temporary Contracts	0.035	0.104	0.081	0.205	0.050	0.143
Seasonal Contracts	0.000	0.007	0.002	0.021	0.001	0.014
Contract Type Missing	0.447	0.276	0.487	0.340	0.487	0.315
Log Wage per Month						
Mean Log Wage	7.996	0.476	7.602	0.472	7.736	0.449
Log Wage at Age 30	7.588	0.292	7.411	0.416	7.493	0.338
... at Age 35	7.859	0.333	7.575	0.460	7.707	0.407
... at Age 40	8.074	0.424	7.691	0.462	7.840	0.464
... at Age 45	8.243	0.485	7.780	0.457	7.922	0.498
... at Age 50	8.385	0.543	7.849	0.462	7.977	0.510
Observations	14,950		1,032,747		1,032,747	

Notes: This table reports person-level summary statistics for the workers who (i) spent at least half of their sample's working years in white-collar positions, (ii) entered the INPS data between ages 14 and 55, (iii) have at least five years of employment records in INPS, and (iv) have a non-missing propensity score, estimated from the bias-corrected firm fixed-effects logistic regression (2.1), which drops firms without at least one matched inventor and one non-inventor at entry. Columns 1–2 report unweighted means and standard deviations for matched inventors, and columns 3–4 the corresponding statistics for non-inventors. Columns 5–6 reweight non-inventors by $w_i = \hat{p}_i/(1-\hat{p}_i)$, where $\hat{p}_i$ is worker i's estimated propensity of ever inventing. Job characteristics, wages, and patenting rates are first averaged within worker across her person-year observations; the table reports means and standard deviations of these worker-level averages.

odds of ever inventing by 78%.Starting in a white-collar job triples the odds, relative to workers whose first job is labeled neither white nor blue collar, while starting in a blue-collar job reduces the odds by 64%. Contract type, by contrast, is a weak predictor: no contract type is associated with meaningfully higher odds of inventing, whether we compare workers across firms within the same industry and province or workers within the same firm. As a robustness check, we also estimate a Poisson regression of Inv_i on observables X_i and first-employer fixed effects; the predicted probabilities from the two models are highly correlated (Pearson correlation = 0.994).

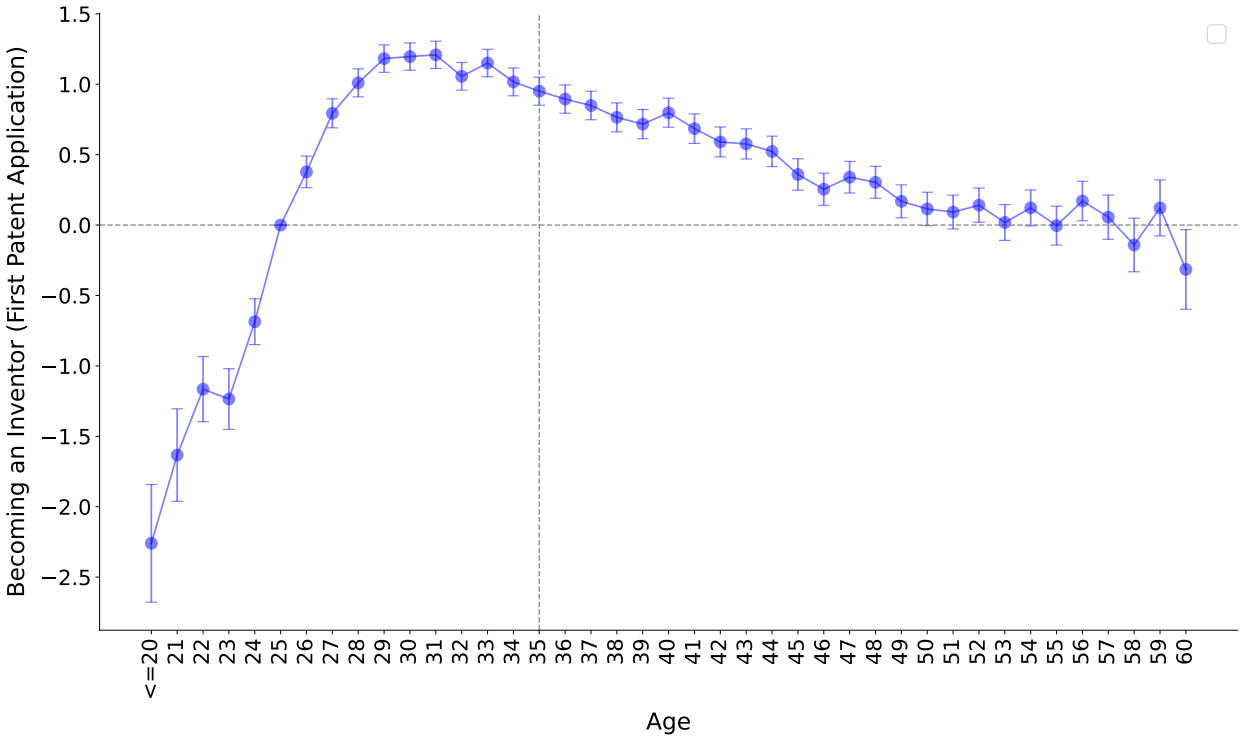
We use the estimated propensity score of each worker i, denoted $\hat{p}_i$, to reweight non-inventors: each non-inventor receives weight $w_i = \frac{\hat{p}_i}{1-\hat{p}_i}$, while matched inventors receive weight 1. Columns 5-6 of Table 1 show that reweighting brings the two groups much closer on average. The female share of non-inventors falls from 30.5% to 9.6%, close to the 9.1% among matched inventors, and 97% of the jobs held by reweighted non-inventors are white-collar. The gap in monthly wages shrinks from 39 to 26 log points, and the wage gap at age 30 falls from 18 to 9 log points.

2.2.2 The Age Profile of Becoming an Inventor

The widening of the wage gap between matched inventors and non-inventors with experience points to diverging career paths. Figure 1 shows the age profile of becoming an inventor, estimated from a logistic regression of filing the first patent application on age dummies, gender, an indicator for the 1987 entry cohort, and calendar year fixed effects. The odds that a worker files her first patent application rise steeply with age, peaking at age 31 at roughly 3.3 times the age-25 level, and then decline steadily, returning to approximately the age-25 level by the mid-fifties. Consistent with this profile, the reweighted wage gap in Table 1 increases from 9 to 15 log points between ages 30 and 35 and remains stable thereafter, suggesting that the earnings of matched inventors and non-inventors diverge around the time matched inventors start patenting. We explore this idea formally in the empirical analysis. Given the age profile in Figure 1, we estimate separate models throughout the paper for workers younger and older than 35. Analyzing younger and older workers separately is also consistent with the employer-learning literature (Altonji

and Pierret 2001; Lange 2007), which finds that employers learn about worker ability mostly within the first decade of a career. Because employers remain uncertain about the ability of young workers, the incentives for and returns to patenting may differ between the two groups.

Figure 1: Becoming an Inventor by Age



Notes: This figure plots the logit coefficients on age dummies from a regression of filing the first patent application, relative to age 25. The estimation sample is a person × year panel of matched inventors and non-inventors, restricted to person-years in which the worker is aged 18–60 and either has not yet filed a patent application or files her first application in that year. The regression controls for age dummies (ages 18–20 grouped together), gender, an indicator for the 1987 entry cohort, and calendar year fixed effects. The vertical dashed line marks age 35, the cutoff we use to separate younger and older workers.

2.3 Firm Heterogeneity

To analyze differences in becoming an inventor at lower-wage versus higher-wage firms, we sort firms into quartiles based on the mean wage of their employees in the previous year. The mean wage serves as a proxy for firm productivity, which we do

not observe directly.[8] This ranking is consistent with the micro-foundations of the AKM model, in which more productive firms set a higher wage premium in an imperfectly competitive labor market (Abowd et al. 1999; Card, Cardoso, Heining, and Kline 2018; Kline 2024).

Denote by $j(i,t)$ the primary employer of worker i in year t, and by $Q(i,t)$ the quartile of the lagged mean wage to which her primary employer belongs. We define the quartile cutoffs on the person-year panel, so each firm enters the ranking weighted by its number of employees. $Q(i,t)$ takes the same value for all employees of a given firm in a given year. Using lagged wages addresses concerns about contemporaneous shocks that affect individual employee wages and the firm ranking simultaneously. It would not, however, address serially correlated productivity shocks.

Table 2 compares firm and worker characteristics across quartiles. Firms in quartile 1 have 22 employees on average, fewer than firms in higher quartiles. Workers in quartile 1 earn about 2,800 euros less per month, in real 1995 euros, than workers in quartile 4. Workers in quartile 1 file fewer patent applications than those in quartiles 2–4, and a smaller share of them apply for any patent.Firms in quartile 1 are also less likely to be located in northern Italy and less likely to operate in pharmaceuticals or electronics and telecommunications.

Based on firm names and tax identifiers, we matched a small subset (about 7%) of the firms in our employer-employee sample to the Bank of Italy's INVIND survey. Firms that can be matched are larger, pay higher wages, and have more patents per employee (Appendix Table A.2). Importantly, firms in quartile 1 have lower revenue and invest less, on average, in R&D and in material and immaterial capital than higher-wage firms. This comparison indicates that firms in quartile 1 are less productive, supporting the use of mean wages as a proxy for firm productivity.

[8]We find supportive evidence that the mean wage is positively correlated with a firm's revenue and investment, based on firms that can be matched to Bank of Italy's INVIND survey (Appendix Table A.2).

Table 2: Firm Characteristics by Quartile of Mean Worker Wages

	Quartile 1		Quartile 2		Quartile 3		Quartile 4	
	mean	sd	mean	sd	mean	sd	mean	sd
Num. Unique Firms	38,152		16,673		12,855		13,781	
Num. Employees	21.960	329.688	84.942	1380.155	128.763	1272.864	77.680	1034.648
Num. Matched Inventors	0.176	1.033	1.115	15.105	2.138	19.754	1.059	10.124
Log Wage	7.154	0.626	7.825	0.353	7.930	0.342	8.206	0.434
Monthly Wage (real 1995)	1,571	1,010	2,863	1,368	3,192	1,498	4,391	3,249
Patent Applications (per worker-year)								
Patent Apps per worker	0.010	0.124	0.033	0.195	0.036	0.194	0.017	0.151
Share of Workers Applying	0.007	0.068	0.021	0.108	0.022	0.103	0.010	0.071
Turnover and Contract Types:								
One-year Turnover	0.084	0.249	0.073	0.216	0.075	0.213	0.082	0.230
Temporary Contracts	0.242	0.406	0.063	0.200	0.046	0.170	0.044	0.176
Firm Location and Industry								
Northern Italy	0.772	0.420	0.842	0.365	0.842	0.364	0.809	0.393
Southern Italy	0.070	0.255	0.044	0.204	0.041	0.198	0.049	0.216
Electronics/Telecom	0.061	0.239	0.103	0.304	0.092	0.290	0.070	0.255
Pharmaceutical	0.016	0.127	0.028	0.165	0.038	0.191	0.055	0.227
Automobile	0.019	0.137	0.032	0.176	0.035	0.183	0.029	0.167
Manufacturing	0.341	0.474	0.493	0.500	0.503	0.500	0.451	0.498
Others	0.563	0.496	0.344	0.475	0.332	0.471	0.396	0.489

Notes: This table summarizes firm characteristics by quartile of the lagged mean wage. Each year, we rank firms by the mean wage of their employees in the previous year and place them into quartiles, with quartile 1 the lowest. Quartile cutoffs are weighted by the number of employees. The table reports means and standard deviations across the unique firms that have ever appeared in each quartile. Firm size and the number of firms are unweighted. Worker characteristics are averaged within each firm over potential inventors: matched inventors receive weight 1, and non-inventors are reweighted by the estimated odds of ever inventing, $w_i = \hat{p}_i/(1-\hat{p}_i)$, where $\hat{p}_i$ is the propensity score from equation (2.1).

3 Patenting Activity at Low-Wage versus High-Wage Firms

We analyze the rates at which workers apply for patents at lower-wage versus higher-wage firms. We split inventors by whether they have applied for patents before and document a significant gap in identifying new inventors between firms in the bottom and top quartiles of mean wages.

The first patent application a worker files at her current employer is an important indicator of innovation activity for two reasons. First, for a worker without prior patenting activity, this application is her first patent submission ever and establishes her status as an inventor. Second, for an experienced inventor who has already applied for patents at a former employer, an application at a new employer marks a continuation of innovation activity, and signals that the new employer allocates R&D tasks to experienced inventors.

We estimate the probability that a worker files her first patent application at her current employer with the following logistic model:

$$\ln\left(\frac{\Pr(\text{Pat}_{it}=1|Q(i,t),X_{it})}{1-\Pr(\text{Pat}_{it}=1|Q(i,t),X_{it})}\right)=\beta_0+\sum_{q<4}\beta_q\times\underbrace{1[Q(i,t)=q]}_{\text{employer } j(i,t) \text{ in quartile } q}+X_{it}'\Gamma \tag{3.1}$$

where Pat_{it} is an indicator for worker i having filed a patent application at employer $j(i,t)$; that is, Pat_{it} is 0 before the worker applies for any patent as an employee of $j(i,t)$ (even if had previously applied from another firm) and becomes 1 thereafter. The coefficients of interest, $\{\beta_q\}_{q<4}$, measure the difference in the log odds of filing this first application when a worker is employed by a firm in quartile q, relative to observationally similar workers at quartile-4 firms, which pay the highest wages. The covariates X_{it} include gender, a cubic polynomial in age (normalized at age 35), and indicators for white/blue collar and permanent/temporary contracts. X_{it} also includes calendar year fixed effects, 2-digit industry fixed effects, and province fixed effects, absorbing common trends and time-invariant heterogeneity in patenting across industries and locations. The estimation sample is a person × year panel that keeps, for each spell of a worker at a primary employer, every year in which they appear in the INPS data, up to and including the year in which Pat_{it} turns from 0 to 1 at that employer.

We fit separate models for workers aged 35 and younger and for those who are

older; we select the age cutoff based on the age profile of becoming an inventor (Figure 1). Within each age group, we estimate the model on three samples: (1) new inventors, that is, matched inventors who have not applied for a patent yet but will do so before 2009; (2) experienced inventors, who have already applied for patents at former employers; and (3) potential inventors, who comprise group (1) together with non-inventors. For group (1), the year in which Pat_{it} turns from 0 to 1 is the year in which a worker becomes an inventor. For group (2), the first patent application at the current employer is not the first application ever. In group (3), non-inventors are reweighted by the estimated odds of ever inventing, $w_i = \hat{p}_i/(1-\hat{p}_i)$, obtained from the model (2.1) in Section 2.2.1, while matched inventors receive a weight of one.

3.1 New Inventors

Matched inventors younger than 35 are 2.8 p.p. ($t = -5.70$) less likely to file their first patent application when employed at a quartile 1 firm than observably similar workers at a quartile 4 firm. This average marginal effect is converted from the 0.34 gap in the log odds of becoming an inventor between quartile 1 and quartile 4, reported in column (1) of Table 3. Relative to the mean patenting rate in quartile 4, 2.8 p.p. amount to a gap of roughly 20%. Workers employed at quartile 2 firms are about 1 p.p. less likely to become inventors than those at quartile 4 firms, while workers at quartile 3 firms become inventors at rates similar to those in the top quartile.

Matched inventors older than 35 are 1.9 p.p. less likely to file a first patent application at quartile 1 firms than at quartile 4 firms, converted from a statistically significant gap of 0.16 in log odds (column (4) of Table 3). Relative to the mean patenting rate in quartile 4, this 11% gap is about half of the 20% gap among younger workers, suggesting that firms matter more for younger workers than for older ones. One plausible explanation is that matched inventors who have not applied for patents by age 35 are negatively selected, so differences in firms' investment in innovation make less difference to their chance of patenting.

Table 3: First Patent Application at Current Employer, by (Lagged) Firm Quartile and Worker Age Group

	First Patent Application at the Current Employer					
	Age ≤ 35			Age > 35		
	(1) New	(2) Experienced	(3) Potential	(4) New	(5) Experienced	(6) Potential
Quartile of Workers' Mean Wage						
quartile 1	-0.3386***	-0.2747	-0.2618***	-0.1674***	0.1618**	0.2018***
	(0.0598)	(0.2360)	(0.0522)	(0.0599)	(0.0824)	(0.0491)
quartile 2	-0.1100**	0.1141	0.0819*	-0.0304	0.3597***	0.1511***
	(0.0510)	(0.1876)	(0.0419)	(0.0538)	(0.0688)	(0.0424)
quartile 3	-0.0081	0.3108*	0.2133***	-0.0584	0.3147***	0.1712***
	(0.0454)	(0.1659)	(0.0385)	(0.0465)	(0.0614)	(0.0388)
quartile 4 (base)	0	0	0	0	0	0
Constant	-4.8029	-4.2433	-7.0347	-3.6615	-7.4508	-7.3794
	(0.4288)	(1.6249)	(0.3836)	(0.6034)	(1.2334)	(0.4342)
Mean | quartile 4	.1391	.0620	.0502	.1738	.0557	.0265
N	51K	5K	4.5M	34K	32K	8.9M
Pseudo R^2	.1439	.0863	.0798	.1079	.0775	.0348

Notes: This table reports the results of equation (3.1), estimating whether a worker files her first patent application at her current employer, on the quartile of that employer, ranked by mean wages in the previous year. The estimation sample is a person × year panel in which each worker is observed at her primary employer, retaining the years up to and including her first patent application at that employer. Columns (1) and (4) are restricted to matched inventors who have not yet applied for a patent but will do so before the end of the sample period. Columns (2) and (5) are restricted to experienced inventors, who have already applied for a patent at a former employer; their first application at the current employer is therefore not their first ever. Columns (3) and (6) add non-inventors to the sample of columns (1) and (4) after reweighting them by $w_i = \hat{p}_i/(1-\hat{p}_i)$, where $\hat{p}_i$ is the estimated propensity of ever inventing from logistic regression (2.1); matched inventors receive a weight of one. All regressions control for gender, a cubic polynomial in age (relative to age 35), indicators for white/blue collar and permanent/temporary contract, and include calendar year, 2-digit industry, and province fixed effects. Standard errors are robust and clustered at the person level. Significance: * 0.10, ** 0.05, *** 0.01.

3.2 Experienced Inventors

The second group comprises experienced inventors, who have already applied for a patent at a former employer. For younger experienced inventors, the estimated gap between the bottom and top quartiles is no longer statistically significant: column (2) shows an imprecisely estimated gap of 0.27 in the log odds, or a 1.4 p.p. lower probability of applying for a patent at the current employer in quartile 1. Older experienced inventors are in fact significantly more likely to continue patenting at bottom-quartile firms than at top-quartile firms (column (5)).

The contrast between new and experienced inventors suggests that lower-wage firms discover new inventors at lower rates than higher-wage firms, but that they do not necessarily provide fewer patenting opportunities to the employees who have already proven their ability. The mean patent application rate in each quartile, shown in Table 2, also indicates that lower-wage firms are almost as likely to patent (per worker-year) as higher-wage firms. According to the regression estimates in Table 3, however, these opportunities are more concentrated among older and experienced inventors than among the workers who have yet to show their innovation ability. Our model of talent discovery in Section 5 allows for this contrast, letting firms invest differently in workers known to be inventors and in workers whose ability is not yet known.

3.3 Potential Inventors without Patenting Experience

An issue with focusing on matched inventors alone is that we are conditioning on an outcome: all matched inventors have at least one patent application in the INPS sample. Adding non-inventor coworkers, who start at firms with matched inventors, allows us to estimate firm heterogeneity in identifying new inventors among workers who look similar at labor market entry. The samples in columns (3) and (6) consist of matched inventors who have not patented yet, together with non-inventors reweighted by the estimated odds of ever inventing, conditional on initial employer and worker characteristics (equation 2.1).

Among younger workers in this combined sample (column 3), there is a 0.26 gap in the log odds of becoming an inventor at quartile 1 firms relative to quartile 4 firms

(statistically significant at the 1 percent level). This translates into a 1 p.p. difference, or a 19% gap relative to the mean patenting rate in quartile 4, which is comparable to the gap estimated in column (1).[9] Workers in the middle quartiles are more likely to start patenting than those in quartile 4, in contrast to the estimates for matched inventors in column (1). These findings suggest that younger workers' chances of becoming inventors are compromised mostly by firms in the bottom quartile.

The estimates for older workers in column (6) also provide a different picture from those for matched inventors in column (4): workers at quartile 1–3 firms are more likely to start patenting than those at quartile 4 firms. Among older workers, the advantage of higher-wage firms that we find for younger workers therefore disappears and even reverses. Employers may have more complete knowledge of a worker's ability after observing her for a decade or so (Altonji and Pierret 2001; Lange 2007), and older workers who have not yet patented could be negatively selected at all firms.

Taken together, the estimates in Table 3 point to a gap in the discovery of new inventors rather than in overall patenting. Younger workers are about 20% less likely to file a first patent application at a quartile 1 firm than similar workers at a quartile 4 firm, and the gap is roughly half as large among workers older than 35. Among experienced inventors, by contrast, there is no longer a disadvantage in patenting when they are employed at a bottom-quartile firm. Lower-wage firms thus offer patenting opportunities but concentrate them on workers who have already showcased their ability to invent.

4 Wage Returns to Patent Applications

Are workers rewarded for filing new patent applications, and does the reward vary across firms? To answer these questions, we estimate the wage return to a worker's first

[9]Note that the mean patenting rate in quartile 4 falls from 0.1391 to 0.0502 once non-inventors are included. Dividing the average marginal effect of working in quartile 1 (−0.0096) by this mean yields the 19% difference.

patent application at an employer in each firm quartile with the following OLS regression:

$$\ln(w_{it}) = \underline{\mu} + \underbrace{\sum_{q<4} \mu_q \times 1[Q(i,t) = q]}_{\text{avg wage diff. rel. to quartile 4}} + \underline{\gamma} \times \text{Pat}_{it} + \underbrace{\sum_{q<4} \gamma_q \times \text{Pat}_{it} \times 1[Q(i,t) = q]}_{\text{excess returns to new patent rel. to quartile 4}} \quad (4.1)$$

$$+ \underbrace{X'_{it}\,\Lambda}_{\text{controls}} + \underbrace{\alpha_i}_{\text{person FE}} + e_{it}$$

in which $\ln(w_{it})$ is the log monthly wage of worker i in year t.[10] As before, the estimation sample is a person × year panel that keeps, for each spell of a worker at a primary employer, every year in which they appear in the INPS data, up to and including the year in which Pat_{it} turns from 0 to 1 at that employer.[11] The coefficient $\underline{\gamma}$ represents the average wage return to the first patent application at the current employer among the workers in the base group (quartile 4). The coefficients $\{\gamma_q : q < 4\}$ represent the excess returns to the first patent application at employers in the other quartiles, relative to the base. The regression includes the same controls X_{it} as in (3.1), which include year, 2-digit industry, and province fixed effects. We also add person fixed effects, α_i, to absorb unobserved worker heterogeneity that matters for wages. The excess returns $\{\gamma_q\}$ can therefore be interpreted as within-person wage changes when a worker files her first patent application at quartile q, relative to the base quartile 4.

4.1 Wage Changes Around First Patent Applications

We estimate (4.1) separately by age group and, within each age group, on three samples: (1) new inventors, who have not applied for a patent yet; (2) experienced inventors, who have already applied at former employers; and (3) group (1) together with reweighted non-inventors.

Firms are ranked by mean worker wages in the previous year, so the quartile assignment on the right-hand side of (4.1) is not mechanically determined by contemporaneous

[10] The monthly wage is computed by dividing workers annual earnings at their primary employer (the employer at which they have the highest earnings) by the number of months they worked there. Using monthly wages avoids a mechanical dip in earnings among the workers who move across firms and do not work for a full year at their primary employer.

[11] An experienced inventor who has already applied for a patent at a former employer re-enters the sample at the new employer.

firm-specific wage shocks. Across all columns, workers at lower-quartile firms earn lower wages. Younger matched inventors who file their first patent application at a quartile 4 firm receive a 2.9 log-point increase in their monthly wage on average (column 1 of Table 4). At firms in the bottom quartile, this return is 6.0 log points higher, closing 55% of the mean wage gap between quartile 1 and quartile 4. The workers at middle-quartile firms do not obtain a significantly higher return to the first patent application than those in quartile 4. Together with the 20% lower probability of becoming an inventor at quartile 1 firms (Table 3), these results suggest that firms in the bottom quartile offer younger workers fewer opportunities to invent, but that those who do become inventors receive significantly higher wage returns than their counterparts at quartile 4 firms.

The wage return to the first patent application is consistent with employer learning: the application sends a positive signal about a worker's ability to invent. As employers revise their beliefs about that ability upward, wages would rise as long as the labor supply facing the firm is not perfectly inelastic.[12]

Older matched inventors who have not applied for patents before do not receive a significant wage increase when they start patenting at quartile 4 firms. At quartile 1 firms, they receive a 2.6 log-point wage increase upon their first patent application, which closes about 28% of the mean wage gap between quartile 1 and quartile 4. The contrast between younger and older first-time inventors also matches existing evidence that employer learning matters more early in a worker's career (e.g., Farber and Gibbons 1996; Altonji and Pierret 2001; Lange 2007; Kahn 2013; Schönberg 2007; Wu 2026). There is less uncertainty about a worker's ability once the labor market has observed her performance and career trajectory for a few years, so a patent application leads to a smaller belief update and a smaller wage adjustment.

In comparison with new inventors, experienced inventors have already demonstrated their ability to invent in earlier patent applications, so there is less room for an upward belief update. Columns 2 and 5 of Table 4 show that experienced inventors receive no significant wage increase, in any quartile, when they file their first patent application at

[12]In a perfectly competitive labor market, wages would increase to fully match a worker's expected marginal revenue product (Altonji and Pierret 2001; Farber and Gibbons 1996).

Table 4: Wage Returns to First Patent Application, by (Lagged) Quartile of Firms and Age Group

	Log Monthly Wages					
	Age ≤ 35			Age > 35		
	(1) New	(2) Experienced	(3) Potential	(4) New	(5) Experienced	(6) Potential
Quartile of Workers' Mean Wage						
quartile 1	-0.1098***	-0.0776***	-0.1432***	-0.0924***	-0.1189***	-0.1148***
	(0.0078)	(0.0153)	(0.0037)	(0.0087)	(0.0116)	(0.0029)
quartile 2	-0.0361***	-0.0493***	-0.0710***	-0.0401***	-0.0476***	-0.0677***
	(0.0066)	(0.0139)	(0.0029)	(0.0065)	(0.0081)	(0.0019)
quartile 3	-0.0141**	-0.0219*	-0.0365***	-0.0190***	-0.0321***	-0.0363***
	(0.0056)	(0.0122)	(0.0024)	(0.0051)	(0.0064)	(0.0013)
Any New Patent Application						
Pat_{it}	0.0291***	0.0131	0.0456***	0.0043	0.0020	0.0371***
	(0.0070)	(0.0180)	(0.0066)	(0.0053)	(0.0095)	(0.0049)
Excess Returns (relative to quartile 4)						
Pat_{it} × quartile 1	0.0601***	-0.0122	0.0740***	0.0259***	0.0211	0.0311***
	(0.0125)	(0.0534)	(0.0125)	(0.0094)	(0.0175)	(0.0095)
Pat_{it} × quartile 2	-0.0019	-0.0033	0.0152*	0.0047	0.0007	0.0165**
	(0.0083)	(0.0344)	(0.0083)	(0.0072)	(0.0136)	(0.0074)
Pat_{it} × quartile 3	-0.0076	-0.0116	0.0042	-0.0022	0.0069	0.0041
	(0.0081)	(0.0246)	(0.0079)	(0.0062)	(0.0120)	(0.0063)
Mean |quartile 4	7.655	7.897	7.697	8.237	8.496	8.149
N	51K	5K	4.6M	34K	32K	8.9M
Adjusted R^2	.750	.813	.723	.905	.875	.861

Notes: This table reports estimates of equation (4.1) of log (monthly) wages on whether a worker applies for a patent and its interactions with the quartile of her current employer, ranked by mean wages in the previous year. The estimation sample is a person × year panel that retains, for each worker-employer spell, the years up to and including their first patent application at that employer. Columns (1) and (4) are restricted to matched inventors who have not yet applied for a patent but will do so before the end of the sample period. Columns (2) and (5) are restricted to experienced inventors, who have already applied for a patent at a former employer; their first application at the current employer is therefore not their first ever. Columns (3) and (6) add non-inventors to the sample of columns (1) and (4), reweighting them by $w_i = \hat{p}_i/(1-\hat{p}_i)$, where $\hat{p}_i$ is the estimated propensity of ever inventing from logistic regression (2.1); matched inventors receive a weight of one. All models control for person-fixed effects, in addition to the covariates listed under Table 3. Standard errors are robust and clustered at the person level. Significance: * 0.10, ** 0.05, *** 0.01.

their current employer.

The findings are robust when we include non-inventors, reweighted on initial job and worker characteristics so that they look observably similar to matched inventors and start their careers at the same set of firms (see Section 3.3). Column (3) shows that younger potential inventors receive a 4.6 log-point wage increase when they file their first-ever patent application at a quartile 4 firm, and a 12 log-point increase when they do so at a quartile 1 firm. The additional 7.4 log points at quartile 1 firms close about half of the mean wage gap between the bottom and top quartiles. Also the potential inventors at quartile 2 firms receive a marginally significant 1.5 log-point higher wage return than their counterparts in the top quartile. Among older workers, the return to a first patent application is smaller: 3.7 log points at quartile 4 firms and 6.8 log points at quartile 1 firms (column (6)). Under employer learning, these lower returns are consistent with smaller updates in employers' beliefs.

In summary, firms reward workers for their first patent applications, even though it takes a few more years to learn whether the patent will be granted. Among younger workers, in both the matched-inventor and the reweighted potential-inventor samples, this return is significantly higher at bottom-quartile firms, which pay the lowest wages on average, than at top-quartile firms.

4.2 Why is there a higher wage return to patenting at lower-wage firms?

Can employer learning explain why the wage return to a first patent application is highest at lower-wage firms? Two channels could produce this pattern. The first is the labor market's prior belief about the ability of workers employed at lower-wage firms, which determines how much a patent application shifts the posterior belief. The second is the turnover these firms face, which determines how much they must pay to retain a worker who has just patented.

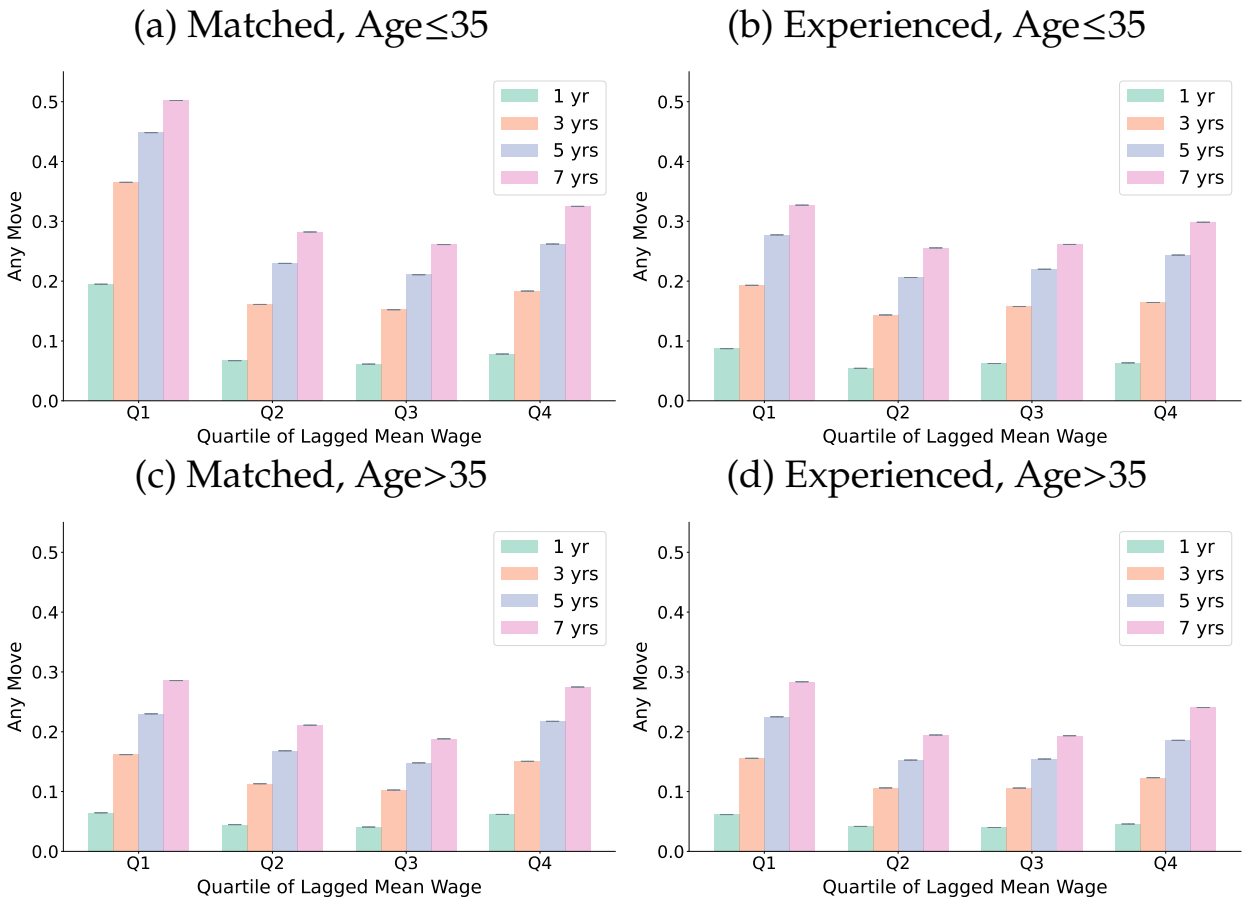
First, the labor market may hold a lower prior on the workers who are initially employed by lower-wage firms. On average, workers at quartile 1 firms earn about 2,800 euros less per month (in real 1995 euros) than workers in quartile 4 (Table 2). Conditional on unobserved individual heterogeneity and time-varying worker and job characteristics,

there remains a 14 log-point wage gap between the young potential inventors in quartile 1 and similar workers in quartile 4 (column 3 of Table 4). The average wage gap at least partly reflects differences in how the market perceives the ability of workers in lower-wage versus higher-wage firms. When the prior belief is lower for a potential inventor in quartile 1, a patent application, as a signal of research ability, generates a larger increase in the employer's posterior belief relative to the prior, and therefore a higher wage return in quartile 1 (Table 4). Consistent with employer learning, experienced inventors in quartile 1 have showcased their ability to invent; an additional patent application barely shifts the market's belief and translates into smaller excess returns that are insignificantly different from zero (columns 2 and 5 of Table 4).

Second, lower-wage firms may have to set a higher wage return to patent applications when they face higher turnover. We find some evidence that supports this hypothesis. About 20% of younger matched inventors in quartile 1 move to a new firm the following year, versus 7–8% in higher-wage quartiles (panel (a) of Figure 2). In seven years, roughly half of the younger matched inventors in quartile 1 have left their original employer, compared with about 30% in the other quartiles. The higher turnover in the bottom quartile also holds for younger potential inventors, who include non-inventors, whether or not they are reweighted (Appendix Figure A.1). By setting a higher wage return to patent applications, firms in quartile 1 may counter the risk of turnover among younger workers and increase the probability of retaining a new inventor. Importantly, the gaps in turnover across quartiles shrink substantially for older workers and experienced inventors, and the resulting weaker retention concerns contribute to more similar wage returns to patent applications across quartiles for these workers.

To understand the relationship between patenting and turnover, we estimate an OLS regression of job mobility between firms on whether a worker files a patent application in

Figure 2: Mobility by Firm Quartiles: Any Move between Firms



Notes: This figure shows the mean mobility of workers at each firm quartile, ranked by the mean wage of their employees in the previous year. Panels (a)-(b) focus on the fraction of workers younger than age 35 at t employed by different firms, $j(i,t+k) \neq j(i,t)$, in $k \in \{1,3,5,7\}$ years. Panels (c)-(d) shows the same for workers older than 35. Matched inventors are the workers who have not yet applied for a patent but will eventually do so. Experienced inventors are the workers who have already applied for a patent at former employers.

year t, interacted with the quartile of her current employer:

$$\text{Move}_{it} = m_0 + \underbrace{\sum_{q<4} m_q \times 1[Q(i,t) = q]}_{\text{average mobility rel. to quartile 4}} + \underbrace{\eta_0 \times \text{Pat}_{it}}_{\Delta \text{ mobility in quartile 4}} + \sum_{q<4} \underbrace{\eta_q \times \text{Pat}_{it} \times 1[Q(i,t) = q]}_{\text{excess mobility response rel. to quartile 4}} + \underbrace{X_{it}\,\Psi}_{\text{controls}} + \varepsilon_{it} \tag{4.2}$$

where $\text{Move}_{it} := 1[j(i,t+1) \neq j(i,t)]$ indicates any move between firms within one year. X_{it} includes the same worker and job characteristics, industry, province, and calendar year fixed effects as in regression (3.1).

The estimated coefficients m_q on the quartile dummies show that, after adjusting for worker and job characteristics and, importantly, industry, province, and calendar year fixed effects, turnover remains 6.6 p.p. higher at quartile 1 firms than at quartile 4 firms among younger matched inventors who have not yet patented (column 1 of Table 5).

Workers who start patenting stay at their current employers longer. In quartile 4, the workers who have just applied for a patent are 4–7 p.p. less likely to move to a different firm the next year (estimated coefficients on Pat_{it}). For younger matched or potential inventors, there is a further 7–8 p.p. decline in turnover after filing their first patent application in quartile 1, which offsets their higher turnover before patenting. It is plausible that younger workers who have not yet patented search harder for firms that provide more patenting opportunities (e.g., Bagger and Lentz 2019). The higher wage return to the first patent application filed in quartile 1 reduces turnover among new inventors, resulting in similar retention across quartiles after patenting.

We do not, however, observe workers' search activity or the arrival of outside offers upon patenting. It is therefore difficult to identify the exact mechanisms that bottom-quartile firms use to retain new inventors. It remains a puzzle why, three years later, new inventors at quartile 1 firms exhibit turnover similar to that of new inventors at quartile 4 firms, by which time the work related to the initial patent application should be complete (Appendix Table A.3). Furthermore, in search models with a single job ladder, such as Postel-Vinay and Robin (2002), low-productivity firms cannot outbid their more productive

Table 5: Between-firm Job Mobility in a Year, by Firm Quartile and Worker Age Group

	Move in 1 Year: $j(i,t+1) \neq j(i,t)$					
	Age ≤ 35			Age > 35		
	(1) New	(2) Experienced	(3) Potential	(4) New	(5) Experienced	(6) Potential
Quartile of Lagged Mean Wage						
quartile 1	0.0663***	0.0133	0.0590***	-0.0135**	-0.0064	0.0040**
	(0.0062)	(0.0158)	(0.0027)	(0.0059)	(0.0065)	(0.0017)
quartile 2	-0.0024	-0.0088	-0.0003	-0.0197***	-0.0044	-0.0080***
	(0.0052)	(0.0144)	(0.0022)	(0.0052)	(0.0056)	(0.0012)
quartile 3	-0.0154***	-0.0081	-0.0083***	-0.0218***	-0.0153***	-0.0097***
	(0.0046)	(0.0130)	(0.0020)	(0.0045)	(0.0048)	(0.0010)
quartile 4	0	0	0	0	0	0
	(.)	(.)	(.)	(.)	(.)	(.)
First Patent Application at $j(i,t)$						
Pat_{it}	-0.0429***	-0.0409	-0.0500***	-0.0487***	-0.0662***	-0.0395***
	(0.0061)	(0.0262)	(0.0051)	(0.0049)	(0.0062)	(0.0035)
Additional effect (relative to quartile 4)						
$\text{Pat}_{it}\times$ quartile 1	-0.0804***	-0.0283	-0.0673***	-0.0053	-0.0148	-0.0196***
	(0.0103)	(0.0466)	(0.0090)	(0.0076)	(0.0101)	(0.0058)
$\text{Pat}_{it}\times$ quartile 2	0.0190**	-0.0086	0.0159**	0.0174***	0.0115	0.0063
	(0.0077)	(0.0369)	(0.0067)	(0.0065)	(0.0111)	(0.0049)
$\text{Pat}_{it}\times$ quartile 3	0.0351***	-0.0253	0.0267***	0.0328***	0.0117	0.0195***
	(0.0077)	(0.0327)	(0.0068)	(0.0063)	(0.0081)	(0.0050)
Mean |quartile 4	.077614	.1057572	.0693738	.0616618	.086912	.0319875
N	52K	5K	4.5M	34K	29K	8.2M

Notes: This table shows the estimated OLS regression (4.2) of any movement between firms in a year ($j(i,t+1) \neq j(i,t)$) on whether a worker applies for a new patent and its interactions with the quartile of her current employer, ranked by the mean wage of its employees in the previous year. The estimation sample is at the person × year level. Columns (1) and (4) are estimated on matched inventors who have not filed for a patent yet. (2) and (5) are restricted to experienced inventors who have applied for patents at former employers. (3) and (6) comprise matched inventors who have not patented and reweighted non-inventors. Standard errors are robust and clustered at the person level. Significance: * 0.10 ** 0.05 *** 0.010.

counterparts, implying stronger upward mobility among workers who generate more positive signals about their ability. We do not find evidence of this prediction: new inventors at quartile 1 firms are no more likely to move up the job ladder after either one or three years (Appendix Tables A.4 and A.5).

In summary, we find that younger potential inventors are 20% less likely to file their first patent application when employed at a quartile 1 firm than observationally similar workers at quartile 4 firms. The gap in patent applications between low-wage and high-wage firms disappears, however, among experienced inventors who have already applied for patents at former employers. Further, there is a significant 3–12 log-point increase in a person's monthly wage when they file her first patent application, and the average wage return is highest at firms in quartile 1.

From the perspective of employer learning, at least part of the wage increase upon workers' first patent applications reflects an upward revision in employers' beliefs about their ability. Given the lack of evidence on the poaching of new inventors, however, positive belief updating alone cannot fully explain why firms in quartile 1 set a higher return to first patent applications. In the next section we formally consider incentive contracts as a complementary explanation, and specify the conditions under which combining them with employer learning can resolve this puzzle.

5 A Model of Talent Discovery

We develop a two-period dynamic model that incorporates employer learning, incentive contracts, and the sorting of forward-looking workers across firms. We lay out the conceptual framework below and solve the model in Appendix B. The equilibrium results help explain why it is harder to become an inventor at low-wage firms, and why low-wage firms reward new inventors more than high-wage firms do.

5.1 Conceptual Framework

The model concerns how firms invest in the innovation activity of workers and design wage contracts under uncertainty about workers' binary ability to invent.

Firms collect revenue from routine activity and from the innovation output of workers. Low-ability workers are as productive as high-ability workers in routine activity, but they cannot invent. In contrast, a high-ability worker can produce a patent application with probability $h(\tau, e)$, which increases in both the investment τ made by her employer and her own effort e (i.e., $h_1 > 0$, $h_2 > 0$).

Within each firm, it is challenging to reach the efficient level of innovation because (1) the firm is uncertain about worker ability until a successful invention, and (2) worker effort is not contractible, a common assumption in principal-agent problems (e.g., Harris and Raviv 1979; Holmström 1979). Given information about a worker at the beginning of each period, firms simultaneously post contracts that comprise a base wage $\underline{w}$, a bonus γ conditional on a successful invention, and an investment τ in the worker's innovation activity.

Information is symmetric across firms, and workers do not have private information about their own ability. At $t = 1$, there is a common prior about each worker's ability. If the worker invents during $t = 1$ (denoted by $p_{i1} = 1$), she is publicly known as high-ability at the beginning of $t = 2$. Otherwise, the market updates its belief given that she has not invented ($p_{i1} = 0$).

Workers who are on the job market observe the contracts posted by potential employers. They choose an employer and then choose their effort conditional on that employer. At $t = 1$, each worker chooses an employer based on the posted contracts, her idiosyncratic preferences over firms $\{\epsilon_{ij1}\}$, and her option value on the labor market at $t = 2$. Workers with a higher belief about their ability to invent have a stronger preference for firms that invest more in innovation (higher τ). Because patenting raises a worker's option value, forward-looking workers exert effort even without a bonus. Workers with a higher belief about their ability to invent are willing to forgo current compensation in return for a higher option value.

We specify the workers' and firms' problems below and solve the model by backward induction in Appendix B. In equilibrium, more productive firms set higher base wages and invest more in innovation. When the complementarity between investment and effort is weak, more productive firms set a lower bonus for an invention, because their higher

investment raises the probability of patenting from a low level, which makes the bonus a costlier way to elicit effort. We then map the model's implications to our empirical findings, in particular the puzzle that higher-wage firms set lower rewards for patenting.

5.2 Model Specification

The model environment features workers who differ in their ability to invent and firms that differ in productivity. For simplicity, information about worker ability is symmetric among all players in each period.

5.2.1 Environment

Workers. A worker's ability to invent, $\alpha_i \in \{H, L\}$, is initially unknown to workers and firms. Let $\pi_{i1} = Pr(\alpha_i = H)$ denote the common prior about worker i at the beginning of $t = 1$. If the worker produces a patent application during $t = 1$ ($p_{i1} = 1$), she is known as an H-ability inventor at the beginning of $t = 2$, so $\pi_{i2} = 1$. Otherwise, firms share the posterior belief $\pi_{i2} = Pr(\alpha_i = H|\pi_{i1},\ j(i,1),\ p_{i1} = 0)$, which conditions on her not producing a patent application during $t = 1$ while employed by $j(i,1)$. The posterior depends on $j(i,1)$ because the probability of patenting depends on the equilibrium investment and effort at that firm.

Firms. Firms, indexed by j, are endowed with a publicly known productivity ϕ_j. At the beginning of each period, they simultaneously post a contract for each worker based on public information about her. A contract comprises a base wage $\underline{w}_{ijt} \in \mathbb{R}^+$, a bonus $\gamma_{ijt} \geq 0$, paid as a proportional wage increase if worker i produces a patent application ($p_{it} = 1$), and an investment in innovation $\tau_{ijt} \geq 0$. Contracts vary across workers because the employer belief about their ability differs from one worker to another. At $t = 2$, contracts also depend on whether the worker is a stayer or a new hire.

Firms operate with constant returns to scale at the match level, so a firm's output is the sum of match outputs across its employees. The expected output from a worker of

ability α, conditional on firm investment τ and worker effort e, is:

$$Y_j(\alpha, \tau, e) = \underbrace{\phi_j}_{\text{productivity}} \cdot \left(\underbrace{1}_{\text{routine}} + \underbrace{\theta \cdot 1[\alpha = H] \cdot h(\tau, e)}_{\text{expected innovation}} \right) - \underbrace{\frac{\zeta}{2}\tau^2}_{\text{innovation cost}} \tag{5.1}$$

where $\theta > 0$ is the return to innovation relative to the firm's productivity ϕ_j. The cost of investing in innovation is convex, governed by the parameter $\zeta > 0$.[13]

Labor Market Dynamics. All workers are on the labor market at $t = 1$. Each worker observes the contracts $\{(\underline{w}_{ij1}, \gamma_{ij1}, \tau_{ij1})\}$ posted by firms and draws idiosyncratic preferences over firms from a type-I extreme value distribution:

$$F(\{\epsilon_{ijt}\}) = exp\left(- \sum_j exp(-\epsilon_{ijt}) \right) \tag{5.2}$$

Worker i chooses the effort e_{ij1} that maximizes her expected utility conditional on joining firm j. Let $j(i, 1)$ denote her employer at $t = 1$, the firm that offers the highest utility given her effort choice and idiosyncratic preferences.

Following a dynamic extension of Card et al. (2018), each worker re-enters the market at $t = 2$ with probability $\lambda \in (0, 1]$, in which case she redraws her preferences over potential employers from (5.2), independently of her preferences at $t = 1$. Workers who do not re-enter stay with their $t = 1$ employer. Because $\lambda > 0$, firms cannot pay stayers a zero wage at $t = 2$. When $\lambda < 1$, however, they can pay stayers less than new hires with the same belief.

We now state the problems of workers and firms in each period. The model is solved by backward induction in Appendix B2.

5.2.2 Workers' Problem

At the beginning of $t = 2$, given the contracts $\{(\underline{w}_{ij2}, \gamma_{ij2}, \tau_{ij2})\}$ posted by employers, a worker with belief π_{i2} who re-enters the job market chooses her employer $j(i, 2)$ as

[13] This cost may include investment in computing power, which often grows convexly as employees spend more time on innovation. It may also include the management costs of moving workers away from routine activities. For example, a firm may have to set up an in-house research lab, hire new managers, and introduce a new performance evaluation system for workers who are increasingly involved in innovation.

follows:

$$j(i,2) = argmax_j \ \{u_{ij2}(\pi_{i2}) + \epsilon_{ij2}\} \tag{5.3}$$

$$\text{where } u_{ij2}(\pi_{i2}) = \underbrace{E[b \cdot ln(w_{ij2})|\pi_{i2}, \tau_{ij2}, e_{ij2}]}_{\text{expected utility from wage}} - \underbrace{c(e_{ij2})}_{\text{effort cost}}$$

$$\text{wage } w_{ij2} = \underline{w}_{ij2} \cdot \left(1 + p_{i2} \cdot \gamma_{ij2}\right)$$

$$\text{effort } e_{ij2} = argmax_e \ \underbrace{\pi_{i2} \cdot h(\tau_{ij2}, e)}_{\text{prob. patent}} \cdot \underbrace{b\, ln(1+\gamma_{ij2})}_{\text{bonus}} - \underbrace{c(e)}_{\text{effort cost}}$$

Her optimal effort maximizes the expected bonus net of the effort cost, given the contract posted by potential employer j. Effort increases in the bonus γ_{ij2} when the probability of producing a patent application increases in effort ($h_2 > 0$) and the effort cost is convex ($c'' > 0$). Furthermore, effort increases in investment τ_{ij2} if and only if worker effort and firm investment are complementary in the production of patents ($h_{12} > 0$).[14]

At $t = 1$, every worker is on the market, observes the initial contracts $\{(\underline{w}_{ij1},\ \gamma_{ij1},\ \tau_{ij1})\}$, and draws preferences over firms from (5.2). Let $\beta \in (0, 1]$ denote the exponential discount factor shared by workers and firms. Workers take into account their option value at $t = 2$ when choosing an employer at $t = 1$. The discrete choice of a worker with prior π_{i1} is:

$$j(i,1) = argmax_j \ \underbrace{E[b \cdot ln(w_{ij1})|\ \pi_{i1},\ \tau_{ij1},\ e_{ij1}]}_{\text{expected utility from wage at t=1}} - c(e_{ij1})$$

$$+ \beta \cdot \underbrace{E[\Omega_j\ (\pi_{i2})\ |\ \pi_{i1},\ \tau_{ij1},\ e_{ij1}]}_{\text{option value at t=2}} + \epsilon_{ij1} \tag{5.4}$$

where her optimal effort at potential employer j solves:

$$e_{ij1} = argmax_e \ \underbrace{\pi_{i1} \cdot h(\tau_{ij1}, e)}_{\text{prob. patent}} \cdot \underbrace{\left(b\, ln(1+\gamma_{ij1}) + \beta\, \triangle_p \Omega_j\right)}_{\text{bonus + change in option value}} - \underbrace{c(e)}_{\text{effort cost}} \tag{5.5}$$

Her option value depends on the posterior belief at $t = 2$:

$$\Omega_j(\pi_{i2}) = \underbrace{(1-\lambda) \cdot u_{ij2}(\pi_{i2})}_{\text{not on market, stay at j}} + \underbrace{\lambda \cdot E[max_{j'}\{u_{ij'2}(\pi_{i2}) + \epsilon_{ij'2}\}]}_{\text{on market}} \tag{5.6}$$

[14] Given any belief $\pi_{i2} > 0$, $\frac{\partial e_{ij2}}{\partial \tau_{ij2}} \propto h_{12}$. See equation (B.3) in Appendix B2.

$\triangle_p \Omega_j$ denotes the difference in option value between having patented at $t = 1$ ($\pi_{i2} = 1$) and not having patented.[15] The higher $\triangle_p \Omega_j$ is, the more effort forward-looking workers put into innovation, even without a bonus from their $t = 1$ employer.[16]

5.2.3 Firms' Problem

Given a public belief π about a worker at the beginning of $t = 2$, firms post contracts that maximize their expected profit at $t = 2$. Let $\delta_{ij} = 1[j(i,1) = j]$ indicate whether worker i is an incumbent employee of firm j ($\delta_{ij} = 1$) or an outside worker ($\delta_{ij} = 0$), written as δ below. Firm j's problem is:

$$v_{j2}^{(\delta)}(\pi) = max_{(\underline{w},\gamma,\tau)} \underbrace{s_{j2}^{(\delta)}(\underline{w},\gamma,\tau)}_{\text{labor supply}} \cdot \underbrace{\left(E_\alpha[Y_j(\alpha,\tau,e)|\pi] - \underline{w}(1+\gamma\cdot\pi\cdot h(\tau,e))\right)}_{\text{expected profit at t=2}} \tag{5.7}$$

$$\text{where } E_\alpha[Y_j(\alpha,\tau,e)|\pi] = \phi_j \cdot (1+\theta\cdot\pi\cdot h(\tau,e)) - \frac{\zeta}{2}\tau^2$$

$$\text{s.t. worker effort } e \text{ solves (5.3) with } \pi_{i2} = \pi.$$

Despite symmetric information across firms, the labor supply of incumbent employees is less elastic than that of outside workers whenever $\lambda < 1$, resulting in lower wages for stayers than for new hires.[17]

At $t = 1$, given a prior π about a worker, firms post contracts that maximize their expected profit at $t = 1$ plus the discounted expected return from an incumbent employee

[15] Recall that the posterior after a patent at $t = 1$ is $Pr(\alpha_i = H|p_{i1} = 1) = 1$. The posterior when $p_{i1} = 0$ is $Pr(\alpha_i = H|\pi_{i1},\ j(i,1),\ p_{i1} = 0) < 1$.

[16] The option value can also be interpreted as a preference for innovation as in Stern (2004). Workers with a higher prior π_{i1} have a stronger preference for higher firm investment τ, which raises the chance of being revealed as H-ability in the next period.

[17] Firms do not observe whether their $t = 1$ employees re-enter the labor market at the beginning of $t = 2$, and therefore cannot price discriminate between incumbents who stay and those who search. Appendix B2 derives the labor supply of incumbent and outside workers in (B.5).

at $t = 2$. Each firm solves:

$$max_{(\underline{w},\gamma,\tau)} \underbrace{s_{j1}(\underline{w},\gamma,\tau)}_{\text{labor supply}} \cdot \left(\underbrace{E_\alpha[Y_j(\alpha,\tau,e)|\pi] - \underline{w}(1+\gamma\cdot\pi\cdot h(\tau,e))}_{\text{expected profit at t=1}} + \beta \cdot \underbrace{E[v_{j2}^{(1)}(\pi_{i2})|\pi,\tau,e]}_{\text{continuation value}} \right) \tag{5.8}$$

s.t. worker effort e solves (5.5) with $\pi_{i1} = \pi$.

A firm's investment in an employee's innovation at $t = 1$ matters not only for patent production today but also for the public information about the worker at $t = 2$. Firms that face higher turnover of inventors have a continuation value that decreases in τ, and they invest less than a myopic firm would. The investment decision is analogous to the provision of general skill training, which is inefficiently low when firms cannot retain workers (Acemoglu and Pischke 1998; Manning 2012; Stevens 1994).

In equilibrium, workers take the posted contracts as given, choose an employer if they are on the market, and choose their effort optimally. Firms take into account how labor supply and worker effort respond to the contracts they post. Each firm also takes as given the contracts posted simultaneously by other firms, conditional on the public information about worker ability. We complete the backward induction in Appendix B2.

5.3 Equilibrium Results and Links to Empirical Findings

We derive three equilibrium results and discuss the assumptions under which they explain two empirical findings: (1) it is harder to become an inventor at lower-wage firms, and (2) the wage return to a first patent application is higher at lower-wage firms. The results rely on the following assumptions.

(A1) *Labor supply.* Labor supply is not perfectly elastic, $0 < b < \infty$. Each firm is small relative to the market and takes the contracts of other firms as given, as in Card et al. (2018).

(A2) *Equilibrium posterior.* The posterior belief about a worker who has not patented at $t = 1$, $\pi_{i2}(0)$, is formed from the equilibrium effort and contract at her $t = 1$ employer.

(A3) *Patent production.* The patent production function $h(\tau, e) \in (0, 1)$ is twice continuously differentiable and quadratic in worker effort, with $h_1 > 0$, $h_2 > 0$, $h_{12} > 0$, and a constant $h_{22} < 0$.

Proposition 1 (Positive Relationship between Firm Productivity and Base Wages) *Under Assumption (A1), for any given bonus γ, investment τ, and change in the worker's option value from patenting, the base wage set by a firm is non-decreasing in firm productivity ϕ_j in both periods.*

Proposition 1 suggests that, holding the rest of the contract fixed, a more productive firm pays a higher base wage, so that ranking firms by their mean wages preserves the ranking of firm productivity. This is an assumption that underlies our empirical analysis. The INPS data do not include a direct measure of firm productivity, but we match 7% of the firms in our sample to the Bank of Italy's INVIND survey by fiscal code or firm name. Appendix Table A.2 shows that lower-wage firms have lower revenue and invest less on average than higher-wage firms, consistent with Proposition 1.

Proposition 2 (Heterogeneity in Firm Investment in Innovation) *Under Assumptions (A1)–(A3), assume also that the optimal investment is interior and satisfies the second-order condition, and that a firm's expected patent output from a known inventor exceeds that from a worker who has not patented. Then, for any positive belief about a worker's ability to invent and any given base wage $\underline{w}$, bonus γ, and change in worker's option value, each firm's investment τ_{jt} is increasing in firm productivity ϕ_j in both periods.*

Assumption (A3) implies that, whenever there is a positive chance that a worker is H-ability, firms are willing to invest in her innovation activity to maximize the expected return to innovation. At $t = 2$, the return to a successful invention is higher at a more productive firm, so the second-period investment τ_{j2} is increasing in firm productivity ϕ_j.[18] At $t = 1$, forward-looking firms also consider how their investment affects the revelation of inventor talent at $t = 2$. The gain in continuation value from a revealed inventor is larger at a more productive firm, so the first-period investment τ_{j1} is also

[18]See the first-order condition with respect to τ, equation (B.9) in Appendix B2.

increasing in ϕ_j. The model does not feature differences in turnover across firms. In Italy, lower-wage firms have higher turnover among workers who have not yet invented (Figure 2), but new inventors appear more likely to stay than to be poached after patenting (Table 5), suggesting that turnover has a limited effect on the investment decision.

We find evidence supporting this proposition among the firms matched to the INVIND survey, which reports investment.[19] The positive relationship between firm productivity and investment in innovation can explain why potential inventors are significantly less likely to file their first patent application at lower-wage firms (Table 3).

Proposition 3 (Wage Bonus for Invention) *Under Assumptions (A1)–(A3) and the regularity conditions (R1)–(R3) in Appendix B3, the bonus γ_{jt} is decreasing in firm productivity ϕ_j in both periods if the worker's probability of patenting is below one half, $\pi h < 0.5$, and the complementarity between investment and effort is weak relative to the effect of investment on the probability of patenting.*

Proposition 3 reflects two opposing effects of firm investment on the bonus. First, the bonus has a net cost to the firm: the expected bonus payment minus the base wage that workers are willing to forgo in return for the bonus. More productive firms invest more (Proposition 2), which raises the probability of patenting and makes the bonus a more costly incentive when $\pi \cdot h < 0.5$. Second, investment and effort are complementary ($h_{12} > 0$), so a higher investment raises the effort that the bonus elicits (see equation 5.5).[20] The bonus is decreasing in firm productivity when the first effect dominates, that is, when the complementarity is weak relative to the effect of investment on the probability of patenting.[21]

Figure 3 illustrates how the bonus varies with firm productivity under different levels of complementarity between firm investment and worker effort. When the complementarity is strong ($h_{12} = 0.15$), more productive firms set a higher bonus, because their higher investment raises the effort that the bonus elicits. When the complementarity is

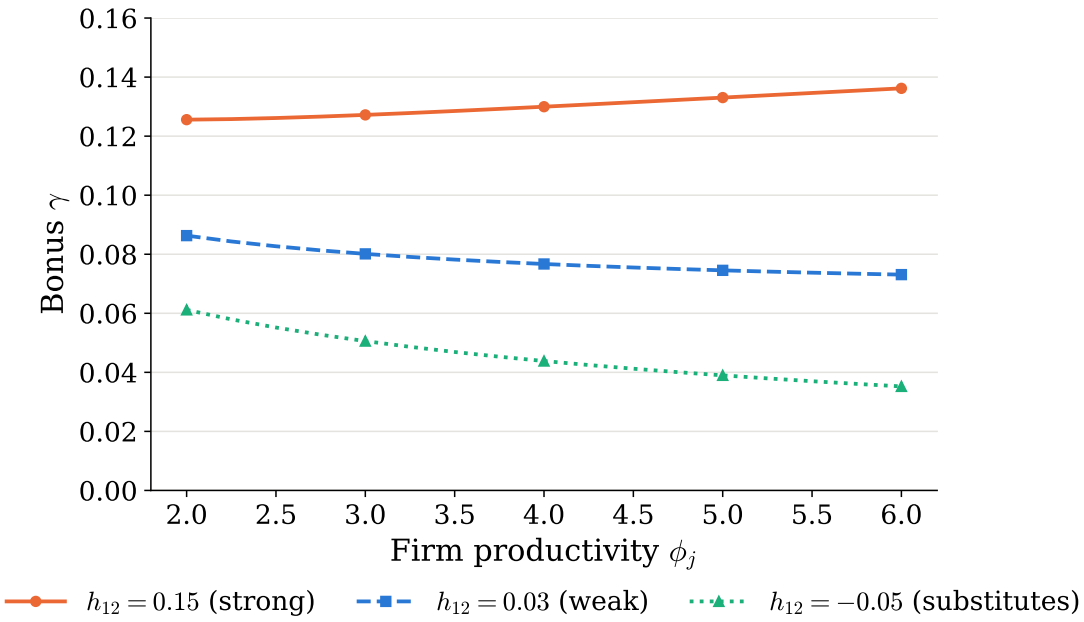
[19] Appendix Table A.2 shows that, among the firms matched to INVIND, lower-wage firms in quartile 1 invest less in R&D, material and immaterial assets, machinery, and housing than higher-wage firms.

[20] From (5.5) we have $\frac{\partial e_{ij1}}{\partial \tau_{ij1}} \propto h_{12} > 0$.

[21] The exact condition is given in (B.20) in Appendix B3, and the derivation in (B.19) and (B.21).

weak ($h_{12} = 0.03$), the net cost of the bonus dominates, resulting in a negative relationship between the bonus and productivity. For comparison, we also include a case outside Assumption (A3), in which investment and effort are substitutes ($h_{12} = -0.05$), and the bonus also decreases with productivity.

Figure 3: Bonus and Firm Productivity under Different Complementarity



Notes: This figure shows how the optimal bonus γ changes with firm productivity ϕ_j at $t = 2$, under three values of the complementarity between investment and effort, h_{12}. All other parameters are the same across the three cases. Details on the calibration and numerical solution are in Appendix B3.

The prediction of Proposition 3 is consistent with our empirical finding that the (log) wage premium for a worker's first patent application is more than twice as high at firms in the bottom wage quartile as at firms in the top quartile (columns 1 and 4 of Table 4). Employer learning alone cannot explain this finding unless new inventors at lower-wage firms are poached by higher-wage firms, which we do not find in the data (Table 5). By considering incentive contracts and firms' investment decisions jointly, we show that the bonus decreases in firm productivity when the probability of patenting is below one half and the complementarity between investment and effort is weak. Through the lens of the model, the higher wage return at lower-wage firms suggests that the net cost of the bonus dominates the complementarity effect in our setting.

In summary, this two-period model with dynamic decisions by firms and workers helps explain the key empirical findings in Sections 3–4:

1. Younger workers are less likely to become inventors at lower-wage firms.
2. Wages increase upon a worker's first patent application.
3. The wage return to a worker's first patent application is higher at lower-wage firms, although new inventors at these firms are not poached at a higher rate.

6 Conclusion

By discovering talent among their own employees, firms play an important role in fostering innovation. Workers with the potential to invent may enter the labor market without a track record that reveals their inventive ability, leaving firms to decide how much to invest in their innovation activity and whether to give them opportunities to develop and demonstrate that ability. Differences across firms in these decisions can therefore shape who becomes an inventor, how quickly inventive talent is discovered, and how workers are rewarded when their ability is revealed.

This paper documents the heterogeneity in the identification and rewarding of inventors across firms in Italy. We find that younger potential inventors are 20% less likely to file their first patent application at a lower-wage firm in the bottom quartile than similar workers in the top quartile. The gap between low- and high-wage firms in patent applications disappears, however, among experienced inventors who have already applied for patents. Furthermore, there is a significant 3–12 log point increase in workers' wage when they file their first-ever patent application, and the wage returns are significantly higher at firms that pay lower wages.

We interpret the empirical findings through a model that combines employer learning and incentive contracts. Heterogeneous firms invest in workers' innovation activity and use a bonus to incentivize workers to invent. More productive firms invest more, but they set a lower bonus when (1) a worker's probability of patenting is low, and (2) the complementarity between firm investment and worker effort is weak. Under these conditions, the higher investment makes the bonus a costlier way to elicit effort. This

model contributes to the employer learning literature by taking into account the dynamic incentives of firms and workers simultaneously.

Our findings on firm heterogeneity have implications for the design of R&D policy. While Bell et al. (2019a) emphasize policies that increase exposure to innovation for groups underrepresented among inventors (e.g., women and children from lower-income families), we focus on young employees with the potential to invent and suggest measures that leverage the underused talent of these "lost Einsteins". Our evidence suggests that many younger workers who could invent are less likely to do so at lower-wage firms. A subsidy tied to the number of young employees involved in innovation could therefore encourage firms to invest more in identifying and developing talent. Such a policy could be particularly beneficial for countries like Italy, where younger workers face worsening economic opportunities amid a prolonged productivity slowdown.

References

Abowd, J. M., F. Kramarz, and D. N. Margolis (1999): "High Wage Workers and High Wage Firms," *Econometrica*, 67, 251–333.

Acemoglu, D. and J.-S. Pischke (1998): "Why Do Firms Train? Theory and Evidence," *The Quarterly Journal of Economics*, 113, 79–119.

Aghion, P., U. Akcigit, A. Hyytinen, and O. Toivanen (2018): "On the Returns to Invention within Firms: Evidence from Finland," *AEA Papers and Proceedings*, 108, 208–12.

——— (2023): "Parental education and invention: the Finnish enigma," *International Economic Review*, 64, 453–490.

Aghion, P. and P. Howitt (1992): "A Model of Growth Through Creative Destruction," *Econometrica*, 60, 323–351.

Akcigit, U. and N. Goldschlag (2025): "Measuring the Characteristics and Employment Dynamics of U.S. Inventors," *Journal of Economic Growth*, 30, 237–269.

Akcigit, U., J. Grigsby, and T. Nicholas (2017): "The Rise of American Ingenuity: Innovation and Inventors of the Golden Age," *American Economic Review: Papers and Proceedings*, 107, 327–331.

Altonji, J. G. and C. R. Pierret (2001): "Employer Learning and Statistical Discrimination," *The Quarterly Journal of Economics*, 116, 313–350.

Antonelli, C., F. Barbiellini Amidei, and C. Fassio (2014): "The mechanisms of knowledge governance: State ownedenterprises and Italian economic growth, 1950–1994," *Structural Change and Economic Dynamics*, 31, 43–63.

Bagger, J. and R. Lentz (2019): "An Empirical Model of Wage Dispersion with Sorting," *The Review of Economic Studies*, 86, 153–190.

Bell, A., R. Chetty, X. Jaravel, N. Petkova, and J. Van Reenen (2019a): "Do Tax Cuts Produce more Einsteins? The Impacts of Financial Incentives Versus Exposure to Innovation on the Supply of Inventors," *Journal of the European Economic Association*, 17, 651–677.

——— (2019b): "Who becomes an inventor in America? The importance of exposure to innovation," *The Quarterly Journal of Economics*, 134, 647–713.

Bianchi, N., G. Bovini, J. Li, M. Paradisi, and M. Powell (2023): "Career Spillovers in Internal Labour Markets," *The Review of Economic Studies*, 90, 1800–1831.

Bianchi, N. and M. Paradisi (2023): "Countries of Old Men: An Analysis of the Wage Age Gap," Tech. rep., mimeo.

Card, D., A. R. Cardoso, J. Heining, and P. Kline (2018): "Firms and Labor Market Inequality: Evidence and Some Theory," *Journal of Labor Economics*, 36, S13–S70.

Daruich, D., S. Di Addario, and R. Saggio (2023): "The Effects of Partial Employment Protection Reforms: Evidence from Italy," *The Review of Economic Studies*, 90, 2880–2942.

Depalo, D. and S. L. Di Addario (2014): "Shedding Light on Inventors' Returns to Patents," Tech. Rep. 375, Development Studies Working Papers, Centro Studi Luca d'Agliano.

DiNardo, J., N. M. Fortin, and T. Lemieux (1996): "Labor Market Institutions and the Distribution of Wages, 1973–1992: A Semiparametric Approach," *Econometrica*, 64, 1001–1044.

Farber, H. S. and R. Gibbons (1996): "Learning and Wage Dynamics," *The Quarterly Journal of Economics*, 111, 1007–1047.

Fernández-Val, I. (2009): "Fixed Effects Estimation of Structural Parameters and Marginal Effects in Panel Probit Models," *Journal of Econometrics*, 150, 71–85.

Goldin, I., P. Koutroumpis, F. Lafond, and J. Winkler (2024): "Why Is Productivity Slowing Down?" *Journal of Economic Literature*, 62, 196–268.

Harris, M. and A. Raviv (1979): "Optimal incentive contracts with imperfect information," *Journal of Economic Theory*, 20, 231–259.

Hirano, K., G. W. Imbens, and G. Ridder (2003): "Efficient Estimation of Average Treatment Effects Using the Estimated Propensity Score," *Econometrica*, 71, 1161–1189.

Holmström, B. (1979): "Moral Hazard and Observability," *The Bell Journal of Economics*, 10, 74–91.

Kahn, L. B. (2013): "Asymmetric Information between Employers," *American Economic Journal: Applied Economics*, 5, 165–205.

Kaltenberg, M., A. B. Jaffe, and M. E. Lachman (2023): "Invention and the life course: Age differences in patenting," *Research Policy*, 52, 104629.

Kline, P. M. (2024): "Firm Wage Effects," Working Paper 33084, National Bureau of Economic Research.

Kline, P. M., N. Petkova, H. Williams, and O. Zidar (2019): "Who profits from patents? Rent sharing at innovative firms," *The Quarterly Journal of Economics*, 134, 1343–1404.

Lange, F. (2007): "The Speed of Employer Learning," *Journal of Labor Economics*, 25, 1–35.

Manning, A. (2012): "Steady-state Equilibrium in a Model of Short-Term Wage-Posting," *CEP Discussion Paper No 1158*.

Postel-Vinay, F. and J.-M. Robin (2002): "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica*, 70, 2295–2350.

Romer, P. M. (1990): "Endogenous Technological Change," *Journal of Political Economy*, 98, S71–S102.

Rosenbaum, P. R. and D. B. Rubin (1983): "The central role of the propensity score in observational studies for causal effects," *Biometrika*, 70, 41–55.

Schönberg, U. (2007): "Testing for Asymmetric Employer Learning," *Journal of Labor Economics*, 25, 651–691.

Stammann, A., F. Heiss, and D. McFadden (2016): "Estimating Fixed Effects Logit Models with Large Panel Data," Working paper.

Stern, S. (2004): "Do Scientists Pay to Be Scientists?" *Management Science*, 50, 835–853.

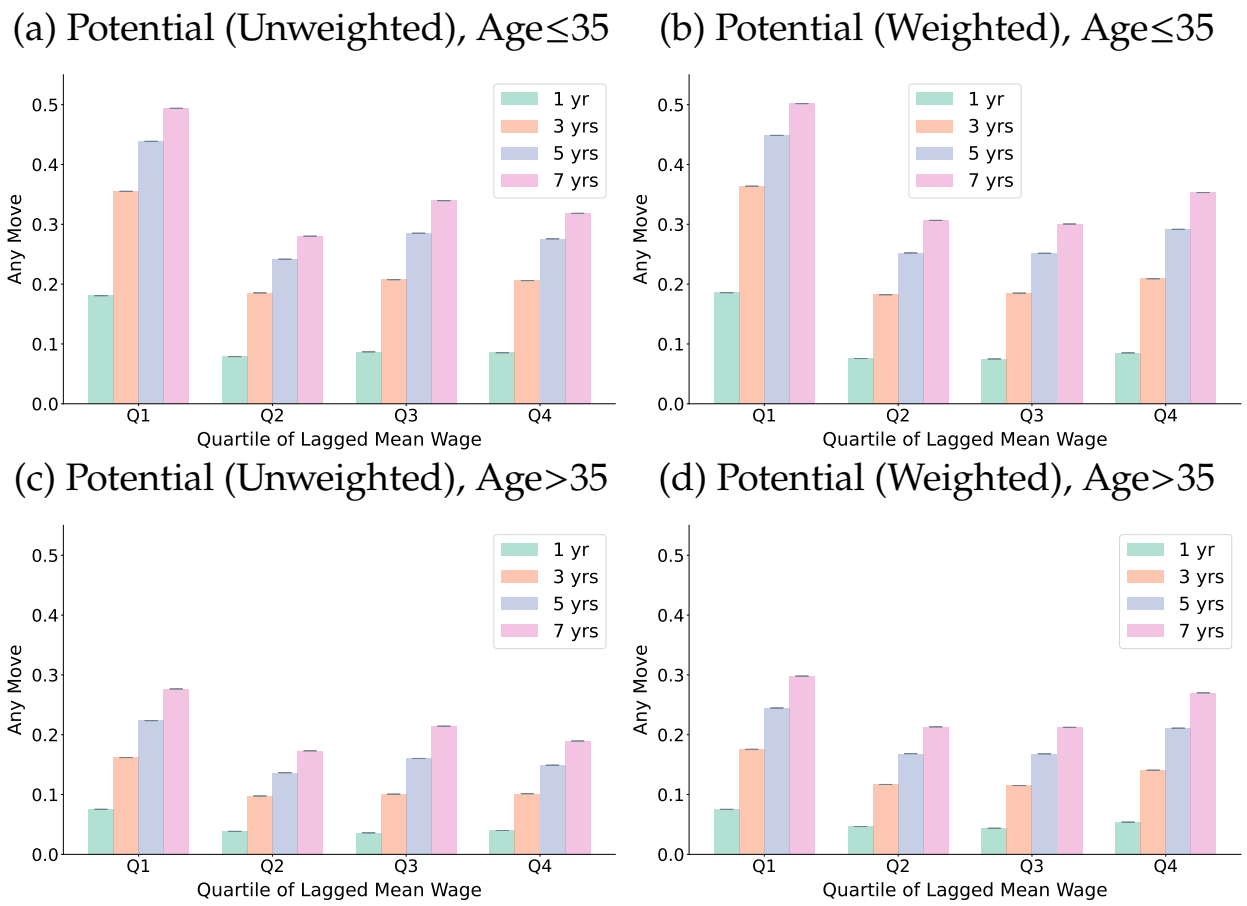
Stevens, M. (1994): "A Theoretical Model of On-the-Job Training with Imperfect Competition," *Oxford Economic Papers*, 46, 537–62.

Toivanen, O. and L. Väänänen (2012): "Returns to Inventors," *The Review of Economics and Statistics*, 94, 1173–1190.

Wu, A. (2026): "Employer Learning, Sorting, and Productivity: Evidence from Computer Scientists," Discussion Paper 214/26, RFBerlin.

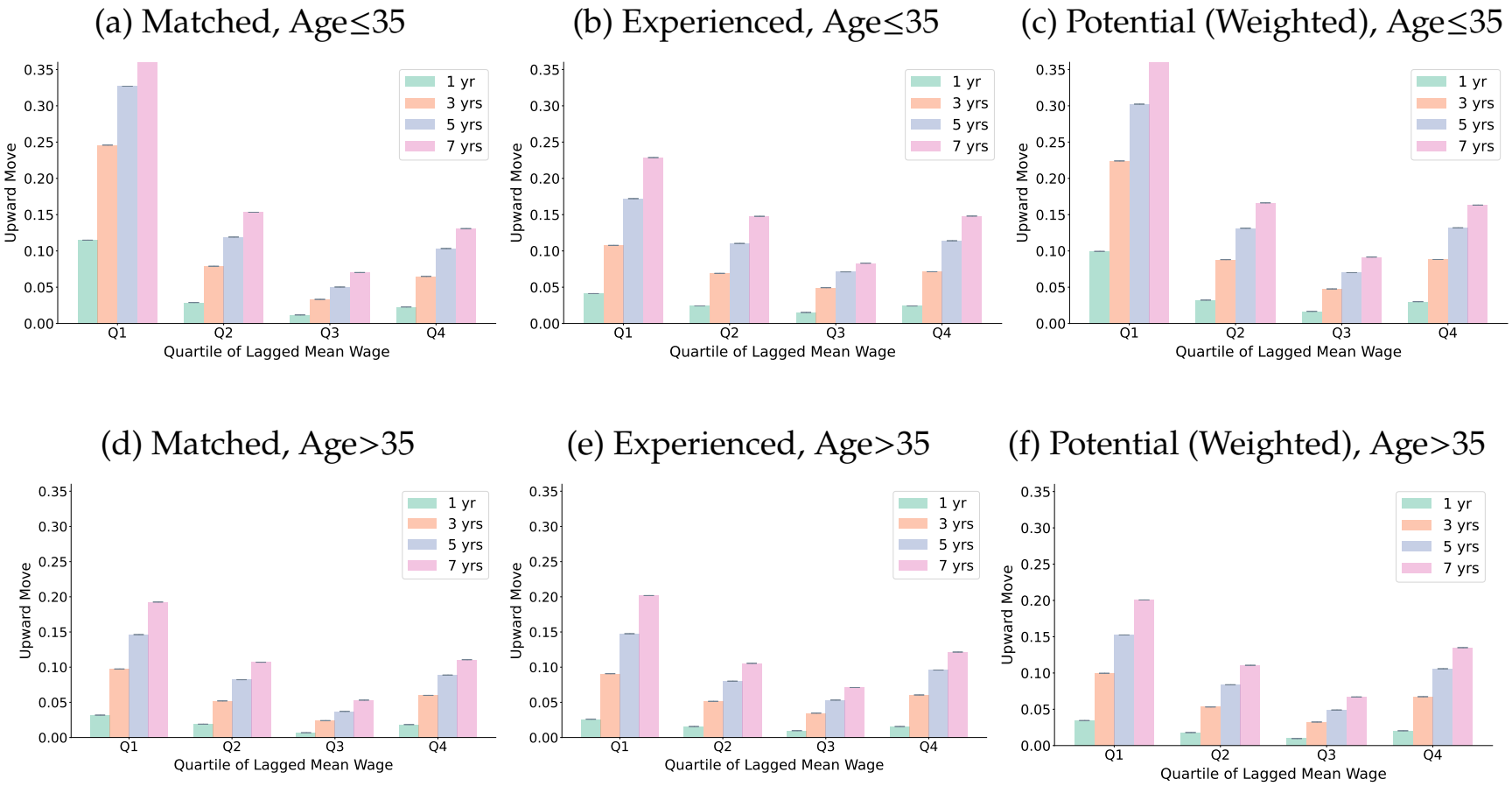
A. Appendix Figures and Tables

Figure A.1: Mobility by Firm Quartiles: Any Move between Firms, Potential Inventors



Notes: This figure shows the mean mobility of workers at each firm quartile, ranked by the mean wage of their employees in the previous year. Panels (a)-(b) focus on the fraction of workers younger than 35 at t employed by different firms, $j(i,t+k) \neq j(i,t)$, in $k \in \{1,3,5,7\}$ years. Panels (c)-(d) shows the same for workers older than 35. Potential inventors include both matched inventors who have not applied for a patent yet and non-inventors. In panels (a) and (c), non-inventors are unweighted. In (b) and (d), non-inventors are weighted by $w_i = \frac{p_i}{1-p_i}$ for each worker with estimated propensity of ever inventing p_i from regression (2.1).

Figure A.2: Upward Mobility by Firm Quartiles: Moving to a Higher Quartile



Notes: This figure shows the mean upward mobility of workers at each quartile of firms, ranked by the mean wage of their employees in the previous year. Upward mobility is the movement of a worker into a higher-quartile firm, $j(i.t+k) \neq j(i,t)$ and $Q(i,t+k) > Q(i,t)$, in $k \in \{1,3,5,7\}$ years. For workers starting at the top, a move between firms within quartile 4 is also coded as an upward move. (a)-(c) focus on the fraction of workers younger than 35 at t employed by different firms, $j(i,t+k) \neq j(i,t)$, in $k \in \{1,3,5,7\}$ years, while (d)-(f) shows the same for workers older than 35.

Table A.1: Logistic Regressions of Ever Inventing

	(1)	(2)
Demographics		
Female	-1.58015	-1.50801
	(0.02881)	(0.03046)
Age (normalized at 35)	-1.90361	-1.48708
	(0.40146)	(0.44697)
Age^2, Age^3	X	X
Job Characteristics		
Blue Collar	-0.76759	-1.02349
	(0.11628)	(0.13107)
White Collar	1.29939	1.11450
	(0.12892)	(0.13919)
Blue/White × Age	X	X
Permanent Contract	0.16407	0.28941
	(0.34119)	(0.37664)
Temporary Contract	0.61473	0.43378
	(0.33733)	(0.37178)
Seasonal Contract	0.58827	-0.14596
	(0.81939)	(0.93999)
Contract Type × Age	X	X
INPS Sample (1987-2009)		
Min(Yr | INPS)=1987	-0.35221	-0.32059
	(0.02902)	(0.03222)
(Min(Yr)=1987) × Age	-0.02368	-0.02382
	(0.03196)	(0.03527)
Constant	-6.80784	
	(0.37223)	
Fixed Effects		
Industry and Province	X	
Initial Firm		X
N	1,533,333	1,047,697
Pseudo R2	0.10997	0.22523

Notes: This table reports logistic regression estimates of Inv_i, an indicator equal to one if person i is a matched inventor. The estimation sample is at the person level, using each worker's characteristics in her first year in the INPS data. Column (1) is estimated on the 1.5 million workers who have at least five years of employment in INPS between 1987 and 2009, entered the sample between ages 14 and 55, and spent at least half of their working years in white-collar positions. Column (2) reports estimates of equation (2.1) in Section 2.2.1, which includes initial firm fixed effects and therefore drops workers starting at firms with no entering matched inventor or no entering non-inventor; the sample falls to just over one million, the sum of matched inventors and non-inventors in Table 1. Age is normalized at 35 and enters as a cubic polynomial, interacted with occupation category and contract type, so the reported coefficients describe a worker entering at age 35. We use the estimated propensity scores from column (2) to reweight non-inventors.

Table A.2: Firm Characteristics by Quartile of Lagged Mean Wage, Restricted to Firms Matched to INVIND

	Quartile 1		Quartile 2		Quartile 3		Quartile 4	
	mean	sd	mean	sd	mean	sd	mean	sd
Num. Unique Firms matched to Invind	887		957		774		486	
Num. Employees	628	5,196	663	4,153	997	4,376	1,290	5,210
Log Wage	7.866	0.376	7.960	0.321	7.997	0.288	8.057	0.324
Mean monthly wage (real 1995 EUR)	3,254	2,281	3,545	2,231	3,616	1,324	3,979	2,855
Patent Apps per Worker-Yr	0.092	0.147	0.103	0.152	0.111	0.157	0.112	0.150
Revenue and Investment (in thousands)								
Revenue	108,320	487,501	130,238	396,525	251,364	1,523,081	353,173	1,909,167
Investment on R&D	688	2,285	2,138	14,886	5,205	37,200	7,053	43,016
Investment on Machinery	2,836	10,010	4,648	16,793	12,395	131,896	16,560	165,953
Investment on Material	101	335	133	419	303	3,375	431	4,273
Investment on Immaterial	1,500	18,147	1,406	16,987	1,789	18,465	2,888	23,941
Investment on Housing	663	3,873	674	1,720	1,002	4,551	1,161	5,501

Notes: This table is restricted to firms that are matched to the Bank of Italy's INVIND survey by fiscal code or by firm name, and displays the mean characteristics of firms (to be compared to those shown in Table 2).

Table A.3: Between-Firm Job Mobility in 3 Years, by Quartile of Firms and Age Group

	Move in 3 Years: $j(i,t+3) \neq j(i,t)$					
	Age $\leq$ 35			Age > 35		
	(1) New	(2) Experienced	(3) Potential	(4) New	(5) Experienced	(6) Potential
Quartile of Lagged Mean Wage						
quartile 1	0.0992***	0.0103	0.0853***	-0.0246**	-0.0136	0.0021
	(0.0112)	(0.0311)	(0.0053)	(0.0120)	(0.0139)	(0.0038)
quartile 2	-0.0027	-0.0100	-0.0069	-0.0415***	-0.0202*	-0.0219***
	(0.0095)	(0.0285)	(0.0044)	(0.0104)	(0.0120)	(0.0029)
quartile 3	-0.0242***	-0.0041	-0.0181***	-0.0444***	-0.0305***	-0.0224***
	(0.0083)	(0.0244)	(0.0039)	(0.0088)	(0.0104)	(0.0023)
quartile 4	0	0	0	0	0	0
	(.)	(.)	(.)	(.)	(.)	(.)
First Patent Application at $j(i,t)$						
Pat_{it}	-0.0283**	-0.1500***	-0.0632***	-0.0739***	-0.1084***	-0.0675***
	(0.0112)	(0.0409)	(0.0100)	(0.0100)	(0.0151)	(0.0075)
Additional effect (relative to quartile 4)						
$\text{Pat}_{it}\times$ quartile 1	-0.1330***	0.0127	-0.1089***	0.0214	-0.0203	0.0006
	(0.0186)	(0.0723)	(0.0170)	(0.0167)	(0.0251)	(0.0141)
$\text{Pat}_{it}\times$ quartile 2	0.0347**	0.0779	0.0379***	0.0473***	0.0391*	0.0298***
	(0.0140)	(0.0594)	(0.0128)	(0.0134)	(0.0232)	(0.0108)
$\text{Pat}_{it}\times$ quartile 3	0.0413***	0.0684	0.0366***	0.0566***	0.0121	0.0342***
	(0.0139)	(0.0546)	(0.0127)	(0.0122)	(0.0188)	(0.0100)
Mean |quartile 4	.1828717	.2577172	.1703986	.1500741	.2131824	.0824841
N	51K	5K	4M	33K	23K	6.5M

Notes: This table shows the estimated OLS regression (4.2) of any movement between firms in 3 years ($j(i,t+3) \neq j(i,t)$) on whether a worker applies for a new patent and its interactions with the quartile of her current employer, ranked by the mean wage of its employees in the previous year. The estimation sample is at the person $\times$ year level. Columns (1) and (4) are estimated on matched inventors who have not filed for a patent yet. (2) and (5) are restricted to experienced inventors who have applied for patents at former employers. (3) and (6) are matched inventors who have not patented plus reweighted non-inventors. Standard errors are robust and clustered at person level. Significance: * 0.10 ** 0.05 *** 0.010.

Table A.4: Upward Mobility in a Year, by Quartile of Firms and Age Group

	Upward Move in 1 Year					
	Age ≤ 35			Age > 35		
	(1) New	(2) Experienced	(3) Potential	(4) New	(5) Experienced	(6) Potential
Quartile of Lagged Mean Wage						
quartile 1	0.0913***	0.0324***	0.0689***	0.0169***	0.0139***	0.0163***
	(0.0041)	(0.0106)	(0.0018)	(0.0035)	(0.0041)	(0.0010)
quartile 2	0.0250***	0.0133	0.0174***	0.0069**	0.0077**	0.0039***
	(0.0031)	(0.0091)	(0.0014)	(0.0030)	(0.0034)	(0.0007)
quartile 3	-0.0045*	-0.0094	-0.0073***	-0.0112***	-0.0091***	-0.0075***
	(0.0024)	(0.0076)	(0.0011)	(0.0022)	(0.0026)	(0.0005)
quartile 4	0	0	0	0	0	0
	(.)	(.)	(.)	(.)	(.)	(.)
First Patent Application at $j(i,t)$						
Pat_{it}	-0.0119***	-0.0198	-0.0187***	-0.0149***	-0.0175***	-0.0172***
	(0.0038)	(0.0172)	(0.0034)	(0.0026)	(0.0047)	(0.0018)
Additional effect (relative to quartile 4)						
$\text{Pat}_{it}\times$ quartile 1	-0.0690***	0.0219	-0.0481***	-0.0115**	-0.0140*	-0.0098***
	(0.0075)	(0.0398)	(0.0067)	(0.0048)	(0.0078)	(0.0038)
$\text{Pat}_{it}\times$ quartile 2	-0.0011	-0.0048	0.0022	0.0021	0.0021	0.0042
	(0.0050)	(0.0228)	(0.0044)	(0.0038)	(0.0080)	(0.0028)
$\text{Pat}_{it}\times$ quartile 3	0.0159***	-0.0103	0.0164***	0.0148***	0.0021	0.0121***
	(0.0046)	(0.0178)	(0.0041)	(0.0032)	(0.0052)	(0.0024)
Mean |quartile 4	.0226718	.0383387	.0264689	.0183291	.0281631	.0134
N	51K	5K	4M	33K	28K	8.1M

Notes: This table shows the estimated OLS regression (4.2) of upward mobility in 1 year ($j(i,t+1) \neq j(i,t)$, and $Q(i,t+1) > Q(i,t)$ or $Q(i,t+1) = Q(i,t) = 4$) on whether a worker applies for a new patent and its interactions with the quartile of her current employer, ranked by the mean wage of its employees in the previous year. The estimation sample is at the person × year level. Columns (1) and (4) are estimated on matched inventors who have not filed for a patent yet. (2) and (5) are restricted to experienced inventors who have applied for patents at former employers. (3) and (6) are matched inventors who have not patented plus reweighted non-inventors. Standard errors are robust and clustered at person level. Significance: * 0.10 ** 0.05 *** 0.010.

Table A.5: Upward Mobility in 3 Years, by Quartile of Firms and Age Group

	Upward Move in 3 Years					
	Age ≤ 35			Age > 35		
	(1) New	(2) Experienced	(3) Potential	(4) New	(5) Experienced	(6) Potential
Quartile of Lagged Mean Wage						
quartile 1	0.1840***	0.0765***	0.1418***	0.0482***	0.0320***	0.0408***
	(0.0079)	(0.0247)	(0.0038)	(0.0080)	(0.0103)	(0.0027)
quartile 2	0.0542***	0.0172	0.0347***	0.0070	-0.0021	0.0024
	(0.0062)	(0.0212)	(0.0029)	(0.0066)	(0.0083)	(0.0019)
quartile 3	-0.0165***	-0.0276	-0.0257***	-0.0300***	-0.0391***	-0.0241***
	(0.0048)	(0.0177)	(0.0023)	(0.0050)	(0.0066)	(0.0014)
quartile 4	0	0	0	0	0	0
	(.)	(.)	(.)	(.)	(.)	(.)
First Patent Application at $j(i,t)$						
Pat_{it}	-0.0007	-0.0806***	-0.0268***	-0.0204***	-0.0524***	-0.0294***
	(0.0077)	(0.0300)	(0.0071)	(0.0066)	(0.0108)	(0.0054)
Additional effect (relative to quartile 4)						
Pat_{it}× quartile 1	-0.1375***	-0.0415	-0.0980***	-0.0166	-0.0065	-0.0108
	(0.0142)	(0.0451)	(0.0130)	(0.0125)	(0.0202)	(0.0108)
Pat_{it}× quartile 2	-0.0030	0.0370	0.0093	0.0126	0.0258	0.0149*
	(0.0098)	(0.0415)	(0.0092)	(0.0092)	(0.0166)	(0.0079)
Pat_{it}× quartile 3	0.0149*	0.0410	0.0224***	0.0261***	0.0224*	0.0233***
	(0.0090)	(0.0369)	(0.0084)	(0.0077)	(0.0128)	(0.0067)
Mean |quartile 4	.0647726	.1291028	.0782539	.0598151	.1052918	.0442733
N	51K	5K	4M	33K	23K	6.5M

Notes: This table shows the estimated OLS regression (4.2) of upward mobility in 3 years ($j(i,t+3) \neq j(i,t)$, and $Q(j(i,t+3)) > Q(i,t)$ or $Q(j(i,t+3)) = Q(i,t) = 4$) on whether a worker applies for a new patent and its interactions with the quartile of her current employer, ranked by the mean wage of its employees in the previous year. The estimation sample is at the person × year level. Columns (1) and (4) are estimated on matched inventors who have not filed for a patent yet. (2) and (5) are restricted to experienced inventors who have applied for patents at former employers. (3) and (6) are matched inventors who have not patented plus reweighted non-inventors. Standard errors are robust and clustered at person level. Significance: * 0.10 ** 0.05 *** 0.010.

B. Model Appendix

B1. Model Timeline & Information Structure

There are two discrete periods in the model.

1. ($\boldsymbol{t = 1}$) All workers are on the labor market.
   (a) Given the common prior $\pi_{i1} = Pr(\alpha_i = H)$ about worker i, firms (indexed by j) simultaneously post contracts $\{(\underline{w}_{ij1}, \gamma_{ij1}, \tau_{ij1})\}$, each comprising a base wage, a proportional bonus upon a patent application, and an investment in the worker's innovation activity.
   (b) Each worker observes the posted contracts, draws idiosyncratic preferences over firms from (5.2), and chooses the employer $j(i,1)$ that maximizes her utility (5.4). Conditional on joining $j(i,1)$, she chooses the effort e_{ij1} that maximizes her expected utility from wages at $t = 1$ plus her option value on the market at $t = 2$, discounted by $\beta \in (0,1]$.
   (c) Whether the worker produces a patent application, $p_{i1} \in \{0,1\}$, is realized at the end of $t = 1$. She receives a bonus of $\gamma_{ij1} \cdot \underline{w}_{ij1}$ if $p_{i1} = 1$, and zero otherwise.
2. ($\boldsymbol{t = 2}$) The public belief about the worker is updated according to her patent production at $t = 1$:

$$\begin{aligned} &p_{i1} = 1 \rightarrow \pi_{i2} = 1 \\ &p_{i1} = 0 \rightarrow \pi_{i2} = Pr(\alpha_i = H | \pi_{i1},\ j(i,1),\ p_{i1} = 0) \end{aligned} \tag{B.1}$$

   A worker who has produced a patent application is publicly known as H-ability. Otherwise, the posterior belief conditions on her not producing a patent application at her $t = 1$ employer $j(i,1)$.
   (a) Given the updated belief, firms simultaneously post new contracts by solving (5.7).
   (b) The worker re-enters the job market with probability $\lambda \in (0,1]$. If she is on the market, she observes the new contracts, draws new preferences from (5.2), and chooses the employer that maximizes her utility (5.3). Otherwise, $j(i,2) = j(i,1)$, and she chooses the effort that maximizes her utility conditional on staying.
   (c) Step 1(c) is repeated. The model concludes.

B2. Backward Induction

We state the problems facing workers and firms in each period in Section 5.2. Now we provide details on the backward induction. To ease notation, we write π for the public

belief about a worker in the current period: $\pi = \pi_{i2}$ at $t = 2$ and $\pi = \pi_{i1}$ at $t = 1$. We assume a quadratic effort cost, $c(e) = \frac{c}{2}e^2$.

Optimization at $t = 2$

Worker's Problem. Given contract $(\underline{w}_{ij2}, \gamma_{ij2}, \tau_{ij2})$ and conditional on being employed by j, the worker chooses the effort that maximizes her expected utility from wages net of the effort cost:

$$\begin{aligned} e_{ij2} &= argmax_e \; b\, ln(\underline{w}_{ij2}) + \pi\, h(\tau_{ij2}, e) \cdot b\, ln(1+\gamma_{ij2}) - \frac{c}{2}e^2 \\ &= \frac{\pi \cdot b\, ln(1+\gamma_{ij2})}{c} \cdot h_2(\tau_{ij2}, e_{ij2}) \end{aligned} \tag{B.2}$$

Applying the implicit function theorem to the first-order condition, we have

$$\begin{aligned} \frac{\partial e_{ij2}}{\partial \tau} &= \frac{\pi\,(b/c)\, ln(1+\gamma) \cdot \boldsymbol{h_{12}}}{1 - \pi\,(b/c)\, ln(1+\gamma) \cdot \boldsymbol{h_{22}}} \\ \frac{\partial e_{ij2}}{\partial \gamma} &= \frac{\pi\,(b/c) \cdot \boldsymbol{h_2}}{1 - \pi\,(b/c)\, ln(1+\gamma) \cdot \boldsymbol{h_{22}}} \cdot \frac{1}{1+\gamma}, \qquad \frac{\partial e_{ij2}}{\partial \underline{w}} = 0 \end{aligned} \tag{B.3}$$

Under the assumption $h_{22} < 0$, the denominators are positive. With $h_2 > 0$, the worker's effort is increasing in the bonus γ; it is increasing in firm investment τ if and only if worker effort and firm investment are complementary ($h_{12} > 0$).

If not on the market, the worker stays at her $t = 1$ employer, $j(i,2) = j(i,1)$. Otherwise, she observes the contracts posted by **all** firms and solves:

$$\begin{aligned} &j(i,2) = argmax_j \; \{u_{ij2}(\pi) + \epsilon_{ij2}\} \\ &\text{where } u_{ij2}(\pi) = b\, ln(\underline{w}_{ij2}) + \pi\, h(\tau_{ij2}, e_{ij2}) \cdot b\, ln(1+\gamma_{ij2}) - \frac{c}{2}e_{ij2}^2, \quad e_{ij2} \text{ solves (B.2)}. \end{aligned}$$

Given the EV-I distributed preferences (5.2), the probability that a worker with belief π chooses firm j under contract $(\underline{w}, \gamma, \tau)$ is the logit share $s_{j2}(\underline{w}, \gamma, \tau) = \frac{exp(u_{ij2}(\pi))}{\sum_{j'} exp(u_{ij'2}(\pi))}$.

Firm's Problem. Given the public belief π at the beginning of $t = 2$, firms post contracts that maximize their expected profit at $t = 2$, separately for incumbent ($\delta = 1$)

and outside ($\delta = 0$) workers (repeating equation 5.7):

$$v_{j2}^{(\delta)}(\pi) = max_{(\underline{w},\gamma,\tau)}\, s_{j2}^{(\delta)}(\underline{w},\gamma,\tau)\cdot\Pi_{j2}(\underline{w},\gamma,\tau) \tag{B.4}$$

$$\text{where } \Pi_{j2}(\underline{w},\gamma,\tau) = \phi_j\cdot(1+\theta\cdot\pi\cdot h(\tau,e)) - \frac{\zeta}{2}\tau^2 - \underline{w}(1+\gamma\cdot\pi h(\tau,e))$$

$$e = e_{ij2} \text{ solves } (B.2).$$

Π_{j2} is the expected profit generated by the employee. The expected labor supply from the worker is:

$$s_{j2}^{(\delta)}(\underline{w},\gamma,\tau) = \begin{cases} 1-\lambda\cdot(1-s_{j2}) & \delta = 1 \\ \lambda\cdot s_{j2} & \delta = 0 \end{cases} \tag{B.5}$$

$$\text{elasticity } \xi_{j2}^{(\delta)} = \frac{\partial\, ln\, s_{j2}^{(\delta)}}{\partial ln(\underline{w})} = \frac{\underline{w}}{s_{j2}^{(\delta)}}\cdot\lambda s_{j2}(1-s_{j2})\frac{b}{\underline{w}} = \frac{\lambda s_{j2}}{s_{j2}^{(\delta)}}\cdot b(1-s_{j2})$$

In both cases, for $x \in \{\underline{w},\gamma,\tau\}$,

$$\frac{\partial\, ln\, s_{j2}^{(\delta)}}{\partial x} = \frac{\lambda s_{j2}(1-s_{j2})}{s_{j2}^{(\delta)}}\cdot\frac{\partial u_{ij2}}{\partial x} = \frac{\xi_{j2}^{(\delta)}}{b}\cdot\frac{\partial u_{ij2}}{\partial x}. \tag{B.6}$$

The first-order condition with respect to base wage yields:

$$\frac{\partial}{\partial \underline{w}} = \frac{\partial\, s_{j2}^{(\delta)}}{\partial \underline{w}}\cdot\Pi_{j2} - s_{j2}^{(\delta)}\cdot(1+\gamma\cdot\pi h(\tau,e)) = 0$$

$$\rightarrow \xi_{j2}^{(\delta)}\cdot\Pi_{j2} = \underline{w}\cdot(1+\gamma\pi h(\tau,e)) \tag{B.7}$$

in which $\partial s_{j2}^{(\delta)}/\partial\underline{w} = \xi_{j2}^{(\delta)}\, s_{j2}^{(\delta)}/\underline{w}$ by (B.6), and effort does not depend on $\underline{w}$.

The first-order condition with respect to bonus is:

$$\frac{\partial}{\partial\gamma} = \frac{\partial\, s_{j2}^{(\delta)}}{\partial\gamma}\cdot\Pi_{j2} + s_{j2}^{(\delta)}\cdot\left(-\underline{w}\pi h + \pi(\phi_j\theta - \gamma\underline{w})\cdot h_2\frac{\partial e}{\partial\gamma}\right) = 0$$

$$\rightarrow\ -\frac{\gamma(1-\pi h)}{1+\gamma}h\,\underline{w} + (\phi_j\theta - \gamma\underline{w})\cdot h_2\frac{\partial e}{\partial\gamma} = 0 \tag{B.8}$$

in which $\partial s_{j2}^{(\delta)}/\partial\gamma$ follows from (B.6) with $\partial u_{ij2}/\partial\gamma = \pi h\, b/(1+\gamma)$, $\xi_{j2}^{(\delta)}\Pi_{j2}$ is substituted from (B.7), and $\frac{\partial e}{\partial\gamma} > 0$ is given in (B.3).

The first-order condition with respect to investment is:

$$\frac{\partial}{\partial \tau} = \frac{\partial s_{j2}^{(\delta)}}{\partial \tau} \cdot \Pi_{j2} + s_{j2}^{(\delta)} \cdot \left(\pi(\phi_j \theta - \gamma \underline{w}) \cdot \frac{dh(\tau, e)}{d\tau} - \zeta \tau \right) = 0$$

$$\rightarrow \pi \, h_1 \, ln(1+\gamma) \cdot \underline{w}(1 + \gamma \pi h) + \pi(\phi_j \theta - \gamma \underline{w}) \cdot \left(h_1 + h_2 \frac{\partial e}{\partial \tau} \right) - \zeta \tau = 0 \tag{B.9}$$

in which $\partial s_{j2}^{(\delta)} / \partial \tau$ follows from (B.6) with $\partial u_{ij2} / \partial \tau = \pi h_1 \, b \, ln(1+\gamma)$, $\xi_{j2}^{(\delta)} \Pi_{j2}$ is substituted from (B.7), and $\frac{\partial e}{\partial \tau}$ is given in (B.3).

Optimization at $t = 1$

Worker's Problem. All workers are on the market at $t = 1$. A worker who chooses firm j at $t = 1$ takes into account her option value at $t = 2$, defined in (5.6) as $\Omega_j(\pi_{i2}) = (1 - \lambda) \cdot u_{ij2}(\pi_{i2}) + \lambda \, E[max_{j'} \{u_{ij'2}(\pi_{i2}) + \epsilon_{ij'2}\}]$, where π_{i2} is the posterior belief at the beginning of $t = 2$. In the first term, u_{ij2} is evaluated at the contract that firm j posts for incumbent workers ($\delta = 1$); in the second term, each $u_{ij'2}$ is evaluated at the contract that firm j' posts for a worker with belief π_{i2} and $\delta = 1[j' = j]$.

Given prior π, the expected option value is $E[\Omega_j(\pi_{i2}) | \pi, \tau, e] = \pi h(\tau, e) \cdot \Omega_j(1) + (1 - \pi h(\tau, e)) \cdot \Omega_j(\pi_{i2}(0))$, where $\pi_{i2}(0)$ is the posterior belief when $p_{i1} = 0$ in (B.1). The change in option value when the worker has a patent application, relative to when she does not, is $\triangle_p \Omega_j := \Omega_j(1) - \Omega_j(\pi_{i2}(0))$.

$$j(i,1) = argmax_j \{u_{ij1}(\pi) + \epsilon_{ij1}\} \tag{B.10}$$

$$\text{where } u_{ij1}(\pi) = b \, ln(\underline{w}_{ij1}) + \pi \, h(\tau_{ij1}, e_{ij1}) \cdot b \, ln(1 + \gamma_{ij1}) - \frac{c}{2} e_{ij1}^2 + \beta \, E[\Omega_j(\pi_{i2}) | \pi, \tau_{ij1}, e_{ij1}]$$

in which the effort solves

$$\begin{aligned} e_{ij1} =& argmax_e \, \pi \, h(\tau_{ij1}, e) \cdot b \, ln(1 + \gamma_{ij1}) - \frac{c}{2} e^2 + \beta \, E[\Omega_j(\pi_{i2}) | \pi, \tau_{ij1}, e] \\ =& \pi \cdot h_2(\tau_{ij1}, e_{ij1}) \cdot \frac{\left(b \, ln(1 + \gamma_{ij1}) \; + \beta \triangle_p \Omega_j \right)}{c} \end{aligned} \tag{B.11}$$

The first-order condition uses $\partial E[\Omega_j(\pi_{i2}) | \pi, \tau, e] / \partial e = \pi h_2 \triangle_p \Omega_j$, which holds because the

worker takes $\pi_{i2}(0)$, and hence $\triangle_p\Omega_j$, as given.[22]

Applying the implicit function theorem to the first-order condition, we have

$$\frac{\partial e_{ij1}}{\partial \tau} = \frac{\pi\,(b\;ln(1+\gamma)+\beta\triangle_p\Omega_j)/c\;\cdot \boldsymbol{h_{12}}}{1-\pi\,(b\;ln(1+\gamma)+\beta\triangle_p\Omega_j)/c\;\cdot \boldsymbol{h_{22}}} \qquad (B.12)$$

$$\frac{\partial e_{ij1}}{\partial \gamma} = \frac{\pi\,(b/c)\;\cdot \boldsymbol{h_2}}{1-\pi\,(b\;ln(1+\gamma)+\beta\triangle_p\Omega_j)/c\;\cdot \boldsymbol{h_{22}}}\;\cdot\frac{1}{1+\gamma}, \qquad \frac{\partial e_{ij1}}{\partial \underline{w}} = 0$$

The signs are the same as at $t = 2$. The option value enters like the bonus: a higher $\triangle_p\Omega_j$ raises effort and strengthens the response of effort to firm investment τ when $h_{12} > 0$.

Given the EV-I distributed preferences (5.2), the probability that a worker with prior π chooses firm j under contract $(\underline{w}, \gamma, \tau)$ is the logit share $s_{j1}(\underline{w}, \gamma, \tau) = \frac{exp(u_{ij1}(\pi))}{\sum_{j'} exp(u_{ij'1}(\pi))}$, with elasticity $\xi_{j1} = \partial\, ln\, s_{j1}/\partial\, ln(\underline{w}) = b(1 - s_{j1})$. For $x \in \{\underline{w}, \gamma, \tau\}$,

$$\frac{\partial\, ln\, s_{j1}}{\partial x} = (1 - s_{j1}) \cdot \frac{\partial u_{ij1}}{\partial x} = \frac{\xi_{j1}}{b} \cdot \frac{\partial u_{ij1}}{\partial x}, \qquad (B.13)$$

in which, by the Envelope Theorem, $\partial u_{ij1}/\partial \underline{w} = b/\underline{w}$, $\partial u_{ij1}/\partial \gamma = \pi h\, b/(1 + \gamma)$, and $\partial u_{ij1}/\partial \tau = \pi h_1 \left(b\; ln(1 + \gamma) + \beta \triangle_p \Omega_j\right)$.

Firm's Problem. Given the common prior π about a worker, firms post contracts that maximize the expected profit at $t = 1$ plus the discounted continuation value from the worker as an incumbent at $t = 2$ (repeating equation 5.8):

$$max_{(\underline{w},\gamma,\tau)}\; s_{j1}(\underline{w}, \gamma, \tau) \cdot \Pi_{j1}(\underline{w}, \gamma, \tau) \qquad (B.14)$$

$$\text{where } \Pi_{j1}(\underline{w}, \gamma, \tau) = \phi_j \cdot (1 + \theta \cdot \pi \cdot h(\tau, e)) - \frac{\zeta}{2}\tau^2 - \underline{w}(1 + \gamma \cdot \pi h(\tau, e)) + \beta\, EV2$$

$$EV2 \coloneqq E[v_{j2}^{(1)}(\pi_{i2})|\pi, \tau, e] = v_{j2}^{(1)}(\pi_{i2}(0)) + \pi\, h(\tau, e) \cdot \triangle_p v_{j2}^{(1)}$$

$$e = e_{ij1} \text{ solves } (B.11)$$

Π_{j1} is the expected profit generated by the employee over both periods, and $\triangle_p v_{j2}^{(1)} \coloneqq v_{j2}^{(1)}(1) - v_{j2}^{(1)}(\pi_{i2}(0))$ is the change in the firm's continuation value when the worker patents during $t = 1$. As at $t = 2$, the firm takes $\pi_{i2}(0)$, and hence $\triangle_p v_{j2}^{(1)}$, as given, so $dEV2/dx = \pi\, \triangle_p v_{j2}^{(1)} \cdot dh/dx$ for $x \in \{\gamma, \tau\}$, in which dh/dx includes the response of effort.

[22] At $t = 2$, the market observes a worker's $t = 1$ employer $j(i, 1)$ and her patent outcome p_{i1}, but not her effort or the contract she received. The posterior belief $\pi_{i2}(0)$ is therefore formed from the equilibrium effort and contract at $j(i, 1)$, and a deviation by a single worker or firm at $t = 1$ does not change it. Workers and firms take $\pi_{i2}(0)$, $\triangle_p\Omega_j$, and $\triangle_p v_{j2}^{(1)}$ as given.

The optimal contract satisfies the first-order conditions (B.15)–(B.17). With respect to the base wage:

$$\frac{\partial}{\partial \underline{w}} = \frac{\partial s_{j1}}{\partial \underline{w}} \cdot \Pi_{j1} - s_{j1} \cdot (1 + \gamma \cdot \pi h(\tau, e)) = 0$$
$$\rightarrow \xi_{j1} \cdot \Pi_{j1} = \underline{w} \cdot (1 + \gamma \pi h(\tau, e)) \tag{B.15}$$

in which $\partial s_{j1}/\partial \underline{w} = \xi_{j1}\, s_{j1}/\underline{w}$ by (B.13), and neither effort nor $EV2$ depends on the base wage $\underline{w}$ at $t = 1$.

With respect to the bonus:

$$\frac{\partial}{\partial \gamma} = \frac{\partial s_{j1}}{\partial \gamma} \cdot \Pi_{j1} + s_{j1} \cdot \left(-\underline{w} \cdot \pi h + \pi(\phi_j \theta - \gamma \underline{w} + \beta \triangle_p v_{j2}^{(1)}) \cdot h_2 \frac{\partial e}{\partial \gamma} \right) = 0$$
$$\rightarrow -\frac{\gamma(1 - \pi h)}{1 + \gamma} \cdot \underline{w} h + (\phi_j \theta - \gamma \underline{w} + \beta \triangle_p v_{j2}^{(1)}) \cdot h_2 \cdot \frac{\partial e}{\partial \gamma} = 0 \tag{B.16}$$

The condition differs from (B.8) at $t = 2$ only in the firm's net return to a patent, which now includes the change in the continuation value, $\beta \triangle_p v_{j2}^{(1)}$.

With respect to investment:

$$\frac{\partial}{\partial \tau} = \frac{\partial s_{j1}}{\partial \tau} \cdot \Pi_{j1} + s_{j1} \cdot \left(\pi(\phi_j \theta - \gamma \underline{w} + \beta \triangle_p v_{j2}^{(1)}) \cdot \frac{dh(\tau, e)}{d\tau} - \zeta \tau \right) = 0 \tag{B.17}$$
$$\rightarrow \pi \left(ln(1 + \gamma) + \frac{\beta}{b} \triangle_p \Omega_j \right) \cdot h_1 \cdot \underline{w}(1 + \gamma \pi h) + \pi \underbrace{(\phi_j \theta - \gamma \underline{w} + \beta \triangle_p v_{j2}^{(1)})}_{\text{net return to a patent}} \cdot \left(h_1 + h_2 \frac{\partial e}{\partial \tau} \right) - \zeta \tau = 0$$

Relative to (B.9), investment at $t = 1$ has two additional effects: it raises labor supply through the worker's option value, and it raises the firm's continuation value through $\triangle_p v_{j2}^{(1)}$.

B3. Proof of Propositions

The proofs use Assumptions (A1)–(A3) stated in Section 5. Under (A1), the labor supply elasticities are $\xi_{j1} = \xi_{j2}^{(0)} = b$, and $\xi_{j2}^{(1)} = \frac{\lambda s_{j2}}{1-\lambda} \cdot b$ if $\lambda < 1$ and $\xi_{j2}^{(1)} = b$ if $\lambda = 1$. Derivatives with respect to ϕ_j are taken along the optimal contract unless stated otherwise.

Proof of Proposition 1: Positive Relationship between Firm Productivity and Base Wages

Proof:

From the first-order conditions (B.7) and (B.15), the base wage in each period is marked down by the inverse labor supply elasticity from the expected value of the worker to the firm:

$$\begin{aligned} \underline{w}_{j2}^{(\delta)} &= \frac{\xi_{j2}^{(\delta)}}{1+\xi_{j2}^{(\delta)}} \cdot \frac{E_\alpha[Y_j(\alpha,\tau,e)|\pi]}{1+\gamma\pi h(\tau,e)} \\ \underline{w}_{j1} &= \frac{\xi_{j1}}{1+\xi_{j1}} \cdot \frac{E_\alpha[Y_j(\alpha,\tau,e)|\pi]+\beta EV2}{1+\gamma\pi h(\tau,e)} \end{aligned} \tag{B.18}$$

Fix γ and τ, and at $t=1$ also $\triangle_p\Omega_j$. Worker effort then does not depend on $\underline{w}$ or ϕ_j by (B.3) and (B.12), so the denominator $1+\gamma\pi h(\tau,e)$ is constant in ϕ_j. Under (A1), $\xi_{j1}=\xi_{j2}^{(0)}=b$ do not depend on ϕ_j, and, if $\lambda<1$, $\xi_{j2}^{(1)}\to 0$ as $s_{j2}\to 0$, so the base wage for incumbent employees at $t=2$ converges to 0 for every ϕ_j and is non-decreasing in ϕ_j. If $\lambda=1$, $\xi_{j2}^{(1)}=b$, and incumbent employees are treated as new hires below. For new hires at $t=2$, $\partial E_\alpha[Y_j(\alpha,\tau,e)|\pi]/\partial\phi_j=1+\theta\pi h>0$ by (5.1), so $\underline{w}_{j2}^{(0)}$ is strictly increasing in ϕ_j. At $t=1$, in addition, $\partial EV2/\partial\phi_j=(1-\pi h)\,dv_{j2}^{(1)}(\pi_{i2}(0))/d\phi_j+\pi h\,dv_{j2}^{(1)}(1)/d\phi_j>0$, because by the envelope theorem applied to (B.4), $dv_{j2}^{(1)}(\pi)/d\phi_j=s_{j2}^{(1)}\cdot(1+\theta\pi h)>0$. Hence $\underline{w}_{j1}$ is strictly increasing in ϕ_j. ■

Proof of Proposition 2: Heterogeneity in Firm Investment in Innovation

Proof:

The optimal investment at $t=2$ solves the first-order condition (B.9). Fix $\underline{w}$ and γ, so that effort $e=e(\tau,\gamma)$ varies only with τ. The left-hand side of (B.9) is decreasing in τ by the second-order condition, and its derivative with respect to ϕ_j at a fixed τ is $\pi\theta\left(h_1+h_2\frac{\partial e}{\partial\tau}\right)$. Under (A3), $h_{12}>0$ and $h_{22}<0$ give $\frac{\partial e}{\partial\tau}>0$ (see B.3), which with $h_1>0$ implies $h_1+h_2\frac{\partial e}{\partial\tau}>0$. By the Implicit Function Theorem, τ_{j2} is increasing in ϕ_j.

At $t=1$, the investment τ_{j1} solves (B.17). Fix $\underline{w}$, γ, and $\triangle_p\Omega_j$, so that effort $e=e(\tau,\gamma,\triangle_p\Omega_j)$ varies only with τ and the labor-supply term in (B.17) does not vary with ϕ_j

at a fixed τ. The derivative of the left-hand side of (B.17) with respect to ϕ_j at a fixed τ is then $\pi\left(h_1 + h_2\frac{\partial e}{\partial \tau}\right) \cdot \frac{d}{d\phi_j}\left(\phi_j\theta + \beta\triangle_p v_{j2}^{(1)}\right)$. It suffices to show that the change in a firm's continuation value, $\triangle_p v_{j2}^{(1)}$, is non-decreasing in ϕ_j. By the envelope theorem applied to (B.4), $dv_{j2}^{(1)}(\pi)/d\phi_j = s_{j2}^{(1)} \cdot (1 + \theta\pi h)$, so

$$\frac{d\triangle_p v_{j2}^{(1)}}{d\phi_j} = s_{j2}^{(1)}(1) \cdot \big(1 + \theta h(1)\big) - s_{j2}^{(1)}(\pi_{i2}(0)) \cdot \big(1 + \theta\pi_{i2}(0)h(\pi_{i2}(0))\big),$$

where $h(\pi)$ denotes the patent probability under the contract posted for a worker with belief π. Under (A1), $s_{j2}^{(1)} = 1 - \lambda(1 - s_{j2}) \approx 1 - \lambda$ for both beliefs, and the expression reduces to $(1 - \lambda)\theta\,[h(1) - \pi_{i2}(0)h(\pi_{i2}(0))]$, which is positive under the assumption that the firm's expected patent output from a known inventor exceeds that from a worker who has not patented. The first-period investment is therefore also increasing in ϕ_j. ■

Proof of Proposition 3: Wage Bonus for Invention

The proof uses the following regularity conditions.

(R1) The optimal bonus is interior, $\gamma_{jt} > 0$, and satisfies the second-order condition.

(R2) Along the optimal contract, investment τ_{jt} is increasing in ϕ_j, and the base wage rises at least in proportion to the firm's gross return to a patent: $\phi_j\theta/\underline{w}$ at $t = 2$ and $(\phi_j\theta + \beta\triangle_p v_{j2}^{(1)})/\underline{w}$ at $t = 1$ are non-increasing in ϕ_j. Under (A1), the second part holds when the bonus is small relative to the share of the investment cost in expected output.

(R3) At $t = 1$, the change in the worker's option value from patenting, $\triangle_p\Omega_j$, is held fixed, as in Proposition 2.

Weak complementarity in Proposition 3 refers to condition (B.20) below.

Proof:

At $t = 2$, the bonus solves the first-order condition (B.8). Define $K = \frac{\partial e_{ij2}}{\partial \gamma} \cdot (1 + \gamma) = \frac{\pi\,(b/c)\,h_2}{1 - \pi\,(b/c)\,ln(1+\gamma)\,h_{22}}$, according to (B.3). The first-order condition (B.8) can be rewritten as

$$\gamma\,\underline{w}\,(h_2K + (1 - \pi h)h) = \phi_j\theta \cdot h_2K \quad \rightarrow \quad ln\,\gamma = ln\frac{\phi_j\theta}{\underline{w}} - ln\,(1 + Q)\,, \qquad Q := \frac{(1 - \pi h)h}{h_2K}.$$

in which $ln\frac{\phi_j\theta}{\underline{w}}$ represents the gross return to patent relative to base wage, and $ln\,(1+Q)$ represents the net cost of the bonus per patent elicited. The numerator of Q is proportional to the expected bonus payment minus the base wage that workers are willing to forgo in return for the bonus. The denominator is the increase in the probability of patenting that the bonus elicits through effort.

Because h_{22} is constant by (A3), $h_2(\tau,e) = h_2(\tau,0) + h_{22}\,e$, so $h_{12} = \partial h_2(\tau,0)/\partial\tau$ does not depend on effort. Substituting the worker's first-order condition (B.2), $e = \frac{\pi b\,ln(1+\gamma)}{c}\,h_2$, gives

$$h_2(\tau,e)\cdot(1-\pi\,(b/c)\,ln(1+\gamma)\,h_{22}) = h_2(\tau,0).$$

Hence $K = \frac{\pi b\,(h_2)^2}{c\,h_2(\tau,0)}$ and $Q = \frac{c\,(1-\pi h)\,h\,h_2(\tau,0)}{\pi b\,(h_2)^3}$.

Along the optimal contract, h and h_2 vary with ϕ_j through investment and through effort:

$$\frac{dh(\tau,e)}{d\phi_j} = \frac{dh}{d\tau}\cdot\frac{d\tau_{j2}}{d\phi_j} + h_2\frac{\partial e}{\partial\gamma}\cdot\frac{d\gamma_{j2}}{d\phi_j}, \qquad \text{where } \frac{dh}{d\tau} = h_1 + h_2\frac{\partial e}{\partial\tau},$$

$$\begin{aligned}\frac{d\,ln\,h_2(\tau,e)}{d\phi_j} &= \frac{d\,ln\,h_2(\tau,0)}{d\phi_j} - \frac{d\,ln\,(1-\pi\,(b/c)\,ln(1+\gamma)\,h_{22})}{d\phi_j}\\ &= \frac{h_{12}}{h_2(\tau,0)}\cdot\frac{d\tau_{j2}}{d\phi_j} + \frac{\pi b\,h_{22}\,h_2}{c\,(1+\gamma)\,h_2(\tau,0)}\cdot\frac{d\gamma_{j2}}{d\phi_j}.\end{aligned}$$

Differentiating $ln\,\gamma$ with respect to ϕ_j:

$$\frac{d\,ln\,\gamma_{j2}}{d\phi_j} = \frac{d\,ln(\phi_j\theta/\underline{w})}{d\phi_j} - \frac{Q}{1+Q}\cdot\frac{d\,ln\,Q}{d\phi_j}$$

Collecting the terms in $d\gamma_{j2}/d\phi_j$, we have

$$\begin{aligned}&\underbrace{\left[\frac{1}{\gamma} + \frac{Q}{1+Q}\left(\frac{1-2\pi h}{(1-\pi h)h}\,h_2\frac{\partial e}{\partial\gamma} - \frac{3\pi b\,h_{22}\,h_2}{c\,(1+\gamma)\,h_2(\tau,0)}\right)\right]}_{>0}\cdot\frac{d\gamma_{j2}}{d\phi_j}\\ &= \frac{d\,ln(\phi_j\theta/\underline{w})}{d\phi_j} - \frac{Q}{1+Q}\cdot\frac{d\tau_{j2}}{d\phi_j}\left[\frac{1-2\pi h}{(1-\pi h)h}\cdot\frac{dh}{d\tau} - \frac{2h_{12}}{h_2(\tau,0)}\right].\end{aligned} \tag{B.19}$$

The coefficient on the left-hand side is positive: $\gamma > 0$ by (R1), $\frac{\partial e}{\partial\gamma} > 0$ by (B.3), $h_{22} < 0$, and $1-2\pi h > 0$ because $\pi h < 0.5$. On the right-hand side, the first term is non-positive and $\frac{d\tau_{j2}}{d\phi_j} > 0$ by (R2). The bracket is positive when complementarity is weak in the following

sense:

$$\frac{1-2\pi h}{(1-\pi h)\,h}\cdot\left(h_1+h_2\frac{\partial e}{\partial \tau}\right)>\frac{2h_{12}}{h_2(\tau,0)}. \tag{B.20}$$

Hence the right-hand side of (B.19) is negative, and γ_{j2} is decreasing in ϕ_j.

The left-hand side of (B.20) captures the effect of investment on the net cost of the bonus: a higher investment raises the probability of patenting h, and the net cost, which is proportional to $(1-\pi h)h$, increases with h if and only if $\pi h < 0.5$. The right-hand side captures complementarity: a higher investment raises the marginal product of effort and hence the effort that the bonus elicits. Condition (B.20) holds when h_{12} (complementarity) is small relative to h_1 (the direct effect of investment).

At $t=1$, the bonus solves the first-order condition (B.16), with $\triangle_p\Omega_j$ fixed by (R3). Define K and Q as at $t=2$, with $\frac{\partial e_{ij1}}{\partial \gamma}$ from (B.12) in place of $\frac{\partial e_{ij2}}{\partial \gamma}$. The first-order condition (B.16) then gives $ln\,\gamma = ln\frac{\phi_j\theta+\beta\triangle_p v_{j2}^{(1)}}{\underline{w}} - ln(1+Q)$. By (B.11), $e=\frac{\pi\left(b\,ln(1+\gamma)+\beta\triangle_p\Omega_j\right)}{c}\,h_2$, so $h_2\left(1-\pi\left(b\,ln(1+\gamma)+\beta\triangle_p\Omega_j\right)h_{22}/c\right)=h_2(\tau,0)$, and K and Q take the same form as at $t=2$. Because $\triangle_p\Omega_j$ is fixed, the derivative of $ln\,h_2$ with respect to ϕ_j is also the same as at $t=2$. Hence

$$\begin{aligned}&\left[\frac{1}{\gamma}+\frac{Q}{1+Q}\left(\frac{1-2\pi h}{(1-\pi h)h}\,h_2\frac{\partial e}{\partial \gamma}-\frac{3\pi b\,h_{22}\,h_2}{c\,(1+\gamma)\,h_2(\tau,0)}\right)\right]\cdot\frac{d\gamma_{j1}}{d\phi_j}\\ &\quad=\frac{d}{d\phi_j}ln\frac{\phi_j\theta+\beta\triangle_p v_{j2}^{(1)}}{\underline{w}}-\frac{Q}{1+Q}\cdot\frac{d\tau_{j1}}{d\phi_j}\left[\frac{1-2\pi h}{(1-\pi h)h}\cdot\frac{dh}{d\tau}-\frac{2h_{12}}{h_2(\tau,0)}\right],\end{aligned} \tag{B.21}$$

in which $\frac{\partial e}{\partial \gamma}$ and $\frac{dh}{d\tau}=h_1+h_2\frac{\partial e}{\partial \tau}$ are evaluated with (B.12). The coefficient on the left-hand side is positive under (R1) and $\pi h<0.5$. Under (R2) and condition (B.20) at the $t=1$ contract, the right-hand side is negative. Hence γ_{j1} is decreasing in ϕ_j. ■

Numerical Illustration of Proposition 3

We illustrate Proposition 3 by solving the firm's problem at $t=2$ for an outside worker ($\delta=0$) on a grid of firm productivity, $\phi_j\in\{2,2.25,\ldots,6\}$. The patent production function is quadratic in effort, as in (A3):

$$h(\tau,e)=0.05+0.6\,\frac{\tau}{1+\tau}+(0.14+h_{12}\,\tau)\,e-2.5\,e^2,$$

so that $h_{22} = -5$ and h_{12} is a constant. We hold the direct effect of investment and h_{22} fixed, and consider three values of the complementarity: $h_{12} = 0.15$, $h_{12} = 0.03$, and $h_{12} = -0.05$. The last case violates (A3) and is shown for comparison. The other parameters are $b = c = \zeta = 1$, $\pi = 0.3$, and $\theta = 2$. Given τ and γ, effort solves the worker's first-order condition (B.2), which has the closed form $e = \frac{\pi b \, ln(1+\gamma) \, h_2(\tau,0)}{c - \pi b \, ln(1+\gamma) \, h_{22}}$ because h_{22} is constant. Under (A1), the labor supply is $s_{j2}^{(0)} \propto exp(u_{ij2}(\pi))$, so maximizing (B.4) is equivalent to maximizing $u_{ij2}(\pi) + ln \, \Pi_{j2}$ over $(\underline{w}, \gamma, \tau)$. We solve this problem numerically for each ϕ_j, and the solution satisfies the first-order conditions (B.7), (B.8), and (B.9).

Figure 3 plots the optimal bonus γ_{j2} against ϕ_j. In all three cases, investment increases with ϕ_j, the probability of patenting πh lies between 0.065 and 0.091, and the bonus lies between 0.035 and 0.136, the range of our estimates in Table 4. When the complementarity is weak ($h_{12} = 0.03$) or investment and effort are substitutes ($h_{12} = -0.05$), the bonus decreases in firm productivity, as in Proposition 3. When the complementarity is strong ($h_{12} = 0.15$), the bonus increases in firm productivity. A stronger complementarity also raises the level of the bonus at all firms, because a higher h_{12} raises the marginal product of effort and hence the probability of patenting that the bonus elicits.